Working Paper

Wine in Your Knapsack?

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Abstract

We pose three Knapsack Problems (KPs) to select the rank-maximizing subset of wines subject to budget and quantity constraints. The first problem seeks the subset of wines, from a single cultivar (zinfandel) that maximizes the sum of rank subject to a budget constraint. We modify this problem by adding an equality constraint on the number of bottles that must be chosen. The third problem seeks to maximize the sum of ranks from three different cultivars (cabernet sauvignon, pinot noir, and zinfandel) subject to a budget constraint and then a budget and minimum bottle constraints for each cultivar. The sum of rank maximization problems may have multiple solutions. We also pose two expenditure minimization problems, subject to achieving the maximum sum of ranks. We also explore how a KP might be formulated when wine is viewed as an investment.

1 Introduction and Overview

The prototype, 0-1, Knapsack Problem has a hiker contemplating which of $j = 1, 2, \ldots n$ items to be included in a knapsack of finite capacity, c. Each item has a utility, $u_j > 0$, and a weight (or size), w_j. If the jth item is selected for inclusion in the knapsack, $X_j = 1$, if not, $X_j = 0$. The 0-1 Knapsack Problem seeks to

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$$\max_{X_j \in \{0,1\}} u = \sum_{j=1}^{n} u_j \times X_j$$

$$Subject \ to \sum_{j=1}^{n} w_j \times X_j \leq c$$

$$X_j \in \{0,1\}, j = 1,2,\ldots,n$$

There are many variations on the prototype, 0-1, KP including the "Bounded Knapsack," "Subset-Sum," "Change-Making," "0-1 Multiple Knapsack," "Generalized Assignment," and "Bin-Packing" problems [1].

From the perspective of computer science, all of these problems are NP-hard. NP-hard stands for "non-deterministic polynomial-time hard." NP-hard is a class of problems for which no polynomial-time solution algorithm has been found. Algorithms that run in polynomial-time are viewed as efficient. This means that NP-hard problems cannot, at present, be solved efficiently. This does *not* mean that a particular "instance" of a 0-1 Knapsack Problem cannot be optimally solved. It simply means that there is no polynomial-time algorithm and as n grows, the problem scales exponentially, taking longer to find a "provably optimal solution." Advances in computing and mixed integer programming (MIP) algorithms now allow one to provably solve KPs with n=1,000,000 on a personal computer [2].

While the history of KPs in operations research and computer science is interesting, this paper is concerned with the application and solution KPs for the selection of wines. The wine enthusiast, with a personal computer and an MIP solver, can quickly determine the best selection of wines for a special event (wedding, anniversary, birthday) or for building a wine cellar. The purpose of this paper is to illustrate and perhaps inspire a more analytical approach to wine selection.

In the next section we present the five KPs of interest to our wine enthusiast; call him Ed. We start with the prototype, 0-1, KP problem and then add a quantity constraint as might arise if Ed is planning a party for a given number of guests. This problem has Ed selecting a given number of bottles from a set of n different wines based on their ranking, price per bottle, and his wine budget for the party. The third problem has Ed selecting a minimum number of bottles from three different cultivars, again subject to a budget constraint. While the items to be chosen in our examples are individual bottles of wine, we could allow X_j to represent a case of a

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[1]See [Martello and Toth.(1990)] or [Kellerer et al.(2004)Kellerer, Pferschy, and Pisinger].

[2]See [Srisuwannapa and Charnsethikul(2007)].

particular wine (with a price including the case discount). Such a formulation may be more appropriate if one is solving a KP to stock a restaurant or wine store with a variety of wines representing different cultivars, wine growing regions, and retail price.

In the third and fourth sections, we illustrate the five problems by selecting the best zinfandels from 64 California, North Coast, zinfandels bottled in 2003. The budget constraint is $1,000. We then modify this problem by adding a quantity constraint where Ed wants precisely 30 bottles for his party and is again willing to spend $1,000. In the third example (instance), Ed wants to select at least 10 bottles from three cultivars, cabernet sauvignon, pinot noir, and zinfandel produced in California, North Coast, all bottled in 2003, to maximize their sum of ranks. We also look at the uniqueness of the solutions found. The fifth section poses a wine investment problem where the optimal portfolio of wines might be determined by solving a KP. The sixth section concludes. In an appendix we provide the AMPL code used to solve the numerical KPs presented in this article.

2   The Wine Selection Problems

Our wine enthusiast, Ed, has just completed a consulting gig and after setting aside a portion of his fee for taxes he has $B = \$1,000$ that he wants to use to add to his collection of California zinfandels. According to wine experts, 2003 was a good year for the California, North Coast, appellation. Ed has identified $n = 64$ zinfandels from this appellation and year which he does not have in his cellar. From the internet he has quickly downloaded the ranking of each wine, denoted r_j, and the per bottle price, p_j, as quoted by a dealer he has used in the past. Instead of purchasing multiple bottles of the same wine, Ed decides to purchase, at most, one bottle of each brand.

These bottles range in their ranking from the low 80s to the high 90s. They range in price from $18 to $140 per bottle. With a budget of $B = \$1,000$, Ed cannot purchase all 64 bottles. The KP in this instance seeks to

$$
\max_{X_j \in \{0,1\}} r = \sum_{j=1}^{n} r_j \times X_j
$$
$$
\text{Subject to} \sum_{j=1}^{n} p_j \times X_j \leq B \tag{1}
$$
$$
X_j \in \{0,1\}, j = 1, 2, \ldots, n = 64
$$

After solving this prototype KP problem, Ed decides to add the quantity constraint, $\sum_{j=1}^{n} X_j = 30$. Ed now wishes to

$$\max_{X_j \in \{0,1\}} r = \sum_{j=1}^{n} r_j \times X_j$$

$$\text{Subject to} \sum_{j=1}^{n} p_j \times X_j \leq B \qquad (2)$$

$$\sum_{j=1}^{n} X_j = 30$$

$$X_j \in \{0,1\}, j = 1, 2, \ldots, n = 64$$

It turns out that the solution to Problem (2) may not be unique. This can arise if there are wines in the choice set with the same rank and price. There may be several 30-bottle combinations that produce the maximum sum of rank, $r^* = \sum_{j=1}^{n} r_j \times X_j^*$.

From the multiple solutions to Problem (2) Ed then seeks a solution which will

$$\min_{X_j \in \{0,1\}} E = \sum_{j=1}^{n} p_j \times X_j$$

$$\text{Subject to} \sum_{j=1}^{n} r_j \times X_j \geq r^* \qquad (3)$$

$$\sum_{j=1}^{n} X_j = 30$$

$$X_j \in \{0,1\}, j = 1, 2, \ldots, n = 64$$

As we will see, there may be multiple solutions to (3) as well. Ed would like to know the number of solutions to both Problems (2) and (3).

Finally, Ed is interested in the best wines from $m = 3$ cultivars: cabernet sauvignon ($k = 1$), pinot noir ($k = 2$), and zinfandels ($k = 3$). Let $X_{j,k}$ denote the jth wine in the kth cultivar class and let n_k be the number of wines (bottles) in the kth class. Suppose Ed wants at least $C_k > 0$ bottles from the kth cultivar so that $\sum_{j=1}^{n_k} X_{j,k} \geq C_k$. In our example we will assume that $C_k = 10$ for $k = 1, 2, 3$. This results in a KP seeking to

$$\max_{X_j \in \{0,1\}} r = \sum_{k=1}^{m} \sum_{j=1}^{n_k} r_{j,k} \times X_{j,k}$$

$$\text{Subject to} \sum_{k=1}^{m} \sum_{j=1}^{n_k} p_{j,k} \times X_{j,k} \leq B$$

$$\sum_{j=1}^{n_k} X_{j,k} \geq C_k = 10 \tag{4}$$

$$X_{j,k} \in \{0,1\}, j = 1, 2, \ldots, n_k, \ k = 1, 2, 3 = m$$

If there are multiple solutions to Problem (4), Ed may need to solve for the least cost solutions which achieve r^*. Specifically he would want to

$$\min_{X_j \in \{0,1\}} E = \sum_{k=1}^{m} \sum_{j=1}^{n_k} p_{j,k} \times X_{j,k}$$

$$\text{Subject to} \sum_{k=1}^{m} \sum_{j=1}^{n_k} r_{j,k} \times X_{j,k} \geq r^*$$

$$\sum_{j=1}^{n_k} X_{j,k} \geq C_k = 10, \tag{5}$$

$$X_{j,k} \in \{0,1\}, j = 1, 2, \ldots, n_k, \ k = 1, 2, 3 = m$$

2.1   Finding all solutions to the problem

A relevant question of interest is to find all indifferent solutions to the aforementioned problems (2)-(5). To answer this question, a new set of dynamic constraints was added, and the problem is solved recursively. These new constraints will identify solutions yielding the same r^* or E^* but which differ in the wines selected. The reason for this is to be able to accommodate the quantity constraints per class. The variables defined are:

[Table 1 here]

The pseudo-algorithm to identify multiple solutions looks as follows (See appendix for AMPL code).

1: $i \leftarrow 1$;
2: Do first run without constraints on past solutions.
3: $i \leftarrow i + 1$;
4: **repeat**
5:    Create dynamic set to store solutions (*cont*). This is implemented as a matrix with increasing number of columns in each iteration (dimensions $P.c \times cont$, where $P.c$ is the number of solutions for all classes, *cont* is the number of solutions found -initially 1).
6:    Store first solution in the first column of the matrix. The implementation places each integer solution per class as a stacked column vector.
7:    **for all** consecutive runs, $i > 1$ **do**
8:      add a constraint such that the squared difference between each element of the new solution and each element of all past solutions is greater than or equal to one. This is equivalent to state that there must be at least one position of the integer solution vector (1's and 0's) that differs.
9:      Store the new solution found in the set of past solutions (*cont*)
10:      $i \leftarrow i + 1$;
11:    **end for**
12: **until** $i = N_{iter}$, run-time specified by user.

Note that the set *cont* is increasing with each iteration, corresponding to new solutions found.

The overall condition evaluated for each candidate solution is spelled as follows:

$$\forall \, i \in cont : \sum_{j \in P} \sum_{k \in c} (X[j,k] - solv[j,i,k])^2 >= 1;$$

3 Cabs, Pinots and Zins

In Table 2 we list the 64 zinfandels comprising the choice set in (1). The ranking and price for each zinfandel may be found at the web site for this article[3] and in [Parker(2003)]. Recall that the budget constraint to (1) was $B = \$1,000$. There are five solutions to this problem, each yielding $r^* = 2,896$. Each solution selects 32 bottles. The solutions differ from one another by one bottle of equivalent price and rank. The bottles selected in each of the five solutions are highlighted and can be identified by moving down a column under a particular solution number. The five solutions share 31 wines in common. Solution 1 includes the zinfandel with Identification Number 22 (Hartford Vineyard Zinfandel) that is not included in Solutions

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[3]The website address will be provided upon request during review process.

2-5. Solution 2 substitutes Identification Number 21 (Hartford's Fanucchi Wood Road Vineyard) for Identification Number 22. Identification Number 19 (Hartford's Dina's Vineyard) is unique to Solution 3, Identification Number 7 (Robert Biale's Monte Rosso) is unique to Solution 4, and Identification Number 3 (Robert Biale's Aldo' Vineyard) is unique to Solution 5. All five solutions completely exhaust the budget and they are the only solutions to the expenditure minimization problem.

[Table 2 here]

Now consider (2) for the same choice set of 64 zinfandels but with the addition of a quantity constraint, $\sum_{j=1}^{n} X_j = 30$. We know from mathematical programming that the value for r^* in (1) must equal or exceed the value of r^* in (2).

There are 50 solutions to (2) all yielding $r^* = 2,751$. We then solve (3) where we identify 15 solutions with $r^* = 2,751$ and where the minimized expenditure is $E^* = \sum_{j=1}^{64} p_j \times X_j^* = 987$, thus saving \$13 from our budget, $B = \$1,000$. The 15 solutions to (3) are shown in Table 3 where the wines contained in a particular solution are again highlighted as one moves down a solution column. The 50 solutions to (2), containing the 15 solutions to (3), can be found at the web site for this article.

[Table 3 here]

The cabs-pinots-zins (CPZ) selection problem is the most complex and the most interesting. The choice set now includes 164 cabernets, 100 pinots, and the same 64 zinfandels contained in Tables 2 and 3. Table 4 provides descriptive statistics for the overall dataset. Overall cabernets exhibit the highest average price,\$106.41, which is more than double the average price for zinfandels (\$44.11) and about 50 percent above the average price for pinots (\$73.23). These wines have a wide range of price and rank variation, making a systematic approach to wine selection under a budget the more important.

[Table 4 here]

Recall that in (4) and (5) there was a budget constraint, $\sum_{k=1}^{m} \sum_{j=1}^{n_k} p_{j,k} \times X_{j,k} \leq B$, and cultivar-quantity constraints, $\sum_{j=1}^{n_k} X_{j,k} \geq C_k = 10$. When we solve (4) we get four solutions all yielding $r^* = 3,262.5$ and each precisely exhausts the budget of $B = \$1,000$. After running the expenditure minimization problem, we determine that there are /no solutions that could achieve $r^* = 3,262.5$ for $\sum_{k=1}^{m} \sum_{j=1}^{n_k} p_{j,k} \times X_{j,k} < B$. The four solutions to (4) and (5) are shown in Table 5.

[Table 5 here]

A careful study of Table 5 provides some interesting insights to the CPZ selection problem. First, only 11 cabs, 10 pinots, and 18 zins are involved in the four solutions in Table 5. Nine cabs with the Identification Numbers 2, 20, 38, 87, 88, 121, 126, 129, and 163 appear in all four solutions. Cab Identification Number 125 appears in Solutions 1 and 2 while cab Identification Number 140 substitutes for 125 in Solutions 3 and 4. Of the 11 cabs involved in the four solutions, six were ranked at 92.5 and five were ranked at 84.5.

The same 10 pinots, with Identification Numbers 15, 17, 28, 41, 48, 49, 51, 52, 86, and 89, appear in all four solutions. Seven out of these 10 pinots have a rank of 84.5 while three were ranked at 92.5.

A total of 18 zins appeared in the four solutions. Common to all four solutions were zins with Identification Numbers 2, 9, 13, 15, 16, 17, 18, 20, 25, 26, 35, 42, 44, 46, 49, and 50. In Solutions 2 and 3 zin Identification Number 8 appears, but is replaced by zin Identification Number 11 in Solutions 1 and 4. This is again an example where multiple solutions can arise in a quantity-constrained wine selection problem. From the point of view of the CPLEX Solver, "wine IN8" is identical to "wine IN11."

Of the 18 zins involved in the four solutions, 8 were ranked at 84.5 while 10 were ranked at 92.5. The fact that the quantity constraint was not binding for the zinfandel selection suggests that it was a cultivar yielding a higher rank per dollar compared to the cabs and pinots. To have more money to spend on the zinfandels, the CPLEX Solver opted to purchase only the minimum number of cabs and pinots and to purchase relatively lower ranked, inexpensive wines. Table 6 provides descriptive statistics for (5).

[Table (6) here]

If we remove the quantity constraints from the CPZ problem, the maximized sum of rank is again $r^* = 3262.5$ but the composition of the selected cabs, pinots, and zins changes significantly. There are 50 solutions to the KP maximizing the sum of rank but only one solution to the expenditure minimization problem subject to $r^* = 3,262.5$ (Table 7). In that solution, 8 cabs are chosen, 9 pinots are chosen, and 20 zins are chosen for an expenditure of $995. With no quantity constraints the CPLEX Solver finds higher rank per dollar wines in the zins and limits the number of cabs and pinots to those that can compete on a rank per dollar basis.

[Table (7) here]

Table 7 displays the solution to (5) after removing all quantity constraints while Table 8 provides descriptive statistics for that solution.

4 Uniqueness of solutions for binding budget constraints

Consider the two knapsack problems above defined in (2) and (3). In the first problem we wish to maximize the sum of the rankings for selected wines, subject to a budget and quantity constraint. The quantity constraint assumes only one bottle of each wine is chosen. As noted, there may be multiple solutions to (2). In Problem (3), the set of solutions found in (2), is searched for the solution or solutions that minimize the expenditure while achieving (or exceeding) r^*.

We claim that the solutions to the overall consumer problem (maximize utility and minimize expenditure) must lie in the intersection of the solution sets for problems (2) and (3). Furthermore, these solutions are the only solutions to the overall problem (i.e. to choose the bundle that minimizes the expenditure of the consumer, subject to a lower bound on the sum of rankings) in cases when the budget constraint is binding.

<u>Claim 1</u> Denote by $SP2$ the set of solutions to problem (2) such that the budget is completely exhausted, and $SP3$ the set of solutions to problem (3). Consider the subset $sp3 = \{\boldsymbol{X} \in SP3 : \sum_{i=1}^{N} p_i \times X_i = B\}$. $\forall \boldsymbol{X} \in sp3, \boldsymbol{X}$ is a solution to the consumer portfolio selection problem.

<u>Proof of Claim 1</u>
The first part is to prove that all solutions of (2) are solutions of (3).[4]

For the sake of contradiction (FSOC) suppose not $\rightarrow SP2 \neq \{\}, \wedge, \exists \boldsymbol{X}^j : \boldsymbol{X}^j \in SP2$ but $\boldsymbol{X}^j \notin SP3$. Let $\boldsymbol{X}^k \in SP3$. Therefore $\sum_{i=1}^{N} p_i \times X_i^k < \sum_{i=1}^{N} p_i \times X_i^j, \wedge, u(\boldsymbol{X}^k) > u(\boldsymbol{X}^j) = r^j = r^*$, the value function for (2) from $\boldsymbol{X}^j$. But then, $\boldsymbol{X}^j$ cannot be a solution to (2) Therefore a contradiction is reached $\square$.

Note that due to the discrete nature of the choices in this problem, the utility function is not continuous.

The above results are derived from construction and definition of problem (3), $SP3 = \{\boldsymbol{X} : \boldsymbol{X} \text{ minimizes } C, \wedge, \sum_{i=1}^{N} r_i \times X_i \geq r^*\}$.

The second part is to prove that the solutions in $sp3$ are the unique solutions to the overall consumer problem -and therefore are solutions to both (2) and (3).

FSOC Suppose not $\rightarrow \exists \boldsymbol{X}^m \in sp3 : \boldsymbol{X}^m \notin SP2$. Let $\boldsymbol{X}^j \in SP2$. Therefore $r^* = r^j = u(\boldsymbol{X}^j) > u(\boldsymbol{X}^m), \wedge, \sum_{i=1}^{N} p_i \times X_i^j < \sum_{i=1}^{N} p_i \times X_i^m = B$. But then,

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[4]This is equivalent to say that If SP2 is a not-empty set $(SP(2) \neq \{\})$, SP(3) is a not empty set.

$\boldsymbol{X}^m$ cannot be a solution to (3), and hence $\boldsymbol{X}^m \notin sp3(sp3 \subseteq SP3)$. Therefore a contradiction is reached $\square$.

The extension to different classes is straightforward and therefore this proof also applies to the problems defined by (4) and (5).

5   Wine as an Investment?

In a recent article, [Masset and Weisskopf(2010)] assess the potential role that wine might play in an investment portfolio. Over the period 1996 - 2009 they find investing in wine would have been beneficial for private investors. Wine returns were not highly correlated with other financial assets, thus providing an opportunity for portfolio diversification, and the indices for wines from Bordeaux, the Rhône Valley, Italy, and the US performed better during the economic downturns in 2001 - 2003 and 2007 - 2009 than the major equity markets.

Determining the size and composition of a wine portfolio relative to other assets is complex and depends on an investor's risk preferences. While fine wines can easily appreciate in value for fifty years or more, determining the optimal time to sell and the expected present value at time of sale is difficult. We could simply assume that the wine investor has a subjective distribution of expected present values for each wine or wine index, and use our previous notation, r_j, to represent the expected net present value from the optimal liquidation of wine asset $X_j = 1$. To develop the problem further, we draw upon the literature concerned with the optimal forest-rotation when price evolves stochastically. For example, see [Clarke and Reed(1989)].

Let $P_j = P_j(t)$ denote the price per unit quality for a bottle of wine j at future instant t. Let $p_j = ln[P_j]$ and we will assume that $dp_j = \mu_j dt + \sigma_j dz_j$ where $\mu_j \geq 0$ is an expected drift rate, $\sigma_j > 0$ is a standard deviation rate, and dz_j is the increment of a standard Wiener process. Using Itô's Lemma, it is well known that $dP_j = (\mu_j + \frac{\sigma_j^2}{2})P_j dt + \sigma_j P_j dz_j$ and that $P_j = P_j(t)$ is log normally distributed with expected value of $E[P(t)] = P(0)e^{(\mu_j + \frac{\sigma_j^2}{2})t}$.

The quality function for wine j at age t is assumed to take the form $Q_j(t) = e^{a_j - b_j/t}$ where $a_j > 0$, $b_j > 0$, $t > 0$. Quality is an S-Shaped curve as a function of time, asymptotically approaching e^{a_j}. If wine j is acquired at a futures auction at $t = 0$, it will then grow in quality according to $Q_j(t) = e^{a_j - b_j/t}$.

Revenue from a sale at t is given by the product $P_j(t)Q_j(t)$. Let $\delta > 0$ denote the instantaneous discount rate. Then, the discounted expected revenue from a sale at future instant t is given by

$$R_j(t) = P_j(0)e^{a_j - b_j/t - (\delta - \mu_j - \sigma_j^2/2)t} \tag{6}$$

where it is assumed that $\delta - \mu_j - \sigma_j^2/2 > 0$. In this case it is optimal to sell the wine at the value of t which maximizes $R_j(t)$. The first-order condition implies

$$t_j^* = \sqrt{b_j/(\delta - \mu_j - \sigma_j^2/2)} \tag{7}$$

Then, $R_j(t_j^*) > 0$ is the discounted expected revenue at the optimal time of sale and might replace r_j in (1) while $P_j(0)$ would replace p_j.

In order for a wine or wine index to even be considered within a portfolio it must be the case that $P_j(0)e^{a_j - b_j/t_j^* - (\delta - \mu_j - \sigma_j^2/2)t_j^*} - P_j(0) > 0$ or

$$e^{a_j - b_j/t_j^* - (\delta - \mu_j - \sigma_j^2/2)t_j^*} > 1 \tag{8}$$

Inequality (8) requires that the discounted expected revenue at the optimal date of sale exceed the initial cost of acquisition.

The above model is highly stylized and does not consider the correlation between wines or the correlation between wines and other assets. Storage costs and commissions paid at time of sale are also not considered. Incorporating these factors would add realism but is beyond the scope of the present paper.

6   Conclusions

Most wine enthusiasts simply enjoy sharing a good bottle of wine and restrict their inventory to a modest wine cellar. The enjoyment and collection of wine does not preclude taking an analytical approach, and in fact the wine selection problems in this article were meant to illustrate how one might get the most out of a modest budget for wine. The wine selection problems in this article are examples of Knapsack Problems, an interesting class of problems in the fields of operations research and computer science. The ability to find optimal solutions to this NP-Hard class of problems has been improved by the development of efficient mixed integer programs (MIPs) so that the analytical wine enthusiast may now pose and quickly solve various wine selection problems on a computer. In our most complex problem, with 164 cabs, 100 pinots, and 64 zins from the California North Coast appellation in 2003, the CPLEX Solver found zins to provide higher rank per dollar than cabs or pinots. With a budget of $1,000 and no quantity constraints, it chose 20 zins compared to 8 cabs and 9 pinots.

The wine enthusiast with strong personal tastes (such as those held by Paul Giamatti's character in the movie 'Sideways') may trump the selection made by a computer, but given the rapid increase in the number of wines and vintages now available at stores, and especially auction sites, the selection of wine from a single cultivar and region (as in our first selection problem) might still benefit from an analytical approach.

There are wine enthusiasts who also view wine as a good investment. It is difficult to accurately represent the discounted expected return from wine in an overall investment portfolio and to then formulate a KP that might be used to select individual wines. Given the recent research by [Masset and Weisskopf(2010)] this problem will likely receive greater attention in the future.

Applications of the KP can be extended to help buyers at restaurants and wine stores make more rational wine procurement decisions. Such buyers often face limited shelf or storage space and need to select a wine assortment (cultivars and regions) to meet customer expectations. Given the proliferation of wine brands, the resulting KPs are likely to be more complex than the applications presented in this study. This "buyer problem" calls for further research.

The fact that wine selection problems are likely to generate multiple solutions (because of similarly ranked and priced wines) is perhaps a good thing. It allows Ed, our not so hypothetical wine enthusiast, to make the ultimate choice based on experience, personal taste, or simply the desire to try a wine not yet tasted.

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Appendix A AMPL Programs

The AMPL code for solving problems (4) and (5) is listed below. For an introduction to AMPL, see [Fourer et al.(2002)Fourer, Gay, and Kernighan]. The first section corresponds to the problem formulation for a single selection. The second part corresponds to the cycle for finding all the solutions to the problem, as explained in subsection 2.1, 'Finding all solutions to the problem'.

A.1 Model for (4)

```
set P;
set c;                          # classes of wine
param r {j in P, k in c} default 0;     # ratings
param p {j in P, k in c} default 0;     # prices
param cont;
param solv{j in P, 1..cont, k in c};    # vector with solutions to problem
param b;                        # budget
param t;
param num{k in c};              # number of wines
var X {j in P, k in c} binary;
maximize Total_Util: sum {j in P, k in c} r[j, k] * X[j, k];
subject to budget: sum {j in P, k in c} p[j, k] * X[j, k] <= b;
subject to numberc {k in c}: sum {j in P} X[j, k] >= num[k];
subject to diff {i in 1..cont}: sum {j in P, k in c} (j*X[j, k]/j - j*solv[j, i, k]/j)^2>=1;
```

A.2 Model for (5)

```
set P;
set c;                          # classes of wine
param r {j in P, k in c} default 0;     # ratings
param p {j in P, k in c} default 0;     # prices
param cont;
param solv{j in P, 1..cont, k in c};    # vector with solutions to problem
param u;                        # utility
param t;
param num{k in c};              # number of wines
var X {j in P, k in c} binary;
minimize Expend: sum {j in P, k in c} p[j, k] * X[j, k];
subject to util: sum {j in P, k in c} r[j, k] * X[j, k] >= u;
subject to numberc {k in c}: sum {j in P} X[j, k] >= num[k];
subject to diff {i in 1..cont}: sum {j in P, k in c} (j*X[j, k]/j - j*solv[j, i, k]/j)^2>=1;
```

A.3 Iterative process for (4)

```
# Iterative cycle for wine problem, 1st stage

reset;                          # reset all information

cd /Applications/ampl;          # change directory file
option solver cplex;
model wineicn.mod;              # set model to run
```

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```
data wineicn.dat;               # set data to run, full data, no class limits
param ncont :=100;              # tries fixed
param loadstart;
param loadend;
param loadtime;
let loadstart:= time();

let cont:= 1;                   # define variables to be filled
for {i in 1..cont}{
    for {j in P}{
        for {k in c}{
            let solv[j, i, k]:= 0;
        }
    }
}
option show_stats 1;
solve;
for {j in P}{
        for {k in c}{
            let solv[j, cont, k]:= X[j, k];
        }
}

for {i in 2..ncont}{
    option show_stats 1;
    solve;
    let cont:= i;
    for {j in P}{
        for {k in c}{
            let solv[j, i, k]:= X[j, k];
        }
    }
    let loadend:= time();
    let loadtime:=loadend-loadstart;
    if loadtime>ncont then{
        break;
    }
}

display solv >> wine1n_out;
```

A.4   Iterative process for (5)

```
# Iterative cycle for wine problem, 2nd stage

reset;                          # reset all information

cd /Applications/ampl;         # change directory file
option solver cplex;
model wine2icn.mod;             # set model to run
data wine2icn.dat;             # set data to run, full data, no class limits
param ncont :=100;             # tries fixed
param loadstart;
param loadend;
param loadtime;
let loadstart:= time();
```

```
let cont:= 1;                          # define variables to be filled
for {i in 1..cont}{
    for {j in P}{
        for {k in c}{
            let solv[j, i, k]:= 0;
        }
    }
}
option show_stats 1;
solve;
for {j in P}{
        for {k in c}{
            let solv[j, cont, k]:= X[j, k];
        }
}

for {i in 2..ncont}{
    option show_stats 1;
    solve;
    let cont:= i;
    for {j in P}{
        for {k in c}{
            let solv[j, i, k]:= X[j, k];
        }
    }
    let loadend:= time();
    let loadtime:=loadend-loadstart;
    if loadtime>ncont then{
        break;
    }
}

display solv >> wine2n_out;
```

7   Tables

Table 1: Sets defined for iterative determination of solutions

Variable		Description
i	:	Solution Index
P	:	Set of choices for each class
c	:	Set of classes (cultivars)
$cont$	:	Set of Past solutions found
$solv$	:	Set of new solutions found

Table 2: Solution to (1)

Zinfandel (i)	i	Solution 1	2	3	4	5
Acorn Zinfandel Heritage Alegria Vineyard	1	0	0	0	0	0
Robert Biale Vineyards Zinfandel	2	**1**	**1**	**1**	**1**	**1**
Robert Biale Vineyards Zinfandel Aldo's Vineyard	3	0	0	0	0	**1**
Robert Biale Vineyards Zinfandel Black Chicken	4	**1**	**1**	**1**	**1**	**1**
Robert Biale Vineyards Zinfandel Grande	5	0	0	0	0	0
Robert Biale Vineyards Zinfandel Old Crane Ranch	6	0	0	0	0	0
Robert Biale Vineyards Zinfandel Monte Rosso	7	0	0	0	**1**	0
Carlisle Winery Zinfandel Tom Feeney Ranch	8	**1**	**1**	**1**	**1**	**1**
Carlisle Winery Zinfandel Riebli Ranch	9	**1**	**1**	**1**	**1**	**1**
Carlisle Winery Zinfandel Rossi Ranch	10	**1**	**1**	**1**	**1**	**1**
Carlisle Winery Zinfandel Fava Ranch	11	**1**	**1**	**1**	**1**	**1**
Carlisle Winery Zinfandel Dry Creek Valley	12	**1**	**1**	**1**	**1**	**1**
Carlisle Winery Zinfandel	13	**1**	**1**	**1**	**1**	**1**
Carlisle Winery Zinfandel Carlisle Vineyard	14	**1**	**1**	**1**	**1**	**1**
Edmeades Estate Zinfandel Ciapusci Vineyard	15	**1**	**1**	**1**	**1**	**1**
Edmeades Estate Zinfandel	16	**1**	**1**	**1**	**1**	**1**
Edmeades Estate Zinfandel Piffero Vineyard	17	**1**	**1**	**1**	**1**	**1**
Franus Zinfandel Brandlin Ranch	18	**1**	**1**	**1**	**1**	**1**
Hartford Hartford Zinfandel Dina's Vineyard	19	0	0	**1**	0	0
Hartford Zinfandel	20	**1**	**1**	**1**	**1**	**1**
Hartford Zinfandel Fanucchi Wood Road Vineyard	21	0	**1**	0	0	0
Hartford Zinfandel Hartford Vineyard	22	**1**	0	0	0	0
Hartford Zinfandel Highwire Vineyard	23	**1**	**1**	**1**	**1**	**1**
J C Cellars Zinfandel Arrowhead Vineyard	24	**1**	**1**	**1**	**1**	**1**
J C Cellars Zinfandel Iron Hill Vineyard	25	**1**	**1**	**1**	**1**	**1**
Robert Keenan Zinfandel	26	**1**	**1**	**1**	**1**	**1**
Mara Zinfandel Reserve Dolinsek Ranch	27	0	0	0	0	0
Mara Zinfandel Reserve Luvisi Ranch	28	**1**	**1**	**1**	**1**	**1**
Martinelli Zinfandel Giuseppe and Luisa	29	**1**	**1**	**1**	**1**	**1**
Martinelli Zinfandel Jackass Hill Vineyard	30	0	0	0	0	0
Martinelli Zinfandel Jackass Vineyard	31	0	0	0	0	0
Louis Martini Zinfandel Monte Rosso Gnarly Vine	32	0	0	0	0	0
Murphy-Goode Winery Snake Eyes Elaine Maria	33	**1**	**1**	**1**	**1**	**1**
Neyers Zinfandel Tofanelli Vineyard	34	0	0	0	0	0
Niebaum-Coppola Edizione Pennino Zinfandel Estate	35	**1**	**1**	**1**	**1**	**1**
Rancho Zabaco Zinfandel Monte Rosso	36	0	0	0	0	0
Rancho Zabaco Zinfandel Monte Rosso Toreador	37	0	0	0	0	0
Rosenblum Cellars Hendry Vineyard Reserve	38	0	0	0	0	0
Rosenblum Cellars Zinfandel Lyons Vineyard Reserve	39	0	0	0	0	0
Rosenblum Cellars Zinfandel Maggie's Reserve	40	0	0	0	0	0
Saddleback Cellar Venge Family Res. Scouts Honor	41	**1**	**1**	**1**	**1**	**1**
Saddleback Cellar Zinfandel Old Vines	42	**1**	**1**	**1**	**1**	**1**
Sausal Zinfandel Old Vine Estate	43	0	0	0	0	0
Sbragia Family Vineyards Zinfandel	44	**1**	**1**	**1**	**1**	**1**
Scherrer Zinfandel Scherrer Vineyard	45	0	0	0	0	0
Storybook Mountain Zinfandel Atlas Peak	46	**1**	**1**	**1**	**1**	**1**
Storybook Mountain Zinfandel Eastern Exposure	47	**1**	**1**	**1**	**1**	**1**
Storybook Mountain Zinfandel Estate Reserve	48	0	0	0	0	0
Storybook Mountain Zinfandel Mayacamas Range	49	**1**	**1**	**1**	**1**	**1**
Summers Ranch Zinfandel Villa Andriana Vineyard	50	**1**	**1**	**1**	**1**	**1**
Trentadue Zinfandel la Storia	51	0	0	0	0	0
Turley Wine Cellars Zinfandel Dragon	52	0	0	0	0	0
Turley Wine Cellars Zinfandel Estate	53	0	0	0	0	0
Turley Wine Cellars Zinfandel Hayne Vineyard	54	0	0	0	0	0
Turley Wine Cellars Mead Ranch Atlas Peak	55	0	0	0	0	0
Turley Wine Cellars Moore (Earthquake) Vineyard	56	0	0	0	0	0
Turley Wine Cellars Zinfandel Rattlesnake Ridge	57	0	0	0	0	0
Turley Wine Cellars Zinfandel Tofanelli Vineyard	58	0	0	0	0	0
Turley Wine Grist Vineyard Bradford Mountain	59	0	0	0	0	0
Turley Wine Cellars Zinfandel Rancho Burro	60	0	0	0	0	0
Vieux-Os Wines Zinfandel Tofanelli Vineyard	61	0	0	0	0	0
Vieux-Os Wines Zinfandel Hell Hole Vineyard	62	**1**	**1**	**1**	**1**	**1**
Vieux-Os Wines Zinfandel Ira Carter Vineyard	63	**1**	**1**	**1**	**1**	**1**
Williams Selyem Zinfandel Bacigalupi Vineyard	64	0	0	0	0	0

Table 3: Solution to (3)

Zinfandel (i)	i	Solution														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Acorn Zinfandel Heritage Alegria Vineyard	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Robert Biale Vineyards Zinfandel	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Robert Biale Vineyards Zinfandel Aldo's Vineyard	3	1	1	0	1	0	1	1	1	0	1	1	1	1	1	1
Robert Biale Vineyards Zinfandel Black Chicken	4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Robert Biale Vineyards Zinfandel Grande	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Robert Biale Vineyards Zinfandel Old Crane Ranch	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Robert Biale Vineyards Zinfandel Monte Rosso	7	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1
Carlisle Winery Zinfandel Tom Feeney Ranch	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Carlisle Winery Zinfandel Riebli Ranch	9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Carlisle Winery Zinfandel Rossi Ranch	10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Carlisle Winery Zinfandel Fava Ranch	11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Carlisle Winery Zinfandel Dry Creek Valley	12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Carlisle Winery Zinfandel	13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Carlisle Winery Zinfandel Carlisle Vineyard	14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Edmeades Estate Zinfandel Ciapusci Vineyard	15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Edmeades Estate Zinfandel	16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Edmeades Estate Zinfandel Piffero Vineyard	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Franus Zinfandel Brandlin Ranch	18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Hartford Hartford Zinfandel Dina's Vineyard	19	1	0	1	0	1	1	1	1	1	1	1	1	1	1	0
Hartford Zinfandel	20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Hartford Zinfandel Fanucchi Wood Road Vineyard	21	0	1	1	1	1	1	1	1	1	1	1	0	1	0	1
Hartford Zinfandel Hartford Vineyard	22	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1
Hartford Zinfandel Highwire Vineyard	23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
J C Cellars Zinfandel Arrowhead Vineyard	24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
J C Cellars Zinfandel Iron Hill Vineyard	25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Robert Keenan Zinfandel	26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mara Zinfandel Reserve Dolinsek Ranch	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mara Zinfandel Reserve Luvisi Ranch	28	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Martinelli Zinfandel Giuseppe and Luisa	29	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Martinelli Zinfandel Jackass Hill Vineyard	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Martinelli Zinfandel Jackass Vineyard	31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Louis Martini Zinfandel Monte Rosso Gnarly Vine	32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Murphy-Goode Winery Snake Eyes Elaine Maria	33	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Neyers Zinfandel Tofanelli Vineyard	34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Niebaum-Coppola Edizione Pennino Zinfandel Estate	35	0	0	0	1	1	1	0	1	1	0	1	1	1	1	1
Rancho Zabaco Zinfandel Monte Rosso	36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rancho Zabaco Zinfandel Monte Rosso Toreador	37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rosenblum Cellars Hendry Vineyard Reserve	38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rosenblum Cellars Zinfandel Lyons Vineyard Reserve	39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rosenblum Cellars Zinfandel Maggie's Reserve	40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Saddleback Cellar Venge Family Res. Scouts Honor	41	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Saddleback Cellar Zinfandel Old Vines	42	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Sausal Zinfandel Old Vine Estate	43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sbragia Family Vineyards Zinfandel	44	1	1	1	1	1	1	1	0	0	1	1	1	0	0	0
Scherrer Zinfandel Scherrer Vineyard	45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Storybook Mountain Zinfandel Atlas Peak	46	1	1	1	0	0	0	1	1	1	1	0	0	1	1	1
Storybook Mountain Zinfandel Eastern Exposure	47	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Storybook Mountain Zinfandel Estate Reserve	48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Storybook Mountain Zinfandel Mayacamas Range	49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Summers Ranch Zinfandel Villa Andriana Vineyard	50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Trentadue Zinfandel la Storia	51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turley Wine Cellars Zinfandel Dragon	52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turley Wine Cellars Zinfandel Estate	53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turley Wine Cellars Zinfandel Hayne Vineyard	54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turley Wine Cellars Mead Ranch Atlas Peak	55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turley Wine Cellars Moore (Earthquake) Vineyard	56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turley Wine Cellars Zinfandel Rattlesnake Ridge	57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turley Wine Cellars Zinfandel Tofanelli Vineyard	58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turley Wine Grist Vineyard Bradford Mountain	59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Turley Wine Cellars Zinfandel Rancho Burro	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Vieux-Os Wines Zinfandel Tofanelli Vineyard	61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Vieux-Os Wines Zinfandel Hell Hole Vineyard	62	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Vieux-Os Wines Zinfandel Ira Carter Vineyard	63	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Williams Selyem Zinfandel Bacigalupi Vineyard	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 4: Descriptive statistics for the CPZ Dataset

		Type			
Rank		Cabernet	Pinot	Zinfandel	Grand Total
84.5	Average Price ($)	58.86	47.11	32.11	49.23
	Max Price ($)	220.00	95.00	55.00	220.00
	Min Price ($)	10.00	19.00	18.00	10.00
92.5	Average Price ($)	102.83	86.34	48.22	86.90
	Max Price ($)	484.50	370.00	140.00	484.50
	Min Price ($)	27.00	25.00	24.00	24.00
97.5	Average Price ($)	350.70	411.25	-	360.79
	Max Price ($)	830.00	490.00	-	830.00
	Min Price ($)	75.00	332.50	-	75.00
98	Average Price ($)	-	-	75.00	75.00
	Max Price ($)	-	-	75.00	75.00
	Min Price ($)	-	-	75.00	75.00
Average Price across all ranks ($)		106.41	73.23	44.11	84.14
Max Price across all ranks ($)		830.00	490.00	140.00	830.00
Min Price across all ranks ($)		10.00	19.00	18.00	10.00

Table 5: Solution to (4)-(5)

Type	Name	i	r	p	Solution 1	2	3	4
Cabernet	Flora Springs Cabernet Sauvignon	2	84.5	26	**1**	**1**	**1**	**1**
Cabernet	Beringer Cabernet Sauvignon	20	84.5	27	1	1	1	1
Cabernet	Chateau Saint Jean Cabernet Sauvignon	38	92.5	27	**1**	**1**	**1**	**1**
Cabernet	Louis Martini Cabernet Sauvignon	87	84.5	17	1	1	1	1
Cabernet	Louis Martini Cabernet Sauvignon	88	84.5	24	**1**	**1**	**1**	**1**
Cabernet	Robert Mondavi Cabernet Sauvignon Napa	121	92.5	27	1	1	1	1
Cabernet	Ruston Family Vineyards Cabernet Sauvignon Stagecoach Vineyard	125	92.5	36	0	0	**1**	**1**
Cabernet	Rutherford Ranch Cabernet Sauvignon	126	92.5	10	1	1	1	1
Cabernet	Sbragia Family Vineyards Cabernet Sauvignon Andolsen	129	92.5	35	**1**	**1**	**1**	**1**
Cabernet	Seavey Caravina	140	92.5	36	1	1	0	0
Cabernet	Zahtila Vineyards Cabernet Sauvignon	163	92.5	33	**1**	**1**	**1**	**1**
Pinot	Beringer Pinot Noir Stanley Ranch	15	84.5	30	**1**	**1**	**1**	**1**
Pinot	Chateau Saint Jean Pinot Noir	17	84.5	19	1	1	1	1
Pinot	Failla Pinot Noir	28	92.5	32	**1**	**1**	**1**	**1**
Pinot	Husch Pinot Noir	41	84.5	21	1	1	1	1
Pinot	La Crema Pinot Noir	48	84.5	29	**1**	**1**	**1**	**1**
Pinot	La Crema Pinot Noir Anderson Valley	49	84.5	29	1	1	1	1
Pinot	La Crema Pinot Noir Russian River	51	84.5	29	**1**	**1**	**1**	**1**
Pinot	Landmark Pinot Noir Grand Detour	52	92.5	25	1	1	1	1
Pinot	Walter Hansel Winery Pinot Noir Estate	86	84.5	29	**1**	**1**	**1**	**1**
Pinot	Walter Hansel Winery Pinot Noir The North Slope	89	92.5	30	1	1	1	1
Zinfandel	Robert Biale Vineyards Zinfandel	2	84.5	27	1	1	1	1
Zinfandel	Carlisle Winery Zinfandel Tom Feeney Ranch	8	92.5	33	**1**	0	**1**	0
Zinfandel	Carlisle Winery Zinfandel Riebli Ranch	9	92.5	30	1	1	1	1
Zinfandel	Carlisle Winery Zinfandel Fava Ranch	11	92.5	33	0	**1**	0	**1**
Zinfandel	Carlisle Winery Zinfandel	13	92.5	25	1	1	1	1
Zinfandel	Edmeades Estate Zinfandel Ciapusci Vineyard	15	92.5	28	**1**	**1**	**1**	**1**
Zinfandel	Edmeades Estate Zinfandel	16	84.5	18	1	1	1	1
Zinfandel	Edmeades Estate Zinfandel Piffero Vineyard	17	84.5	28	**1**	**1**	**1**	**1**
Zinfandel	Franus Zinfandel Brandlin Ranch	18	92.5	32	1	1	1	1
Zinfandel	Hartford Zinfandel	20	92.5	30	**1**	**1**	**1**	**1**
Zinfandel	J C Cellars Zinfandel Iron Hill Vineyard	25	92.5	30	1	1	1	1
Zinfandel	Robert Keenan Zinfandel	26	84.5	26	**1**	**1**	**1**	**1**
Zinfandel	Niebaum-Coppola Edizione Pennino Zinfandel Estate	35	84.5	25	1	1	1	1
Zinfandel	Saddleback Cellar Zinfandel Old Vines	42	92.5	32	**1**	**1**	**1**	**1**
Zinfandel	Sbragia Family Vineyards Zinfandel	44	84.5	25	1	1	1	1
Zinfandel	Storybook Mountain Zinfandel Atlas Peak	46	84.5	25	**1**	**1**	**1**	**1**
Zinfandel	Storybook Mountain Zinfandel Mayacamas Range	49	84.5	27	1	1	1	1
Zinfandel	Summers Ranch Zinfandel Villa Andriana Vineyard	50	92.5	24	**1**	**1**	**1**	**1**

Table 6: Statistics, Solution Found to (5)

Rank			Cabernet	Pinot	Zinfandel	Grand Total
			Type			
84.5		Number	5.00	7.00	8.00	20.00
		Min Price ($)	10.00	19.00	18.00	10.00
		Average Price ($)	20.80	26.57	25.13	24.55
		Max Price ($)	27.00	30.00	28.00	30.00
		Expenditure ($)	104.00	186.00	201.00	491.00
		Exp. Percentage	50.00	70.00	47.06	54.05
92.5		Number	5.00	3.00	9.00	17.00
		Min Price ($)	27.00	25.00	24.00	24.00
		Average Price ($)	31.60	29.00	29.33	29.94
		Max. Price ($)	36.00	32.00	33.00	36.00
		Expenditure ($)	158.00	87.00	264.00	509.00
		Exp. Percentage	50.00	30.00	52.94	45.95
Total Number			10.00	10.00	17.00	37.00
Total Min Price ($)			10.00	19.00	18.00	10.00
Total Average Price ($)			26.20	27.30	27.35	27.03
Total Max Price ($)			36.00	32.00	33.00	36.00
Total Expenditure ($)			262.00	273.00	465.00	1000.00

Table 7: Solution to (4)-(5) without quantity constraints

Type	Name	i	r	p
Cabernet	Flora Springs Cabernet Sauvignon	2	84.5	26
Cabernet	Beringer Cabernet Sauvignon	20	84.5	27
Cabernet	Chateau Saint Jean Cabernet Sauvignon	38	92.5	27
Cabernet	Louis Martini Cabernet Sauvignon	87	84.5	17
Cabernet	Louis Martini Cabernet Sauvignon	88	84.5	24
Cabernet	Robert Mondavi Cabernet Sauvignon Napa	121	92.5	27
Cabernet	Rutherford Ranch Cabernet Sauvignon	126	92.5	10
Cabernet	Zahtila Vineyards Cabernet Sauvignon	163	92.5	33
Pinot	Chateau Saint Jean Pinot Noir	17	84.5	19
Pinot	Failla Pinot Noir	28	92.5	32
Pinot	Husch Pinot Noir	41	84.5	21
Pinot	La Crema Pinot Noir	48	84.5	29
Pinot	La Crema Pinot Noir Anderson Valley	49	84.5	29
Pinot	La Crema Pinot Noir Russian River	51	84.5	29
Pinot	Landmark Pinot Noir Grand Detour	52	92.5	25
Pinot	Walter Hansel Winery Pinot Noir Estate	86	84.5	29
Pinot	Walter Hansel Winery Pinot Noir The North Slope	89	92.5	30
Zinfandel	Robert Biale Vineyards Zinfandel	2	84.5	27
Zinfandel	Robert Biale Vineyards Zinfandel Black Chicken	4	92.5	34
Zinfandel	Carlisle Winery Zinfandel Tom Feeney Ranch	8	92.5	33
Zinfandel	Carlisle Winery Zinfandel Riebli Ranch	9	92.5	30
Zinfandel	Carlisle Winery Zinfandel Fava Ranch	11	92.5	33
Zinfandel	Carlisle Winery Zinfandel	13	92.5	25
Zinfandel	Edmeades Estate Zinfandel Ciapusci Vineyard	15	92.5	28
Zinfandel	Edmeades Estate Zinfandel	16	84.5	18
Zinfandel	Edmeades Estate Zinfandel Piffero Vineyard	17	84.5	28
Zinfandel	Franus Zinfandel Brandlin Ranch	18	92.5	32
Zinfandel	Hartford Zinfandel	20	92.5	30
Zinfandel	J C Cellars Zinfandel Iron Hill Vineyard	25	92.5	30
Zinfandel	Robert Keenan Zinfandel	26	84.5	26
Zinfandel	Neyers Zinfandel Tofanelli Vineyard	34	84.5	29
Zinfandel	Niebaum-Coppola Edizione Pennino Zinfandel Estate	35	84.5	25
Zinfandel	Saddleback Cellar Zinfandel Old Vines	42	92.5	32
Zinfandel	Sbragia Family Vineyards Zinfandel	44	84.5	25
Zinfandel	Storybook Mountain Zinfandel Atlas Peak	46	84.5	25
Zinfandel	Storybook Mountain Zinfandel Mayacamas Range	49	84.5	27
Zinfandel	Summers Ranch Zinfandel Villa Andriana Vineyard	50	92.5	24

Table 8: Statistics, Solution Found to (5) without quantity constraints

r			type			
			Cabernet	Pinot	Zinfandel	Grand Total
84.5		Number	5.00	6.00	9.00	20.00
		Min Price ($)	10.00	19.00	18.00	10.00
		Average Price ($)	20.80	26.00	25.56	24.50
		Max Price ($)	27.00	29.00	29.00	29.00
		Expenditure ($)	104.00	156.00	230.00	490.00
		Exp. Percentage	62.50	66.67	45.00	54.05
92.5		Number	3.00	3.00	11.00	17.00
		Min Price ($)	27.00	25.00	24.00	24.00
		Average Price ($)	29.00	29.00	30.09	29.71
		Max. Price ($)	33.00	32.00	34.00	34.00
		Expenditure ($)	87.00	87.00	331.00	505.00
		Exp. Percentage	37.50	33.33	55.00	45.95
Total Number			8.00	9.00	20.00	37.00
Total Min Price ($)			10.00	19.00	18.00	10.00
Total Average Price ($)			23.88	27.00	28.05	26.89
Total Max Price ($)			33.00	32.00	34.00	34.00
Total Expenditure ($)			191.00	243.00	561.00	995.00

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