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## **On the Design of an Optimal Transfer Schedule with Time Inconsistent Preferences**

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# On the Design of an Optimal Transfer Schedule with Time Inconsistent Preferences

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## Abstract

This paper incorporates the phenomenon of time inconsistency into the problem of designing an optimal transfer schedule. It is shown that if program beneficiaries are time inconsistent and receive all of the resources in just one payment, then the equilibrium allocation is always inefficient. In the spirit of the second welfare theorem, we show that any efficient allocation can be obtained in equilibrium when the policymaker has full information. This assumption is relaxed by introducing uncertainty and asymmetric information into the model. The optimal solution reflects the dilemma that a policymaker has to face when playing the roles of commitment enforcer and insurance provider simultaneously.

**JEL Classification:** D910, H390, H550.

**Keywords:** *Time Inconsistency, Transfer Schedule, Hyperbolic Discounting, Self-Control Problems, Consumption, Uncertainty.*

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# 1 Introduction

Public transfers constitute a very important policy tool in developed and developing societies alike. From anti-poverty programs to unemployment insurance benefits, they play a very important role as a welfare enhancing mechanism.

Mainstream public economics analyzes the problem of designing an optimal transfer schedule based on the assumption that individuals have an abundance of psychological resources: unboundedly rational, forward looking, and internally consistent.<sup>1</sup> Particularly, it is assumed that individuals are unbounded in their self-control and optimally follow whatever plans they set out for themselves. In this paper, we investigate the optimal design of a transfer schedule when individuals have self-control problems.

Economic theories of intertemporal choice generally assume that individuals discount the future exponentially. In other words, the choices made between today and tomorrow should be no different from the choices made between the days 200 and 201 from now, all else equal. However, experimental evidence suggests that many individuals have preferences that reverse as the date of decision making nears. Research on animal and human behavior has led scientists to conclude that preferences are roughly hyperbolic in shape, implying a high discount rate in the immediate future, and relatively lower rate over periods that are further away (Ainslie 1992; Lowenstein and Thaler 1989). Moreover, there exists field evidence of present-biased preferences and time inconsistent behavior (DellaVigna and Malmendier 2003; Fang and Silverman 2004). Angeletos et al (2001) calibrate the hyperbolic and exponential models using US data on savings and consumption, finding that the former model better matches actual consumers' behavior. They noticed that, in contrast to the expo-

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<sup>1</sup>An example of such approach applied in a dynamic setting is the article on unemployment insurance written by Shavell and Weiss (1979). They characterize the time sequence of benefits that maximizes the expected utility of the unemployed. Blackorby and Donaldson (1988) study second best allocations in a static model where government lacks full information about consumer types.

ponential discounting model, hyperbolic households exhibit a high level of comovement between predictable changes in income and changes in consumption. This type of behavior has also been found in empirical studies that show how consumption is often very sensitive to an income transfer in the very short-run.<sup>2</sup> Similar results have been found in developing countries, particularly the development of commitment devices to face the time inconsistency problem (Rutheford 1999; Ashraf, Gons, Karlan, and Yin 2003; Ashraf, Gons, Karlan, and Yin 2006).

In this paper we present a very simple model that captures this phenomenon within the context of designing an optimal transfer schedule. We refer to this type of policy tool as a consumption maintenance program (CMP). The dynamic economic environment we study has two actors: a policymaker whose goal is to allocate an exogenous budget in order to maximize some welfare function, and an agent who takes consumption-savings decisions over time and is borrowing constrained. The policymaker is fully committed to his plan once it is established. In contrast, the beneficiary may be time-inconsistent and may not follow up his original consumption plan in the future.

Following a tradition in public economics, we begin the analysis with a first-best approach. We show that if program beneficiaries are time inconsistent and receive all the benefits in just one payment, then the equilibrium consumption allocation is *always* inefficient. In other words, it could be possible, in principle, to strictly increase the beneficiary's welfare at some point in time, without decreasing his welfare in other time periods. On the other hand, if the policymaker has total flexibility in the way he can allocate the public budget over time and can impose negative lump-sum transfers,

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<sup>2</sup>Stephens (2003) and Stephens (2002) study the consumption response to monthly paycheck receipt in the United States and the United Kingdom, respectively. Under the standard life-cycle/permanent income hypothesis, household consumption should not respond to paycheck arrival. Nevertheless, he finds an excessive response to paycheck receipt. In the case of the US, he shows how the sensitivity is higher for households for which Social Security represents an important proportion of their total income. In a similar study and using data on the consumption patterns of food stamp recipients in the US, Shapiro (2005) presents evidence of declining caloric intake over the 30-day period following the receipt of food stamps.

any efficient consumption allocation can be obtained in equilibrium. Intuitively, the CMP is used as a commitment mechanism by the policymaker in order to impose time consistency for some previously chosen efficient consumption plan. We also characterize the set of feasible consumption allocations when lump-sum transfers are non-negative and the beneficiary has access to an exogenous and deterministic income flow. Therefore, we can find an analogy between the CMP and Laibson's golden eggs model (Laibson 1997a), where the commitment technology takes the form of an illiquid asset.

In a more realistic scenario, the assumption that the beneficiary's relevant information is public seems to be too strong. Income often cannot be observed by the policymaker, especially in developing countries where the informal sector is pervasive. Moreover, fully committing to some transfer schedule is not the best policy ex-ante in an uncertain environment. Therefore, not only the policymaker should consider his role as a commitment "enforcer", but also as an insurer that helps beneficiaries face the potential risk of receiving a negative income shock. We introduce this concern into our model by assuming that while the policymaker can observe the distribution of income shocks, he cannot observe their actual realizations. We approach this problem from a mechanism design perspective. The solution we found represents the existent tradeoff between a more committed versus a more flexible transfer schedule.

The plan of this paper is as follows. Section 2 introduces a simple dynamic model with quasi-hyperbolic discounting into the problem of designing a transfer schedule. Section 3 studies the problem from a first-best perspective, assuming the policymaker has full information and lump-sum transfers are feasible. Section 4 characterizes the optimal transfer schedule when the policy maker only knows the distribution of income shocks. Section 5 concludes. Most of the mathematical details are in the Appendix.

## 2 The Model

Consider the following economy. Time is discrete and indexed by  $t = 1, 2, \dots, T$ . There is one agent who lives for  $T \geq 3$  periods and one policymaker or planner. There is one consumption good  $x$ . The instantaneous utility function  $u : \mathbb{R}_+ \rightarrow \mathbb{R}$  of the agent is assumed to satisfy the following conditions:  $u(\cdot)$  is  $C^2$  over  $(0, \infty)$ ,  $u'(x) > 0$ , and  $u''(x) < 0$ .

In period  $t$ , preferences over consumption streams  $x = (x_1, \dots, x_T) \in \mathbb{R}_+^T$  are representable by the utility function

$$U_t(x) = u(x_t) + \beta \sum_{\tau=t+1}^T \delta^{\tau-t} u(x_\tau)$$

where  $(\beta, \delta) \in (0, 1] \times (0, 1]$ . There exists a linear storage technology with gross return  $R > 0$ . The agent is liquidity constrained in the sense that he can save but not borrow.

The type of preferences represented by this model incorporates the so-called quasi-geometric discounting<sup>3</sup>. The parameter  $\delta$  is called the *standard discount factor* and it represents the long-run, time consistent discounting; the parameter  $\beta$  represents a preference for immediate gratification and is known as the *present-biased factor*. For  $\beta = 1$  these preferences reduce to exponential discounting. For  $\beta < 1$ , the  $(\beta, \delta)$  formulation implies discount rates that decline as the discounted event is moved further away in time.<sup>4</sup>

In the present analysis, we assume that the agent is sophisticated in the sense that she is fully aware of her time inconsistency problem. When preferences are dynamically inconsistent, it is standard practice to formally model the agent as a sequence of temporal selves making choices in a dynamic game. Similar to Strotz (1956), Peleg and Yaari (1973), Goldman (1980), Laibson (1997b), Laibson (1998), O'Donoghue

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<sup>3</sup>This type of preferences was originally proposed by Phelps and Pollack (1968).

<sup>4</sup>See Frederick, Loewenstein, and O'Donoghue (2002), for review of the  $(\beta, \delta)$  formulation and its relation to hyperbolic discounting.

and Rabin (2001), and O'Donoghue and Rabin (1993), we model this problem by thinking of the agent as consisting of  $T$  autonomous selves whose intertemporal utility functions are given by

$$\begin{aligned}
U_1 &= u(x_1) + \beta\delta u(x_2) + \beta\delta^2 u(x_3) + \dots + \beta\delta^{T-2} u(x_{T-1}) + \beta\delta^{T-1} u(x_T) \\
U_2 &= u(x_2) + \beta\delta u(x_3) + \beta\delta^2 u(x_4) + \dots + \beta\delta^{T-3} u(x_{T-1}) + \beta\delta^{T-2} u(x_T) \\
U_3 &= u(x_3) + \beta\delta u(x_4) + \beta\delta^2 u(x_5) + \dots + \beta\delta^{T-4} u(x_{T-1}) + \beta\delta^{T-3} u(x_T) \\
&\vdots = \vdots \\
U_t &= u(x_t) + \beta\delta u(x_{t+1}) + \dots + \beta\delta^{T-t} u(x_T) \\
&\vdots = \vdots \\
U_T &= u(x_T)
\end{aligned}$$

The government implements a consumption maintenance programme (CMP hereafter), which consists of allocating an exogenous budget  $B > 0$  to the individual through a transfer schedule  $\{\tau_t\}_{t=1}^T$ . The government allocates this budget over time in order to maximize the "long-run" welfare of the agent represented by the function<sup>5</sup>

$$W(x_1, \dots, x_t) = \sum_{t=1}^T \delta^{t-1} u(x_t) \tag{1}$$

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<sup>5</sup>Three main approaches to evaluate welfare when preferences are time inconsistent can be found in the literature. The first approach, extensively applied in the consumption-savings literature by Goldman (1979) and Phelps and Pollak (1968), emphasizes the application of a Pareto criterion to evaluate equilibrium allocations. O'Donoghue and Rabin (1999) advocate maximizing welfare from a "long-run perspective". It involves the existence of a "... (fictitious) period 0 where the person has no decision to make and weights all future periods equally." This approach incorporates the fact that most models of present-biased preferences try to capture situations in which people pursue immediate gratification. Moreover, they consider the Pareto criterion as "too strong" because it often refuses strategies that are preferred by almost all incarnations of the agent. In that sense, ranking strategies becomes complicated since "... the Pareto criterion often refuses to rank two strategies even when one is much preferred by virtually all period selves, while the other is preferred by only one period self." Finally, there is a third approach that privileges a subset  $\mathcal{C} \in 2^T$  of players. For instance, welfare may be evaluated with respect to current self's perspective. This "dictatorship of the present" approach has been applied by Cropper and Laibson (1998), and Cropper and Koszegi (2001), where the goal of the policy maker at time  $t$  is to maximize the welfare of self- $t$ .



Intuitively, this welfare function represents the policymaker’s preference for smoother consumption paths.<sup>6</sup> Following a tradition in the income maintenance program literature, we set aside the revenue-raising implications to finance this budget. We have in mind a world in which the budget  $B$  is financed by the non-target population or by some other exogenous source of funding. For the purposes of the present study, we abstract from the process of identifying the target population, focusing exclusively on the allocation of benefits.

### 3 First-Best Consumption Maintenance Programs

In this section, we establish a benchmark case by characterizing the optimal CMP when the beneficiary’s income flow  $\{y_t\}_{t=1}^T$  can be observed by and lump-sum transfers are feasible to the policymaker. Formally, the set of feasible transfer schedules is given by

$$\mathcal{B}^F = \{(\tau_1, \dots, \tau_T) \in \mathbb{R}^T : \sum_{t=1}^T R^{1-t}\tau_t = B\}$$

In contrast to a time-inconsistent beneficiary, we assume that once the policymaker decides which transfer schedule will be implemented, he is fully committed to that program. We can formally model this problem as a two-stage game where the players are the policymaker and the  $T$  different incarnations of the agent. In stage 1, the policymaker announces the transfer schedule to be implemented. In stage 2, the different incarnations of the agent play a consumption-savings game.

Before proceeding with the analysis, we introduce some useful concepts and definitions as well as the equilibrium concept we will employ in this section. Let  $\omega_t$  be cash on hand. This variable evolves according to

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<sup>6</sup>This type of analysis, where the policymaker has an objective function that is different from that of the agent, is not new in public economics. As noticed by Kanbur, Pirttila, and Tuomala (2004) “...there is a long tradition of non-welfarist welfare economics...where the outcomes of individual behavior are evaluated using a preference function different from the one that generate the outcomes.”

$$\omega_{t+1} = R(\omega_t - x_t) + y_{t+1} + \tau_{t+1}$$

with  $\omega_1 = \tau_1 + y_1$ . In the present study, we will focus on consumption strategies in which the past influences current play only through its effect on cash on hand, so the equilibrium concept for the consumption-savings game is that of *Markov perfect equilibrium*. A feasible consumption strategy for player  $t \in \{1, \dots, T\}$  is given by the function  $s_t : \omega_t \rightarrow [0, \omega_t]$ .<sup>7</sup> We say that an equilibrium allocation  $x^*(\tau)$  is *induced* by a transfer schedule  $\tau \in \mathcal{B}^F$  if it is supported by some Markov perfect equilibrium of the consumption-savings game. A first-best CMP is derived from the solution to the two-stage game described above:

**Definition 1**  $\tau^* = (\tau_1^*, \dots, \tau_T^*) \in \mathcal{B}^F$  is a first-best CMP if  $W(x(\tau^*)) \geq W(x(\tau))$  for every  $\tau \in \mathcal{B}^F$ , where  $x(\tau^*) \in \mathbb{R}_+^T$  and  $x(\tau) \in \mathbb{R}_+^T$  are equilibrium allocations induced, respectively, by the transfer schedules  $\tau^*$  and  $\tau$ .

### 3.1 Transfer Schedules without Commitment: The One-Payment CMP

In this section, we study the behavioral implications and welfare outcomes of *one-payment* CMP. We assume that the policy maker is constrained to transfer all of the resources in period 1, where by all resources we mean the public budget  $B$  plus the present value of the beneficiary's future income flow.<sup>8</sup> In some circumstances, this is equivalent to giving access to capital markets to the beneficiary, so he would be able to borrow money against his future income stream. Besides being a benchmark case for comparisons, this seems to be the natural setup for the analysis: administrative costs, technological constraints, and other types of impediments may prevent the policymaker from distributing the budget with more flexibility.

<sup>7</sup>More formally, we could denote by  $S_t$  the set of all feasible strategies for player  $t \in \{0, 1, \dots, T\}$  and by  $S_1 \times S_2 \dots \times S_T$  the joint strategy space of all players.

<sup>8</sup>This implicitly implies that negative transfers can be implemented. We will weaken this assumption later on.

One implication of assuming that the beneficiary is time consistent ( $\beta = 1$ ) is that the optimal consumption path from self 1's perspective can be implemented in equilibrium: his future incarnations will consume and save the amounts he wants them to. Moreover, because the beneficiary and the policy maker share the same intertemporal preferences, an optimal CMP is to transfer the total budget in period 1. On the other hand, if the individual is time inconsistent ( $\beta < 1$ ), this may not be an efficient policy because, as we will see below, it could be possible for the policymaker to weakly improve the welfare of the beneficiary in all periods, and to strictly increase his welfare at some period. The strategic interaction of his different incarnations might generate a coordination failure with a suboptimal outcome as a result.

In the present setting, it can be shown that for all  $\beta < 1$  the equilibrium allocation  $x^* \in \mathbb{R}_{++}^T$  is inefficient from a long-run perspective: we can always find a period  $t < T$  such that reallocating consumption from  $t$  to some  $j > t$  implies a welfare improvement. In other words, by transferring consumption from period  $t$  to period  $j$ , not only could it be possible to increase the welfare of self  $t$ , but also the welfare of their past and future incarnations.<sup>9</sup> Notice that if the agent were time consistent, this behavior should not be observed in equilibrium. Having time inconsistent preferences is what opens the possibility of an inefficient equilibrium.<sup>10</sup>

Proposition 2.1 establishes that, for the one-payment CMP, if the beneficiary is time inconsistent, then the equilibrium allocation is inefficient<sup>11</sup>

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<sup>9</sup>Therefore, this result also implies that the consumption allocation is not Pareto optimal.

<sup>10</sup>In the context of a consumption-savings problem, Laibson(1996) shows how damaging in terms of welfare the type of behavior implied by quasi-hyperbolic discounting could be when the agent has a constant relative risk aversion utility function. Based on his own calibration, he argues that inadequate access to optimal savings policies translates in a welfare cost of at least  $\frac{9}{10}$  of one year income. He discusses the positive effects of some policies to increase not only savings but also the welfare of each of the different selves when the agent faces a time inconsistency problem.

<sup>11</sup>Although there has been some progress in the characterization of equilibria with quasi-hyperbolic discounting, the analysis of the welfare properties of those equilibria has been limited to the case of constant relative risk aversion. Intuitively, it is clear that the inefficient property of the equilibrium of the game should not be a consequence of assuming CRRA preferences. Under very general conditions, Goldman (1979) shows that an interior equilibrium consumption allocation is efficient if

**Proposition 1** *In the one-payment CMP with a time-inconsistent beneficiary, the consumption allocation,  $x^*(\tau) \in \mathbb{R}_{++}^T$ , arising in equilibrium is inefficient.*

PROOF: See Appendix.

The intuition behind this result is very simple: if the policymaker transfers all of the resources in just one payment, a time-inconsistent beneficiary will find himself in a situation of overconsumption. In particular, it can be shown that self T-2 will always be overconsuming in the sense that it would be possible to increase his welfare by transferring resources to the future. Since preferences are separable and monotone, the equilibrium allocation is also inefficient from a long-run perspective.

Interestingly, this result may sound counterintuitive for those who consider that providing *more* liquidity to the poor is *always* the best policy. Our conjecture is that a final answer much depends on the specific goals of a CMP. For instance, if the primary goal of a CMP is to smooth consumption over time, then a one-payment CMP may not be an optimal policy if the beneficiary is time inconsistent, other things constant. On the other hand, if the objective of the policymaker is to help beneficiaries to better face some form of risk such as income shocks, then transferring financial support in as few payments as possible seems to be a better policy, particularly if insurance markets do not work efficiently. This type of dilemmas will be analyzed more formally in section 4 where we create a second-best environment in which the policymaker must face potential tradeoffs implied by more committed, though less flexible, transfer schedules.

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and only if it is best for the first generation (self 1 in the current setting). Therefore, our result is a kind of corollary to the main proposition in Goldman's paper. More precisely, the stronger result we have obtained is a direct consequence of assuming intertemporal separability as well as concavity and differentiability of the instantaneous utility function.

### 3.2 Reestablishing Efficiency through Transfer Schemes

In the present setting, we have shown that any equilibrium allocation is inefficient when the policymaker transfers all of the resources in just one payment. In a first-best scenario where the policymaker has full information, it seems reasonable to expect that the best allocation from a long-run perspective can be obtained in equilibrium.<sup>12</sup> In fact, we will show that it is possible for the policymaker to implement *any* Pareto efficient allocation  $x^* \in \mathbb{R}_+^T$  by doling out transfers such that cash on hand is equal to the optimal consumption path: i.e.  $\omega_t = \tau_t + y_t = x_t^*$ , for all  $t$ . Lemma 2.1 establishes this result more formally:

**Lemma 1** *If the policymaker has full information and there is total flexibility in the way transfers can be allocated over time, then any efficient consumption allocation can be obtained in equilibrium. Moreover, there exists a unique perfect equilibrium supporting this allocation.*

PROOF: See Appendix.

We prove this proposition by applying the following line of logic. First, notice that for any efficient allocation  $x^* \in \mathbb{R}_{++}^T$  the beneficiary is not overconsuming at any time period. Second, since he is not overconsuming, he has no incentive to transfer resources to the future *even if he actually could choose the point in time at which these resources will be consumed*. From here we obtain the result that the efficient allocation arises as the equilibrium allocation. Intuitively, the policymaker provides, through the lump-sum transfer scheme, a mechanism that makes the beneficiary commit to follow up an optimal consumption path.<sup>13</sup>

Since the best allocation from a long-run perspective is efficient, the following result is a direct consequence of Lemma 2.1:

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<sup>12</sup>Notice that the best allocation from a long-run perspective corresponds to the allocation that would be chosen by the beneficiary if he were time consistent.

<sup>13</sup>As a corollary to this Proposition, notice that it is always possible for the policy maker to implement a CMP that Pareto dominates the one-payment CMP.

**Proposition 2** *In a first-best setting where negative transfers can be implemented, the best allocation from a long-run perspective can be obtained in equilibrium.*

In a more realistic scenario, lump-sum transfers should be restricted to be non-negative. Most consumption maintenance programs do not impose any type of negative income transfer to their beneficiaries. We formally incorporate this feature by defining a new set  $\mathcal{B}_+^F$  of feasible transfers:

$$\mathcal{B}_+^F = \{(\tau_1, \dots, \tau_T) \in \mathbb{R}^T : \sum_{t=1}^T R^{1-t} \tau_t \leq B; \tau_t \geq 0 \forall t\}$$

The following corollary is a simple extension of Proposition 2 to the case with non-negative transfers.<sup>14</sup>

**Corollary 1** *Given an efficient consumption profile  $x^* \in \mathbb{R}_+^T$ , if  $y_t \leq x_t^* \forall t$ , then  $x^*$  can be implemented with non-negative transfers.*

Intuitively, the larger the budget  $B$  is with respect to the present value of the beneficiary's income flow  $\sum_{t=1}^T R^{1-t} y_t$ , the more control the policymaker has over the flow of post-transfer income  $\sum_{t=1}^T R^{1-t} (y_t + \tau_t)$ . In consequence, the set of efficient consumption schedules that can be implemented expands as  $B$  gets larger.

## 4 Second-Best Consumption Maintenance Programs

The assumption that the policymaker has full information with respect to the beneficiary's income sequence, though helpful to establish a benchmark case to compare with, is clearly not representative of a more realistic CMP design. Incomes are far from being perfectly observable, especially in developing countries. Moreover, the assumption that the income process is deterministic does not seem to be a reasonable one since the poor are likely to face a highly uncertain economic environment. In

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<sup>14</sup>This result is very easily obtained as an extension of Proposition 2 by setting  $\omega_t = x_t^*$  for all  $t$ . Since  $y_t \leq x_t^*$ , the policymaker sets  $\tau_t = x_t^* - y_t$  for all  $t$ . This is a feasible choice since  $\sum_t R^{1-t} (x_t^* - y_t) = \sum_t R^{1-t} \tau_t = B$

this context, an optimal CMP should consider the existent tradeoff between bringing commitment to the beneficiary with self-control problems and providing an insurance mechanism that help him overcome the ups and downs of everyday life. In other words, an optimal CMP should offer a package balancing both insurance and commitment motives.

We introduce uncertainty into the model by assuming that income is independently and identically distributed over time with probability distribution

$$y_t = \begin{cases} y_L & \text{with probability } \gamma \\ y_H & \text{with probability } 1 - \gamma \end{cases}$$

where  $y_H > y_L$ . We say that the beneficiary receives a negative income shock at time  $t$  if  $y_t = y_L$ . Analogously, we say the beneficiary receives a positive income shock at time  $t$  if  $y_t = y_H$ . For tractability and to keep the analysis as simple as possible we assume that the instantaneous utility function is exponential  $u(x_t) = -\exp(-\alpha x_t)$ . Let  $E_t$  be the expectation operator conditional on all information available at  $t$ , and let  $E(-u(y_t)) = \mu < \infty$

It is assumed that, while the policymaker knows the distribution of income shocks, income realizations are not public information. Therefore, the efficient allocation of resources is impeded by the problem of incentive compatibility: if reporting a negative income shock in period  $t$  implies the reception of a higher transfer, then it is very likely that the beneficiary has an incentive to misreport his current income shock when it is positive.

Based on the revelation principle, the policymaker can restrict attention to direct revelation mechanisms with the property that the beneficiary truthfully reports her true income  $y_t$ .

For any period  $t$ , let  $\tau_i$  represent the transfer at time  $t$  when the beneficiary reports income shock  $i \in \{H, L\}$ , and let  $\tau'_i$  be the corresponding budget left at  $t + 1$ . In

period  $T - 1$ , the policymaker solves the problem

$$\max_{\tau_L, \tau_H, \tau'_L, \tau'_H} \gamma[u(\tau_L + y_L) + \delta E_{T-1}u(\tau'_L + y_T)] + (1 - \gamma)[u(\tau_H + y_H) + \delta E_{T-1}u(\tau'_H + y_T)]$$

subject to the following incentive-compatibility and resource constraints

$$u(\tau_L + y_L) + \beta\delta E_{T-1}u(\tau'_L + y_T) \geq u(\tau_H + y_L) + \beta\delta E_{T-1}u(\tau'_H + y_T) \quad (2)$$

$$u(\tau_H + y_H) + \beta\delta E_{T-1}u(\tau'_H + y_T) \geq u(\tau_L + y_H) + \beta\delta E_{T-1}u(\tau'_L + y_T) \quad (3)$$

$$\tau_L + R^{-1}\tau'_L \leq B_{T-1} \quad (4)$$

$$\tau_H + R^{-1}\tau'_H \leq B_{T-1} \quad (5)$$

where  $B_{T-1}$  is the budget left at time  $T - 1$ . Define by  $v_{T-1}(B_{T-1})$  the value function of this problem. By standard arguments,  $v_{T-1}(B_{T-1})$  is strictly concave and differentiable.

Next, take any period  $t$  and suppose  $v_{t+1}(B_t)$  is strictly concave and differentiable. Although the policymaker and the beneficiary disagree on the amount of discounting applied between  $t$  and  $t + 1$ , they both agree on the utility obtained from  $t + 1$  on. By applying a standard induction argument, we have that for all  $t$  the planner solves the problem:

$$\max_{\tau_L, \tau_H, \tau'_L, \tau'_H} \gamma[u(\tau_L + y_L) + \delta v_{t+1}(\tau'_L)] + (1 - \gamma)[u(\tau_H + y_H) + \delta v_{t+1}(\tau'_H)]$$

subject to the following incentive compatible and budget constraints:

$$u(\tau_L + y_L) + \beta\delta v_{t+1}(\tau'_L) \geq u(\tau_H + y_L) + \beta\delta v_{t+1}(\tau'_H) \quad (6)$$

$$u(\tau_H + y_H) + \beta\delta v_{t+1}(\tau'_H) \geq u(\tau_L + y_H) + \beta\delta v_{t+1}(\tau'_L) \quad (7)$$

$$\tau_L + R^{-1}\tau'_L \leq B_t \quad (8)$$

$$\tau_H + R^{-1}\tau'_H \leq B_t \quad (9)$$



In what follows, we will characterize the equilibrium arising in the current setting. First, we introduce the following result that states that when the beneficiary receives a "negative" income shock he must be transferred at least the same amount than in the case where he receives a "positive" income shock in order to have an incentive-compatible equilibrium.

**Lemma 2**  $\tau_L \geq \tau_H$  in equilibrium.

PROOF: See Appendix.

The policymaker faces a tradeoff: on the one hand, he must take into account the fact that the beneficiary has a self-control problem, implying that he has an incentive to report  $y_L$  when he actually received a positive income shock. On the other hand, the policymaker plays the role of an insurer who should provide a higher transfer when the agent receives a negative income shock. In other words, the policymaker considers both benefits and costs of implementing a more "committed", though less flexible, CMP.

It was argued above that the self control problem can be parameterized by  $\beta$ : the lower this parameter, the stronger the preference for immediate gratification. In his role of insurer, the policymaker should consider some measure of risk that considers somehow the dispersion of income shocks. We define the following measure of risk:

$$\psi = -u(y_H - y_L) = \exp(-\alpha(y_H - y_L))$$

This measure integrates a constant  $\alpha > 0$ , a measure of the degree or risk aversion of the beneficiary, and a measure of the dispersion of the income shock  $y_H - y_L$ . This measure is based on the idea that a beneficiary's sense of well being depends on the risk he faces. We have the following result

**Proposition 3** *If income shocks are unobservable, then the optimal CMP is designed as follows*

- i) If  $\beta \leq \psi$ :  $\tau_H = \tau_L$  (pooling equilibrium).*

ii) If  $\beta > \psi$ :  $\tau_H < \tau_L$  (*separating equilibrium*).

PROOF: See Appendix.

Proposition 2.3 establishes that if the beneficiary's self-control problem (parameterized by  $\beta$ ) is relatively more serious than the vulnerability problem he faces (parameterized by  $\psi$ ), then the policymaker optimally opts for a pooling equilibrium where he transfers  $\tau^*$  independently of the value that the income shock takes, where  $\tau^*$  satisfies  $u'(\tau^*) = v'_{t+1}(B_t - \tau^*)$ .

Income reports are a mechanism to extract private information that may be helpful for the design of a more efficient transfer schedule in the presence of risk. Specifically, having information on actual realizations of income shocks makes consumption smoothing an easier task for the policymaker. However, if the degree of self control is too low, the policy maker's optimal response is to offer a non-contingent transfer schedule. This is equivalent to commit to a transfer schedule at period 0, before the consumption-savings game starts. Therefore, the value of information is zero for low levels of self-control.

## 5 Conclusions

We have analyzed the problem of designing an optimal transfer schedule when the beneficiary is a dynamically-inconsistent decision maker. When he has total control over the resources from the beginning of the period under consideration, the outcome is generally inefficient. This questions the traditional view that providing more liquidity to the poor and making capital markets work more efficiently are sufficient conditions to generate efficient outcomes. In a world with imperfect individuals, perfect markets may not generate the best possible equilibrium.

If the policymaker has full information and lump-sum transfers are not restricted to be non-negative, then any efficient consumption allocation can be obtained in

equilibrium. By imposing constraints on future cash-on-hand, the policymaker is able to influence the pattern of expenditure in future periods and, in consequence, to reestablish efficiency. Obviously, the set of efficient allocations that can be obtained in equilibrium is more restricted when lump-sum transfers cannot be negative: the policymaker has less influence on the final arrangement of the income flow. However, for many, if not most, CMP the budget  $B$  represents an important proportion of the total amount of resources available to the the beneficiary. This fact provides the policymaker with more degrees of freedom for reallocating resources and obtaining more efficient outcomes by means of exercising more control over the beneficiary's income flow. In this sense, a transfer schedule is a kind of commitment mechanism.

One potential drawback of this first-best approach is that, although helpful to establish a benchmark case, it does not provide an accurate description of the circumstances that a policymaker usually has to face when allocating benefits to the poor or the unemployed. Information is far from being public, and many characteristics of the beneficiary, particularly income, are hidden information. Another problem is that a reasonable goal of a CMP is to help beneficiaries to face certain types of risk such as income shocks. This means that the policymaker faces a dilemma since an optimal transfer schedule explicitly designed for dealing with risky environments should be as flexible as possible. However, if the beneficiary has self-control problems, the role of the policymaker as an insurer may imply important trade-offs with its role as a commitment provider. In fact, if the self-control problem is relatively serious with respect to the degree of income uncertainty, the value of obtaining information through income reports is likely to be very low, or even negative if implementing such a mechanism implies some sort of cost such as administrative and data collection costs.

Our analysis has several limitations and possible extensions. First, we do not explicitly consider the possibility of social commitment mechanisms. This type of

mechanisms are likely to arise in small communities where individuals are closer to each other and information is semi-public. In some communities, insurance mechanisms among their members naturally arise. Should we expect the same for social commitment devices such as peer pressure? Second, there may exist less interventionist commitment technologies. For instance, the policymaker could provide the beneficiary with an illiquid instrument a la Laibson (1997). He could also offer a more sophisticated mechanism where the beneficiary has the option to choose a transfer schedule from a menu. If he is aware of his self-control problem, the final consumption allocation would be the best from a current perspective, and hence efficient. Third, the second-best results of this paper could be extended to preferences outside the neighborhood of constant absolute risk aversion. Fourth, it could be assumed that income shocks are not i.i.d., following instead another type of random process. In reality, income realizations may not be independent: a bad draw may generate a series of bad draws. In fact, the analysis of poverty traps in development economics is based on this type of dynamic mechanism. It would be very interesting to find out what the behavior of time inconsistent beneficiaries could be in such a scenario as well as to study the optimal response of the policy maker. Finally, we could introduce naivete into the model and design an optimal mechanism that takes into account the possibility of facing a mixture of sophisticated and naive individuals within the target population.

## 6 Appendix: Proofs

**Lemma 3** *Let  $x^* \in \mathbb{R}_+^T$  be some equilibrium consumption allocation. If there exist periods  $j$  and  $t$ ,  $j > t$ , such that  $u'(x_t^*) < \beta\delta^{j-t}R^{j-t}u'(x_j^*)$ , then the allocation is inefficient.*

PROOF: First, we will show that there exists  $\varepsilon > 0$  such that  $u'(x_t^* - \varepsilon) \geq \beta\delta^{j-t}R^{j-t}u'(x_j^* + \varepsilon)$ . Define the function  $\phi(\varepsilon, \beta) = u'(x_t^* - \varepsilon) - \beta\delta^{j-t}R^{j-t}u'(x_j^* + \varepsilon)$ . Since  $u(\cdot)$  is concave and twice differentiable, we have

$$\phi'(\varepsilon, \beta) = -u''(x_t^* - \varepsilon) - \beta\delta R^{j-t}u''(x_j^* + \varepsilon)$$

which is strictly positive for all  $\varepsilon \geq 0$ . Since  $\phi(\varepsilon, \beta)$  is continuous, there exists some  $\varepsilon > 0$  such that  $\phi(0, \beta) < \phi(\varepsilon, \beta) \leq 0$ . Hence, by concavity of the utility function, it follows that transferring  $\varepsilon$  to period  $i$  strictly increases the welfare of selves  $t$  to  $j$  keeping the welfare of selves  $j+1$  to  $T$  constant since preferences are strictly monotone and separable. Next, we claim that the welfare of selves 1 to  $t-1$  strictly increases by transferring such amount of consumption from period  $t$  to period  $j$ . By a similar argument to the one presented above, it suffices to show that  $\phi(0, \beta) < 0$  implies that  $\delta^{t-\tau}u'(x_t^*) - \delta^{j-\tau}R^{j-t}u'(x_j^*) < 0$ , or equivalently  $\delta^{t-\tau}\phi(0, 1) < 0$ , and that for any  $\varepsilon > 0$  satisfying  $\phi(\varepsilon, \beta) \leq 0$  we have  $\delta^{t-\tau}\phi(\varepsilon, 1) \leq 0$ , for all  $\tau \in \{1, \dots, t-1\}$ . This follows immediately since  $\delta^{t-\tau}\phi(\varepsilon, 1) \leq \phi(\varepsilon, \beta)$  for all  $\varepsilon \geq 0$  and  $\beta \in (0, 1]$ .  $\square$

**Lemma 4** *An allocation  $x \in \mathbb{R}_+^T$  satisfying*

$$u'(x_{T-2}) \geq \max[\beta\delta R u'(x_{T-1}), \beta\delta^2 R^2 u'(x_T)]$$

*cannot be an equilibrium allocation.*

PROOF: Define the set  $\Phi = \{\lambda \in R^3 \mid \lambda_{T-2} = 1, \lambda_{T-1} \leq 0, \lambda_T \leq 0, \lambda_{T-1} + \lambda_T = -1\}$ , and the function  $\varphi(\tau) = u(x_{T-2}^* + \tau\lambda_{T-2}) + \beta \sum_{t=T-1}^T \delta^{t-T+2} u(x_t^* + R^{T-2-t}\tau\lambda_t)$ . Taking the second derivative of the function  $\varphi(\cdot)$  we have

$$\varphi''(\tau) = u''(x_{T-2}^* + \tau) + \beta R^2 \lambda_{T-1}^2 u''(x_{T-1}^* + R\lambda_{T-1}\tau) + \beta R^3 \lambda_T^2 u''(x_T^* + R\lambda_T\tau)$$

which is clearly strictly negative for all  $\lambda \in \Phi$ , and, in consequence, strictly concave. Hence,  $\tau(\lambda) = \arg \max_{\tau \in \mathbb{R}_+} \varphi(\tau)$  is a continuous function on  $\Phi$  by the Maximum theorem. Taking the first derivative of  $\varphi(\tau)$  and evaluating it at  $\tau = 0$ , we have

$$\begin{aligned} \varphi'(0) &= u'(x_{T-2} + \beta R \lambda_{T-1} u'(x_{T-1}^*) + \beta R^2 \lambda_T u'(x_T^*)) \\ &= u'(x_{T-2}^* + \lambda_{T-1} \beta \delta R u'(x_{T-1}) + \lambda_T \beta \delta^2 R^2 u'(x_T)) \\ &> u'(x_{T-2}^*) - \max[\beta \delta R u'(x_{T-1}), \beta \delta^2 R^2 u'(x_T)] \\ &\geq 0 \end{aligned}$$

Where the last inequality follows from the initial hypothesis. This shows that the optimum is strictly positive on  $\Phi$ : i.e.  $\tau(\lambda) > 0$ , for all  $\lambda \in \Phi$ . Since  $\Phi$  is a compact set,  $\tau(\lambda)$  attains its minimum on  $\Phi$  by Weierstrass theorem. Let  $v = \min_{\lambda \in \Phi} \tau(\lambda)$ , and take any  $\bar{\tau} \in (0, v)$ .

Let  $s_t(\omega)$  be the consumption strategy of self  $t$  when cash on hand at that period is equal to  $\omega$ , and define  $\Delta_{T-1} = s_{T-1}^*(\omega_{T-1} - \bar{\tau}) - s_{T-1}^*(\omega_{T-1})$ . By the argument above,  $s_{T-2}^*(\omega_{T-2}) + \bar{\tau}$  is an optimal deviation if  $(1, \frac{\Delta_{T-1}}{\bar{\tau}}, \frac{\Delta_{T-1}}{\bar{\tau}} - 1) \in \Phi$ . In period  $T$ , the agent will consume all resources left. Thus, her equilibrium strategy is given by  $s_T(\omega) = \omega$ , so all we need to show is that  $\frac{\Delta_{T-1}}{\bar{\tau}} \in [0, 1]$ . In period T-1, there is no dynamic inconsistency, so the optimal strategy is obtained by solving

$$s_{T-1}(\omega) = \arg \max u(x_{T-1}) + \beta \delta u(x_T)$$

subject to the constraint  $x_{T-1} + R^{-1}x_T = \omega$ . First order conditions are given by

$$u'(s_{T-1}(\omega)) = \beta \delta R u'(\omega - s_{T-1}(\omega))$$

Differentiating with respect to  $\omega$  at both sides of the equality, and after some algebraic manipulations, we obtain

$$\frac{s'_{T-1}(\omega)}{1-s'_{T-1}(\omega)} = \frac{u'(\omega-s_{T-1}(\omega))}{u'(s_{T-1}(\omega))}$$

Since  $u(\cdot)$  is strictly concave, we must have  $s'_{T-1}(\omega) \in (0, 1)$ . By the Mean Value Theorem, there exists  $\eta \in (\omega_{T-1} - \bar{\tau}, \omega_{T-1})$ , such that  $\frac{\Delta_{T-1}}{\bar{\tau}} = s'(\eta) \leq 1$ . Hence  $(1, \frac{\Delta_{T-1}}{\bar{\tau}}, \frac{\Delta_{T-1}}{\bar{\tau}} - 1) \in \Phi$  and the result follows.  $\square$

**Proof of Proposition 2.1:** The result is a direct consequence of Lemmas 2.3 and 2.4.  $\square$

**Proof of Lemma 2.1:** First, we show that  $\tau = x^*$  arises as the equilibrium allocation. Let  $s^*$  be a Markov perfect equilibrium of the post-transfer game. In order to prove the result, it suffices to show that  $s_t^*(\tau_t) = \tau_t$  for all  $t$ , where  $\tau = x^* \in \mathbb{R}_+^T$  is some efficient allocation. For period  $T$ , it is trivially true that this is the best strategy since player  $T$  will consume all resources on hand. Next, assume  $s_j^*(\tau_j) = \tau_j$  for all  $j > t$ . I claim that optimal strategy for player  $t$  implies  $s_t^*(\tau_t) = \tau_t$ . Assume, towards a contradiction, that  $s_t^*(\tau_t) = \tau_t - \varepsilon$ , for some  $\varepsilon > 0$ , and let  $(\tau_t - \varepsilon, x'_{t+1}, \dots, x'_T)$  be the new consumption allocation from period  $t$  to  $T$ . Since players  $t + 1$  to  $T$  are playing the strategy  $s_j(\tau_j) = \tau_j$  by hypothesis, notice that, for any  $\varepsilon > 0$ , all of them have the option of obtaining a utility of at least  $U_j(\tau_j, \tau_{j+1}, \dots, \tau_T)$ , and hence  $U_j(x'_j, x'_{j+1}, \dots, x'_T) \geq U_j(\tau_j, \tau_{j+1}, \dots, \tau_T)$ , for all  $j \in \{t, \dots, T\}$ , with strict inequality for player  $t + 1$  since  $u(\cdot)$  is strictly monotone and  $\omega_{t+1} = \tau_{t+1} + \varepsilon$ . This implies that  $(\tau_1, \dots, \tau_{t-1}, \tau_t - \varepsilon, x'_{t+1}, \dots, x'_T)$  Pareto dominates  $(\tau_1, \dots, \tau_T)$ , a contradiction. Next, we show uniqueness. I claim that any efficient allocation  $x^* \in R_+^T$  satisfies:

$$u'(x_t^*) \geq \beta(\delta R)^\tau u'(x_{t+\tau}^*)$$

for all  $t, \tau \geq 1$ . Assume, towards a contradiction, that there exist  $\tau'$  and  $t'$  such that this condition does not hold. Then

$$u'(x_{t'}^*) < \beta(\delta R)^\tau u'(x_{t'+\tau'}^*)$$

This implies that  $x^*$  cannot be an efficient allocation since, by concavity of  $u(\cdot)$ , there exists  $\varepsilon > 0$  satisfying

$$u'(x_{t'}^* - \varepsilon) \leq \beta(\delta R)^\tau u'(x_{t+\tau'}^* + \varepsilon)$$

Which is clearly a Pareto improvement. Uniqueness follows from Theorem 1 in Laibson (1997b).  $\square$

**Proof of Lemma 2.2.** Define  $V(x) = \beta\delta v_{t+1}(x)$ , and notice that  $\tau_i' = R(B_t - \tau_i)$  and  $u(\tau_i + y_i) = -u(\tau_i)u(y_i)$ ,  $i = L, H$ . Assume, towards a contradiction, that  $\tau(\theta_L) > \tau(\theta_H)$ . From the incentive compatible constraints we have

$$u(y_H) \leq \frac{V(\tau_H') - V(\tau_L')}{u(\tau_H) - u(\tau_L)}$$

and

$$u(y_L) \geq \frac{V(\tau_H') - V(\tau_L')}{u(\tau_H) - u(\tau_L)}$$

Hence  $u(y_H) \leq u(y_L)$ , a contradiction.  $\square$

**Proof of Proposition 2.3.** It is easier to analyze the problem if we first define some variables. Let  $\mu = E_j[-u(y_t)]$ ,  $\varphi_L = -u(y_L)$ , and  $\varphi_H = -u(y_H)$ . Moreover, make the following change of variables: instead of having  $\tau_L$  and  $\tau_H$  as our decision variables, let the decision variables be  $u_H = u(\tau_H)$ ,  $u_H' = v(\tau_H')$ ,  $u_L = u(\tau_L)$  and  $u_L' = v(\tau_L')$ . Since an exponential utility function can be decomposed as  $u(y + x) = -u(y)u(x)$  and after some algebraic manipulations, the problem becomes

$$\begin{aligned} \max \quad & \gamma[\varphi_L u_L + \delta u_L'] + (1 - \gamma)[\varphi_H u_H + \delta u_H'] \\ \text{s.t.} \quad & \end{aligned}$$



$$\begin{aligned}
\varphi_L u_L + \beta \delta u'_L - \varphi_L u_H - \beta \delta u'_H &\geq 0 \\
u_H + \beta \delta u'_H - \varphi_H u_L - \beta \delta u'_L &\geq 0 \\
B - V_1(u_L) - R^{-1} V_2(u'_L) &\geq 0 \\
B - V_1(u_H) - R^{-1} V_2(u'_H) &\geq 0
\end{aligned}$$

where  $V_1$  and  $V_2$  are the inverse functions of  $u(\cdot)$  and  $v(\cdot)$ , respectively. The Lagrangean for this problem is given by the function

$$\mathcal{L} = \phi(u_L, u_H, u'_L, u'_H) + \sum_{i=1}^4 \lambda_i \phi_i(u_L, u_H, u'_L, u'_H)$$

where  $\phi(\cdot)$  represents the objective function, while  $\phi_i$  corresponds to constraint  $i$ , starting from above. Because  $u(\cdot)$  and  $v(\cdot)$  are concave functions,  $V_i : R \rightarrow R$ ,  $i = 1, 2$ , is convex. Hence,  $\phi_i$ ,  $i = 1, \dots, 4$  is concave.

Notice that the objective function is linear, so by the Theorem of Kuhn and Tucker,  $u^* = (u_H, u'_H, u_L, u'_L)$  is a solution to the problem above if and only if there is  $\lambda^* = (\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*) \in R_+^4$ , where  $\lambda_i^*$  is the corresponding multiplier for constraint  $i$ ,  $i = 1, \dots, 4$ , such that the following Kuhn-Tucker first order conditions hold:

$$\gamma \varphi_L + \lambda_1 \varphi_L - \lambda_2 \varphi_H - \lambda_3 V'_1(u_L) = 0 \tag{10}$$

$$\gamma + \lambda_1 \beta - \lambda_2 \beta - \lambda_3 R^{-1} \delta^{-1} V'_2(u'_L) = 0 \tag{11}$$

$$(1 - \gamma) \varphi_H - \lambda_1 \varphi_L + \lambda_2 \varphi_H - \lambda_4 V'_1(u_H) = 0 \tag{12}$$

$$(1 - \gamma) - \lambda_1 \beta + \lambda_2 \beta - \lambda_4 R^{-1} \delta^{-1} V'_2(u'_H) = 0 \tag{13}$$

In a pooling equilibrium the policymaker solves

$$\max_{\tau} \gamma [u(\tau + y_L) + \delta v_{t-1}(R(B - \tau))] + (1 - \gamma) [u(\tau + y_H) + \delta v_{t-1}(R(B - \tau))]$$

After some algebraic manipulations, this problem is equivalent to solve

$$\max_{\tau} \mu u(\tau) + \delta v_{t-1}(R(B - \tau))$$

The solution is implicitly given by  $\mu u'(\tau) = \delta R v'(\tau')$ , where  $\tau' = R(B - \tau)$ . This implies

$$\delta^{-1} R^{-1} V_2' = \mu^{-1} V_1'$$

Given this condition, the Kuhn-Tucker first order conditions for a pooling equilibrium can be rewritten as follows

$$\gamma \varphi_L + \lambda_1 \varphi_L - \lambda_2 \varphi_H = v_1 \lambda_3 \quad (14)$$

$$\gamma + \lambda_1 \beta - \lambda_2 \beta = \mu^{-1} v_1 \lambda_3 \quad (15)$$

$$(1 - \gamma) \varphi_H - \lambda_1 \varphi_L + \lambda_2 \varphi_H = v_1 \lambda_4 \quad (16)$$

$$(1 - \gamma) - \lambda_1 \beta + \lambda_2 \beta = \mu^{-1} v_1 \lambda_4 \quad (17)$$

From equations 14)-16) or 17)-18) we have:

$$\lambda_2 = \frac{\gamma(\mu - \varphi_L)}{\mu\beta - \varphi_H} + \frac{\mu\beta - \varphi_L}{\mu\beta - \varphi_H} \lambda_1 \quad (18)$$

From where it can be concluded that necessary and sufficient conditions for having positive  $\lambda_1$  and  $\lambda_2$  multipliers are

$$\varphi_L > \mu\beta \quad (19)$$

$$\lambda_1 \geq \frac{\gamma(\mu - \varphi_L)}{\varphi_L - \mu\beta} \quad (20)$$

Positive  $\lambda_3$  and  $\lambda_4$  are obtained if and only if the following conditions are satisfied

$$\gamma\varphi_L + \lambda_1\varphi_L - \lambda_2\varphi_H \geq 0 \quad (21)$$

$$\gamma + \lambda_1\beta - \lambda_2\beta \geq 0 \quad (22)$$

$$(1 - \gamma)\varphi_H - \lambda_1\varphi_L + \lambda_2\varphi_H \geq 0 \quad (23)$$

$$(1 - \gamma) - \lambda_1\beta + \lambda_2\beta \geq 0 \quad (24)$$

equivalently

$$\lambda_2 \leq \frac{\varphi_L}{\varphi_H} + \frac{\varphi_L}{\varphi_H}\lambda_1 \quad (25)$$

$$\lambda_2 \leq \frac{\gamma}{\beta} + \lambda_1 \quad (26)$$

$$\lambda_2 \geq -(1 - \gamma) + \frac{\varphi_L}{\varphi_H}\lambda_1 \quad (27)$$

$$\lambda_2 \geq -\frac{1 - \gamma}{\beta} + \lambda_1 \quad (28)$$

Condition (25) implies condition (26). Therefore, expressions 18)-20), 25), and 27)-28) together provide a set of necessary and sufficient parametric restrictions for a pooling equilibrium to exist.

Conditions 18) and 25) imply:

$$\lambda_1 \geq \frac{\varphi_L\mu\beta - \varphi_H}{\beta\mu(\varphi_H - \varphi_L)} \quad (29)$$

which is trivially satisfied for any  $\lambda_1 \geq 0$ . Conditions 18) and 27)-28) are satisfied if and only if:

$$\lambda_1 \leq (1 - \gamma) \frac{\varphi_H}{\varphi_H - \varphi_L} \frac{1 - \beta}{\beta} \quad (30)$$

Therefore, all of these conditions above are satisfied if and only if

$$(1 - \gamma) \frac{\varphi_H}{\varphi_H - \varphi_L} \frac{1 - \beta}{\beta} \geq \frac{\gamma(\mu - \varphi_L)}{\varphi_L - \mu\beta} \quad (31)$$

and condition (19) are satisfied.

Define the function

$$\phi(\beta) = \frac{1 - \beta}{\beta}(\varphi_L - \mu\beta)$$

Condition (31) can be rewritten as follows

$$\phi(\beta) \geq \frac{\gamma}{1 - \gamma} \frac{\varphi_H - \varphi_L}{\varphi_H} (\mu - \varphi_L) \quad (32)$$

Define by  $\beta^*$  the value of  $\beta$  that makes equation (31) hold with equality. After some manipulations, we have  $\beta^* = \frac{\varphi_L}{\varphi_H}$ . Since  $\phi(\cdot)$  is strictly decreasing in the set  $[0, \frac{\varphi_L}{\mu})$ , and  $\beta^* < \frac{\varphi_L}{\mu}$ , we have that condition (32) is satisfied if and only if  $\beta \leq \beta^*$ .

This completes the proof.  $\square$

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