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Characterizing Distributions of Class III Milk Prices: Implications for Risk Management

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Abstract

Descriptive statistics and time-series econometric models are used to characterize the behavior of monthly fluid milk prices. Prices in April, May and June appear to be more variable than those in subsequent months, and the spring-time prices are perhaps skewed. Econometric models can capture the historical behavior of spot prices, but forecasts converge to the marginal distribution of the sample prices in about six months. Futures prices for Class III milk have the expected time-to-maturity effect and converge to the respective monthly distributions of the cash prices at contract maturity (as they must, since the contracts are cash settled). Thus, econometric models and futures quotes provide similar information about price behavior at contract maturity. Routine hedges in futures, especially those made four or more months prior to maturity, reduce the variance of returns, but over a period of years, lock-in an "average" return. While econometric models and futures quotes provide imprecise forecasts, they can be used in conjunction with historical data to determine whether expected prices are high relative to past experience. This may assist with making decisions about selective hedging. Likewise, historical evidence may be useful in evaluating expected returns from the use of put options. Results from simple hedging strategies using either futures or puts are illustrated, but more work is needed to evaluate "optimal" portfolios for dairy farmers.

Keywords: hedging, marketing strategies, milk futures, milk prices, risk management

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Implications for Risk Management

A variety of policies have been used by the federal government to help stabilize dairy farmers' incomes, but with the reduction of support levels in the late 1980s, milk prices became more variable. Indeed, this variability has increased from 1988 to the present. Given pressures to limit expenditures on farm subsidies and to liberalize trade, increased market interventions by the federal government to stabilize milk prices seems unlikely. Thus, a potential demand exists for low-cost risk management tools that can be used by farmers. Futures, options, and forward contracts may be such tools.

An important step in evaluating risk management strategies is to understand the nature of price risk faced by producers. Hence, a major objective of this paper is to model monthly class III milk prices and to obtain conditional probability distributions of these prices. Since the principal futures contract for milk is settled on the class III cash price, the maturity month price of each futures contract will be the same as the corresponding cash price. But, the distributions of futures prices for a given contract will differ for the months prior to its maturity. For example, a time-to-maturity effect likely exists, so that the variance of futures prices increases as maturity approaches. Thus, this paper will also characterize the distributions of prices of futures contracts over their life cycles.

A second objective is to evaluate the efficacy of using futures and options contracts to manage milk price risk, given our characterization of price risk.

The paper is arranged as follows. The next section describes the data used in this analysis and their unconditional statistics. An econometric model and its justification are then discussed and empirical results are presented. Next, the implications of the results for price risk management are presented. A final section discusses the need for future research.

Data Description

The research uses class III milk prices from 1988 onward, as it is the major mover of the “mail box” prices received by many dairy producers. Emphasis will be placed on monthly prices from 1988 to 2004 as spot prices change monthly. The analysis takes account of the changing definitions of prices over the sample period. Prior to May 1995, the Minnesota-Wisconsin (M-W) price is relevant; from May 1995 to December 1999, the Basic Formula (BFP) is used; from January 2000 onward, the Class III price is used[2]. Three regimes are defined correspondingly.

The cash-settled Basic Formula Price (BFP) futures contracts began trading in 1996. Due to the changes of Federal Milk Marketing Order pricing system, the BFP contracts were converted to Class III milk contracts in the year 2000. From July 2000 onward, Chicago Mercantile Exchange (CME) is the only exchange trading dairy product futures and options. Among all the dairy-related futures contracts, Class III fluid milk is the most active one. Futures prices are available daily. We use futures price observations from September 1997 through December 2004, but because of the limited trading volume and the change from BFP to Class III, futures prices from 2000 onward are the main focus of this analysis.

Descriptive statistics of the Class III cash price data from January 1988 to December 2004 are listed in Table 1. The first part of Table 1 includes the mean, variance,

[2] The Class III price formula has also changed 3 times since 2000. The pricing formulas are modified according to the Tentative Final Decision and the Rules of the 11/07/02 Final Decision. These changes are supposed to be minor and no additional regimes are defined correspondingly.

skewness, kurtosis, and coefficient of variation (standard deviation/ mean) for the entire sample period and three sub-regimes. The same statistics are calculated for each month. The mean of the Class III price is higher in the second regime (corresponding to the definition of BFP) and the variance is higher in the last two regimes (the regimes of BFP and Class III).

Test statistics for hypotheses about the means and variances are listed in Table 2. Basically, the null hypotheses are for equal means and equal variances for the various sub-periods as defined in the Table. Not surprisingly, the variances are not equal. The results imply that the mean of prices is significantly different in the middle period from the other two periods, but the means are not significantly different in periods I and III. There is significant skewness in sub-period prices, when the data are pooled over months.

When the observations are disaggregated by month and yearly sub-periods, definitive conclusions are difficult to reach because there are so few observations per sub-period. For example, skewness appears to be important when the data are aggregated over all of the months, but is typically not important for individual months, except for a few associated with the full sample. Also, while in general, the variance of prices has increased through time, this is not true for every month. For the full 1988-2004 sample, the variance is largest in the Spring months, April, May and June. These months are the "planting season" with milk production information. Skewness also exists for these months.

Monthly price distributions are plotted in Figure 1, and a Gamma distribution is fitted by month for the pooled data (17 observations per month). If one assumes that the different definitions of prices affected only the mean, then one can compare the distributions around the deviations from the sub-means. The monthly prices, when de-meaned by the regime means, are plotted in Figure 2 using normal distribution, as a basis for comparison. Also, distributions are shown by month for the 2000-04 sample (only five observations per month) in Figure 3.

The results imply collectively that the April, May and June prices have similar distributions. In general, the variances by month are largest in the recent period, though this is not true in January and February. From the viewpoint of price risk, groups of months seem to have similar risks, with April through June being largest, July through December being next largest, and January and February smallest. This conclusion can be verified by the coefficeints of variation that are shown in the last column of Table 1. The standard deviation/mean ratios within the three groups are similar. This statistic is arround 0.30, 0.20 and 0.10 for each group respectively.

Figure 4 illustrates the time-to-maturity[3] effect for four different futures contracts: April 2003, July 2003, April 2004 and July 2004[4]. When the time to maturity is greater than 180 days (6 months), the futures price is nearly equal to its historical mean for the contract delivery month cash price. At about 180 days to maturity, the futures price starts to become more volatile and converges to the cash settlement price of the delivery month. In 2003, the expected price more than 240 days (8 months) to maturity is higher than the

[3] USDA announces the Class III on the 5th of the month or the Friday prior to the 5th of the month following the month for which the prices are being applied. For example, the July Class III price is announced on August 5 or the nearby Friday. The Class III futures terminate one business day immediately preceding the day on which the USDA announces the price for that contract month. The time-to-maturity days are calculated as the total number of days betweens announcement day and the day on which futures price is quoted. The time-to-maturity months are calculated as the contract month minus the month for which the futures price is quoted.

[4] April and July are chosen because they seem to belong to different groups if we classify the 12 month data into smaller groups from the results of data description.

cash price at maturity, while the expected price more than 240 days (8 months) to maturity is lower for the 2004 contracts (Figure 4).

Using daily prices of April and July futures contracts as two examples, the distributions of prices of the futures contracts over their life cycle are showed in Table 3. The distributions of these prices show that the mean of the distribution converges to the settlement cash price and the variances increases as the time to maturity decreases. The price volatility of the April contract at maturity is larger than the volatility of July contract, which is consistent with the cash price statistics. But, over the contracts life cycle, the volatility of July contract appears to be larger than the April contract. Normal distributions are fitted for the April contracts at 0, 6 and 10 months time-to-maturity (see note 1 for the definition of months to maturity), shown in Figure 5.

The properties of the milk price data can be summarized as follows: (1) for the cash price series, the mean is larger in the second period, which may be associated with the definition of BFP prices. (2) The variance increases in the second and third period. (3) The pooled prices have positive skewness for Spring months, and the distributions are not symmetric, that is, the variance in April, May and June are larger than other months and positive skewness exists in these months. The fatter right tail of the price distribution implies possible spikes in these months. (4) The results imply collectively that the April, May and June prices have similar distributions. From the viewpoint of price risk, groups of months seem to have similar risks, with April through June being largest, July through December being next largest, and January and February smallest. (5) Futures prices have a time-to-maturity effect: when maturity is longer than 6 months away, the futures prices appear similar to a long-run historical mean. Subsequently, the variance increases and price converges to the settlement (cash) price. The futures prices one month from maturity are very close to the settlement price of the delivery month.

Econometric Models

An econometric model able to reproduce the properties of cash prices may provide more insights about price behavior that are important for managing price risk. Therefore, in this section, a time-series econometric approach is used to capture the relevant characteristics of milk prices: autocorrelation, price spikes, seasonality, and changing mean and variance over the sample with different price definitions. Clearly, milk price behavior is complex, because of animal cycle dynamics, the effects of inventory behavior of manufactured products, and of structural changes associated with changing government programs (Liu et al. 1991; Holt and Craig 2004; Miranda and Hayenga 1993; Rosen 1987; Sun et al 1994). Given successful estimation of an appropriate time-series model, conditional mean, variance, and skewness estimates can be obtained. And forecasting from the model can be compared to the unconditional information from the historical data.

A large literature focuses on inventory behavior to explain seasonal and inter-year price relations (Working, 1949 and Brennan, 1958). Williams and Wright (1991) rational expectations competitive storage model imposes nonlinearity in storage. Aggregate storage cannot be negative. The prediction from this model would be that prices follow a two-regime process depending on whether or not inventories are held. Spikes in the price series are the result of total or close to full stock-out.

Our econometric models are obtained using a general-to-specific approach. We start from a full model using the full sample. Simplifications from the full model are reestimated. Since our risk management problem is more relevant to the last regime (2000-04), the simplified model specification from the full sample is also applied to the sub-sample of 2000-04. Finally, a simplified model for the sub-sample Regime III is obtained.

Autoregressive models with seasonality, dummy variables for the regimes and a stocks variable are estimated (see next section). The inverse of dairy stock is one possible specification for the non-linear effect of stocks[5]. Another possible way to detect nonlinearity and regime changes is using Tong and Lim's (1980) Threshold Autoregressive (TAR) model, which assumes that the regime that occurs at time t can be determined by an observable variable q_t, relative to a threshold value γ. The nonlinearity tests and model estimation details are in Appendix I. Dairy stocks are used as the potential splitting variable.

Econometric Results

Overall Estimation

The full model is fitted to the 1988-2004 sample of monthly observations. The specification involves six harmonic variables (annual, half year, and quarterly) to account for possible seasonality, an AR(12) structure, two dummy variables to account for the

[5] Logarithm of stocks gives similar results as the inverse of stocks. Stock/production ratios are also used. The estimation and out of sample forecasts using stock/production ratios are relatively worse than using the inverse of stocks, which seems different from expection since production variable is added and should provide more explainablity of the price model. The results using the inverse of the stock are reported in this paper.

three different price definitions (regimes), the one-month lagged inverse of milk-equivalent stocks, and interaction terms. The specific definitions of variables are provided in Table 4, and the results are listed in Table 5. The residuals of the resulting specification appear to be white noise. A simplified specification, fitted to the full sample, is presented in Table 6; it contains AR(1,2,4,8,9) variables, two harmonic variables, the two regime-change dummies, interaction between the dummies and seasonality, and the inverse of the inventory variable. The resulting residuals still appear to be white noise.

The coefficients of the dummy variables and the cosine variable are not statistically important, but the interaction of the cosine with one of the dummies is important. These variables are retained. The lagged inventory variable clearly is important. Recalling that this variable is specified as an inverse, a smaller inventory variable is associated with a larger inverse. Thus, the positive coefficient is logical; smaller stocks are associated with higher prices. The partial derivative of price with respect to the stocks is the inverse of stocks to the power of 2 with a negative sign times the estimated coefficient. The range of stocks from 1988 to 2004 is [3.96, 21.47] billion pounds. The partial derivative of price with respect to the stocks evaluated at the minimum and maximum are 1.70 and 0.06, $/cwt, respectively. When the stocks increase 1 billion pounds, the price decreases in the range of [0.06, 1.70] dollars.

Sub-regime Estimation

A shorter sample with 60 observations (2000.01-2004.12) is used to estimate with the same model specification as the model in Table 6. The results are in Table 7. The sub-regime estimation can be further simplified as an AR(2) model with 2 harmonic terms (annual sine and cosine) and the one-month lagged inverse of milk-equivalent stocks. The residuals of the model are shown to be white noise (Table 7). The sine variable is statistically important, and the lagged inventory variable is important.

Threshold Specification Results

Tong and Lim's (1980) Threshold Autoregressive (TAR) model was tried to improve the model for the full sample. We hope to capture the nonlinear dynamics and structural changes in the series using dairy stocks as the potential splitting varible.

The following model assumes that nonlinearity depends on the level of inventories. Instead of using inverse of milk-equivalent stocks as a regressor in the overall model, the stock variable is specified as a threshold variable. The non-linear model has two regimes: one regime is when the stock level is lower or equal to some threshold level $q_{t-d}=\gamma$, d=1, 2 and 3, where q is milk-equivalent stock measured in billion pounds (MEFATBIL) and d is the delay parameter, and the other regime is when the stock level is higher than this critical level γ. Estimates of the parameters are obtained within each regime defined by the level of the stocks.

The same specification as the model in Table 6, excluding the inverse of stocks is used. No significant reduction of sum square of residuals is obtained. But a simplified AR(2) model with 2 harmonic terms (annual sine and cosine) also fits the three regime data reasonably well. This simplified specification is estimated using lagged dairy stocks as the potential splitting variable.

$$p_t = (\alpha_0 + \alpha_1 p_{t-1} + \alpha_2 p_{t-2} + \alpha_3 SY + \alpha_4 CY) 1(MEFATBIL_{t-3} \le \gamma)$$
$$+(\beta_0 + \beta_1 p_{t-1} + \beta_2 p_{t-2} + \beta_3 SY + \beta_4 CY) 1(MEFATBIL_{t-3} > \gamma) + \varepsilon_t$$

Lagged dairy stock (milk-equivalent dairy stock 3 months ago, MEFATBIL3) is an important splitting variable, with a bootstrap p-value of 0.098. The critical stocks level is 5.43 billion pounds (Figure 6). The 95% confidence interval is [5.21, 5.43]. No further splitting is detected at the 10% significant level using MEFATBIL with d=1, 2, and 3. As shown in Figure 7, all of the inventories less than 5.43 occur in the second regime. This effect is equivalent to regime dummy variable effect. The estimation results are listed in Table 8.

The same specification was also applied to the last regime. No significant reduction of sum square of residual is obtained using lagged milk-equivalent dairy stocks. The nonlinearity test results imply that as long as the dairy stocks are not too low (below 5.43 billion), the simplified model for regime III is a sufficient model.

Implication for Price Risk Management

Forecasting

Prices are clearly more variable in recent years, and therefore regime III is most relevant for price risk management. The simplified model for 2000-04 includes two autoregressive terms, seasonal cycle and the one-month lagged milk-equivalent stocks variable. Since the inverse of stock is included in the model, a model for forecasting stocks is estimated first. Strong seasonality exists for the dairy stocks. Production of milk is large from March through June, the "flush" season, while seasonal patterns of consumption for all dairy products are almost counter-cyclical with milk production. Eleven monthly dummy variables and an AR(1,6) terms are included to capture the seasonality and autocorrelation of the stocks. The estimates of the stocks for the period from January 2000 to December 2004 are listed in Table 9a. And the 24-period ahead (two years) forecasts of the stocks are in Table 9b.

In the second step, estimated stocks are used as an input variable to forecast Class III milk price. These forecasts are based on maximum likelihood estimates of the simplified Regime III model. Table 10 lists the results of our forecasts for 24 months, January 2005 to December 2006. The forecasts and 95% confident intervals are plotted in Figure 8. Six observations are available out of the sample (January 2005 to June 2005). In goodness-of-fit terms, the model predicts these six months reasonably well: the price of January 2005 is within two standard deviations of the forecast, while the prices of February to June 2005 are within one standard deviation.

The mean squared error associated with the forecast increases with the forecast horizon. As the forecast horizon goes farther into the future, the forecast approaches the unconditional mean of the series and the MSE approaches the unconditional variance of the series[6]. Table 10 shows that our empirical forecasts converge to a standard deviation of 2.88 after 9 months.

If stochastic input variables are used, the forecast standard errors and confidence limits of the response should also depend on the estimated forecast error variance of the predicted inputs (Feldstein, 1971). Because ancillary forecasts for stock are included, the standard errors of the Class III price forecasts will be underestimated, since stock values are assumed to be known with certainty.

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[6] An ARMA process can be written as an MA(∞) representation $(Y_t - \mu) = \psi(L)\varepsilon_t$ with ε_t white nosie and $\psi(L) = \sum_{j=0}^{\infty} \psi_j L^j$, L is the lag operator. Then the mean squared error assocatied with this forecast is $E(Y_{t+s} - \hat{E}(Y_{t+s} \mid \varepsilon_t, \varepsilon_{t-1}, ...])^2 = (1 + \psi_1^2 + \psi_2^2 + ... + + \psi_{s-1}^2)\sigma^2$.

Implications

The conditional forecasts have non-decreasing forecast standard errors. Hence, the uncertainty of using these forecasts increases. As the forecast horizon increases, the forecast error approaches a constant consistent with the unconditional variance of the sample, while the point forecast has a seasonal component. In our example, forecasts more than half a year in advance provide no more information than the unconditional mean and variance of the monthly statistics. These results can help setup marketing plans and evaluate if the current CME Class III futures and options contracts offer good price risk management tools.

Numerous alternative strategies are available to farmers to help them price their milk. The strategy appropriate to a particular farmer will, of course, depend on his/her level of aversion to risk, debt to asset ratio, and other factors related to the individual farm situation. In this subsection, we connect the information about price behavior to some general strategies, and subsequently we illustrate the outcomes of some simple strategies.

We categorize strategies as follows. (1) To manage price risk, a time-limit order could be used which involves routinely selling one or more futures contracts at some fixed interval, like six, eight or ten months prior to contract maturity. This strategy would take advantage of the time-to-maturity effect in the variance of futures and should reduce the variability of returns. This strategy presumably assures an “average” return for the respective maturity months. (2) Futures can also be used to establish an absolute target price. In this case, futures are sold selectively, when the contract’s price reaches a specified trigger level. The trigger level might be related to the cost of production or other appropriate indicator. (3) Or, a relative target can be established by reference to the historical probability distribution of prices for the individual month, e.g., based on the top 20 percent of Class III cash prices in the sample. (4) Alternatively, a combination of strategy (1) with (2) or (3) may be possible. In (1), the emphasis is on reducing the variance of returns, while in (2) and (3) the emphasis is on establishing a price for the milk marketed in a particular month that meets a pre-specified target; of course, there can be many months in which no futures position is established.

The econometric forecasts might help establish the trigger level, and the distribution of historical prices would be useful in setting up relative price targets. The econometric forecasts and the futures price quotations can be compared to the historical distributions as a benchmark for making hedging decisions.

The forecasting error and/or the monthly unconditional distribution determine the risk of the market. They tell the potential downside or upside of the market. If the residuals of the econometric model are normally distributed or the unconditional price distribution is normal, then approximately 67 percent of the time, the market price could be within one standard deviation around the mean and approximately 95 percent of the time, the market price could be within two standard deviations around the mean.

For Class III milk prices, the distributions of prices appear to be skewed in the Spring months (April to June). The distributions have fatter tails to the right. The skewness implies that price spikes in those months are more probable than in other months. And the upside risk and downside risk are different. Asymmetric distributions imply that options contracts have a potentially important role to play in risk management

(Lapan et al 1991; Vercammen 1995; Hanson et al 1999). The unconditional distrubution can be useful in evaluating the downside risk that would be covered by a put option, while the forecasting error will give biased measure of the risk.

For example, on December 17, 2004, the closing futures prices for the following 12 months contracts are listed in Table 11 row 1; our forecasts from the econometric model are listed in row 2; the unconditional monthly means are listed in row 3 and row 4; the forecast errors and the unconditional standard deviations are listed in row 5 to 7; the premiums for a strike closest to the futures prices are listed in row 8. Row 9 to row 11 are percentiles based on the distribution of prices for the past 17, 10 and 5 years. The percentiles shown are the probabilities based on the area in the distribution below the quoted futures price.

The December 17 futures quotes for 2005 delivery months are above the average prices prevailing in the five years, 2000-04. In contrast, the point forecasts from the econometric model are above the futures quotes January to July, but the differences are not statistically significant. Since the futures quotes are above the historical average and since this relationship seems to be confirmed by the econometric forecasts, one might conclude that hedging can lock in a relatively favorable price. Of course, while the foregoing helps illustrate the historical context for making a hedging decision, actual decisions about the level of hedging require additional information. Likewise, one can compare the option premia with the historical probability distributions to obtain insights about the potential benefits of using a put option. On December 17, the price of at-the-money puts ranged from $0.4 for nearby delivery, $0.7 for 3-6 months delivery to $0.8 per cwt for distant delivery. For example, using the Gamma distribution to fit the 2000-04 observations, the results imply that hedger would have paid \$0.7 or \$0.8 to get the benefit of a 40% probability of a price increase above the current futures price level.

Simple Hedging Strategy and the Effectiveness of the Hedging

Thus, without considering more information, the efficacy of fully hedging by selling futures and buying put options are evaluated for the most recent period (2000-04). We assume one unit of production and 100% hedges. Each transaction is made 14 days after the USDA announcement date for the current cash price (Friday on or before the 5^{th} of each month), i.e. near the middle of each month. The return of no hedging is compared to the returns from selling futures at different times to maturity in Table 12. For month i, the return $R_i = P_i \times 1 + (F_{i-t} - F_i) \times 1$, where t is the time to maturity, and t=0, 4, 6, 8 and 10. The average return for month i is the average of R_i over the period of 2000-04. The last row in Table 11 is the annual return, $\pi = \sum_{i=1}^{12} R_i /12$ averaged over 5 years (2000-04).

Table 13 shows the returns from buying put options at different times to maturity, t equals 0, 4, 6, 8 and 10 respectively. Again, a full hedge with unit production is assumed. For month i, the return $R_i = P_i \times 1 + \max(K_{i-t} - P_i, 0) \times 1$ and the annual return is $\pi = \sum_{i=1}^{12} R_i /12$. The numbers are the returns averaged over 5 years (2000-04).

The hedging results show: (1) Futures contracts reduce the variance of the returns. Even 4-months ahead hedging reduces the variance, and the longer the time in advance, the larger the reduction of variance. (2) In view of hedging efficacy, groups of months seem to have similar patterns. April through June, July through November and January through February can be three different groups. Selling futures in April, May and June is less effective than buying put options, which is consistent with the finding that asymmetry exists in these months. Limitation of the analysis includes: production costs and transaction costs are not included in the analysis; and the number of available observations is small.

Conclusions and Future Work

The results help provide a deeper understanding of the behavior of milk prices under relatively competitive market regimes. Given this understanding, it should be possible to help dairy producers (or their cooperatives) to improve their risk management strategies.

Future work is needed to improve our understanding of alternative marketing strategies. The foregoing empirical analysis can be placed in a conceptual framework, with varying definitions of optimal hedges. Such a framework can, in principle, accommodate the use of both futures and options markets and the effects of transaction costs. A related issue is accomodating a portfolio of positions for the different months.

References

Brennan, M. J. “The Supply of Storage.” *The Amer. Econ. Rev.* 48(1958): 50-72.

Feldstein, M. S. "The Error of Forecast in Econometric Models When the Forecast-Period Exogenous Variables Are Stochastic." *Econometrica* 39(1971): 55-60.

Hansen, B. E. "Inference in Threshold Models." *Studies in Nonlinear Dynamics and Econometrics*, Quarterly Journal, 2(1997): 1-14.

Hansen, B. E. “Sample Splitting and Threshold Estimation.” *Econometrica*, 68(2000): 575-603.

Hanson, S.D., R.J. Myers, and J.H. Hilker. “Heging with Futures and Options under a Truncated Cash Price Distribution.” *J. Agr. and Appl. Econ.* 31(1999): 449-59.

Holt, M.T., and L.E. Craig. “A Nonlinear Model of the U.S. Hog-Corn Cycle.” A Festschrift in Honor of Stanley R. Johnson. J.-P. Chavas and M.T. Holt, eds. Berkeley: *Berkeley Electronic Press*, 2004.

Lapan, H.E., G. Moschini, and S.D. Hanson. “Production, Hedging, and Speculative Decisions with Options and Futures Markets.” *Amer. J. Agr. Econ.* 73 (1991): 66-74.

Liu, D.J., H. M. Kaiser, O. Forker, and T. Mount. "Modeling the U.S. Dairy Sector Under Government Intervention." *W. J. Agr. Econ*. 16(1991): 360-73.

Miranda, M.J., and J.W. Glauber. “Estimation of Dynamic Nonlinear Rational Expectations Models of Primary Commodity Markets with Private and Government Stockholding.” *Rev. of Econ. and Statist.* 75 (1993): 463-70.

Sun, C.H., D. J. Liu, and H.M. Kaiser. "Estimating Market Conduct of the US Dairy Industry Within a Switching Regime Framework." *Amer. J. Agr. Econ.* 76(1994): 1266-67.

Teräsvirta, T. “Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models.” *J. Amer.Statist.Assoc.*89(1994):208-18 .

Vercammen, J. “Hedging with Commodity Options when Price Distributions are Skewed.” *Amer. J. Agr. Econ.* 77(1995): 935-45.

Working, H. “The Theory of Price of Storage.” *The Amer. Econ. Rev.* 39(1949): 1254-1262.

Wright, B. D., and J. C. Williams. “A Theory of Negative Prices for Storage.” *The Journal of Futures Markets* 9(1989): 1-13.

Table 1. Statistics For Class III Milk Prices, Selected Time Periods

Time Period	*N*	*Mean*	*St.d.*	*Skewness*	*Kurtosis*	*Mean/St.d.*
1988:01 To 2004:12	204	12.14	1.93	1.16***	2.23	0.16
1988:01 To 1995:05	89	11.75	1.02	0.54**	0.59	0.09
1995:06 To 1999:12	55	12.90	1.81	0.58*	-0.19	0.14
2000:01 To 2004:12	60	12.01	2.74	1.11***	0.85	0.23
January						
1988:01 To 2004:12		11.81	1.64	1.23*	2.27	0.14
1988:01 To 1995:05		11.66	1.15	1.00	1.50	0.10
1995:06 To 1999:12		13.55	1.89	1.52	2.60	0.14
2000:01 To 2004:12		10.66	0.99	0.60	-3.03	0.09
February						
1988:01 To 2004:12		11.29	1.12	0.03	-1.04	0.10
1988:01 To 1995:05		11.28	0.82	-0.01	-1.01	0.07
1995:06 To 1999:12		12.16	1.32	-1.49	2.76	0.11
2000:01 To 2004:12		10.60	1.10	0.38	-2.91	0.10
March						
1988:01 To 2004:12		11.47	1.36	0.29	0.20	0.12
1988:01 To 1995:05		11.26	0.90	0.40	-0.49	0.08
1995:06 To 1999:12		12.41	0.54	-1.66	2.75	0.04
2000:01 To 2004:12		11.04	2.13	1.31	1.71	0.19
April						
1988:01 To 2004:12		11.84	2.29	2.60***	8.98	0.19
1988:01 To 1995:05		11.44	1.00	0.10	-0.87	0.09
1995:06 To 1999:12		12.09	0.71	1.34	2.24	0.06
2000:01 To 2004:12		12.28	4.27	1.88	3.66	0.35
May						
1988:01 To 2004:12		11.92	2.57	2.60***	8.34	0.22
1988:01 To 1995:05		11.46	0.94	0.08	-1.30	0.08
1995:06 To 1999:12		11.66	1.44	1.85	3.45	0.12
2000:01 To 2004:12		12.86	4.66	1.56	2.12	0.36
June						
1988:01 To 2004:12		11.99	2.11	1.34**	2.04	0.18
1988:01 To 1995:05		11.61	1.04	0.44	-0.70	0.09
1995:06 To 1999:12		12.12	1.33	0.61	-1.83	0.11
2000:01 To 2004:12		12.40	3.73	0.87	-1.71	0.30
July						
1988:01 To 2004:12		12.30	1.81	0.39	-0.99	0.15
1988:01 To 1995:05		11.73	0.98	0.80	0.19	0.08
1995:06 To 1999:12		12.99	1.83	-0.39	-2.96	0.14
2000:01 To 2004:12		12.42	2.66	0.17	-2.43	0.21
August						
1988:01 To 2004:12		12.70	1.89	0.20	-0.97	0.15
1988:01 To 1995:05		11.91	0.77	0.34	-1.27	0.06
1995:06 To 1999:12		13.88	1.92	-0.51	-2.86	0.14

2000:01 To 2004:12	12.61	2.63	-0.33	-2.58	0.21
September					
1988:01 To 2004:12	13.09	1.85	0.28	-0.90	0.14
1988:01 To 1995:05	12.19	0.51	0.70	1.13	0.04
1995:06 To 1999:12	14.33	1.78	-0.42	-2.39	0.12
2000:01 To 2004:12	13.12	2.62	-0.43	-2.60	0.20
October					
1988:01 To 2004:12	12.74	1.62	0.19	-0.38	0.13
1988:01 To 1995:05	12.22	1.00	-0.18	2.25	0.08
1995:06 To 1999:12	13.42	1.74	0.83	0.44	0.13
2000:01 To 2004:12	12.78	2.22	-0.64	-2.97	0.17
November					
1988:01 To 2004:12	12.25	2.06	0.36	0.42	0.17
1988:01 To 1995:05	12.30	1.33	0.49	2.20	0.11
1995:06 To 1999:12	12.81	2.59	0.86	1.68	0.20
2000:01 To 2004:12	11.62	2.58	0.18	-1.74	0.22
December					
1988:01 To 2004:12	12.24	2.21	0.94	0.65	0.18
1988:01 To 1995:05	12.10	1.47	1.09	2.41	0.12
1995:06 To 1999:12	12.90	2.87	0.86	1.34	0.22
2000:01 To 2004:12	11.78	2.69	1.28	1.79	0.23

*--10% level; **--5% level ***--1% level

Table 2. Tests for the Equality of the Means and Variances, Three Regimes

	Regime I Regime II, III	*Regime I and Regime II*	*Regime I and Regime III*	*Regime II and Regime III*
F value for the test of Equal Mean	6.61***	24.05***	0.71	4.13**
Levene's Test for Homogeneity Variance	14.62***	19.11***	23.65***	5.74**
Welch's Test for Equal Mean Allowing Hetersdasticity	9.34***	18.76***	0.53	4.27**

*--10% level; **--5% level ***--1% level

Table 3. Time-to-Maturity Effects For Milk Futures Prices, April and July Contracts, Pooled Daily Observations, 2000-04

Months to Maturity	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
April Contract													
N	102	109	96	102	125	121	133	121	133	127	127	100	84
Mean	12.22	11.72	11.06	11.04	11.53	11.38	11.27	11.48	11.58	11.62	11.54	11.64	11.75
Standard Deviation	3.72	2.81	1.25	0.72	0.94	0.77	0.70	0.59	0.54	0.52	0.43	0.27	0.32
July Contract													
N	105	105	107	103	109	96	102	125	121	133	121	116	86
Mean	12.33	12.61	12.70	13.08	12.75	12.40	12.53	12.54	12.34	12.22	12.23	12.13	12.36
Standard Deviation	2.33	2.25	1.74	2.12	1.69	1.08	0.65	0.74	0.70	0.67	0.52	0.49	0.32

Table 4. Definitions of Variables, 1988.01-2004.12

Variable	*Definition*
P	Dependent variable
AR1 to AR12	Autoregressive terms, lag 1 to lag 12
SY and CY	Seasonality, sine and cosine terms for annual cycle
SY2 and CY2	Seasonality, sine and cosine terms for half-year cycle
SY3 and CY3	Seasonality, sine and cosine terms for quarterly cycle
D2 and D3	Dummy variables for the changes of price definition: D2=1 for observations from May 1995 to December 1999; otherwise D2=0; D3=1 for observations from January 2000 to December 2004; otherwise D3=0.
D2SY, D2CY D2SY2, D2CY2 D2SY3, D2CY3	Interaction terms between D2 and annual, half-year and quarterly cycles
D3SY, D3CY D3SY2, D3CY2 D3SY3, D3CY3	Interaction terms between D3 and annual, half-year and quarterly cycles
INVMEFATBIL1	MEFAT is the milk equivalent end-of-month total stocks employing fat-based accounting; INVMEFATBIL1 is one-month lagged inverse of milk equivalent dairy stocks measured in billion pounds.

Table 5. Parameter Estimates, Full Model, 1988.01-2004.12

Parameter	*Estimate*		*St.d.*
MU	9.972	***	0.758
AR1	1.195	***	0.077
AR2	-0.516	***	0.120
AR3	0.043		0.129
AR4	0.138		0.130
AR5	0.005		0.126
AR6	-0.048		0.124
AR7	0.002		0.124
AR8	0.324	***	0.124
AR9	-0.292	**	0.129
AR10	0.041		0.134
AR11	-0.071		0.133
AR12	-0.066		0.090
SY	-0.581	***	0.220
CY	0.023		0.226
SY2	-0.084		0.153
CY2	0.040		0.156
D2	-0.270		0.634
D3	0.123		0.575
D2SY	-0.414		0.343
D2CY	-0.371		0.361
D2SY2	0.282		0.245
D2CY2	-0.437	*	0.252
D3SY	-0.231		0.337
D3CY	-0.923	***	0.342
D3SY2	-0.076		0.249
D3CY2	-0.345		0.253
SY3	0.004		0.054
CY3	0.015		0.053
D2SY3	0.088		0.089
D2CY3	0.106		0.086
D3SY3	0.089		0.085
D3CY3	-0.075		0.084
INVMEFATBIL1	19.091	***	7.150
Constant Estimate	2.450		
Variance Estimate	0.689		
Std Error Estimate	0.830		
AIC	534.944		
SBC	647.593		
Number of Residuals	203		
White Noise Residual Check			
Lag	ChiSq	Pr>Chisq	
12	--	--	
18	1.59	0.953	
24	3.87	0.986	
30	5.06	0.999	
36	18.06	0.800	

***-1% level; **-5% level;*-10% level

Table 6. Parameter Estimates, Simplified Model, 1988.01-2004.12

Parameter	*Estimate*		*St.d.*
MU	9.169	***	0.833
AR1	1.236	***	0.064
AR2	-0.526	***	0.073
AR4	0.168	***	0.045
AR8	0.231	***	0.067
AR9	-0.268	***	0.070
SY	-0.812	***	0.189
CY	-0.257		0.218
D2	-0.432		0.616
D3	0.190		0.721
D3SY	-0.038		0.350
D3CY	-0.927	***	0.334
INVMEFATBIL1	26.731	***	6.499
Constant Estimate	1.468		
Variance Estimate	0.691		
Std Error Estimate	0.831		
AIC	516.549		
SBC	559.62		
Number of Residuals	203		

White Noise Residual Check		
Lag	ChiSq	Pr>Chisq
6	1.54	0.215
12	3.13	0.872
18	7.37	0.882
24	13.06	0.835
30	18.37	0.826
36	23.86	0.816

***-1% level; **-5% level;*-10% level

Table 7. Parameter Estimates, Simplified Models, 2000.01-2004.12

Parameter	*Estimate*		*St.d.*	*Estimate*		*St.d.*
MU	9.235	***	1.952	8.350	***	2.129
AR1	1.274	***	0.119	1.335	***	0.119
AR2	-0.539	***	0.136	-0.479	***	0.121
AR4	0.169	**	0.075			
AR8	0.287	**	0.139			
AR9	-0.387	**	0.156			
SY	-0.833	**	0.337	-1.365	*	0.795
CY	-1.172	***	0.449	-0.580		0.731
INVMEFATBIL1	26.071		17.647	36.851	**	17.966
Constant Estimate	1.810			1.204		
Variance Estimate	0.964			1.182		
Std Error Estimate	0.982			1.087		
AIC	180.741			188.166		
SBC	199.591			200.732		
Number of Residuals	60			60		
			White Noise Residual Check			
Lag	ChiSq	Pr>Chisq		Lag	ChiSq	Pr>Chisq
6	1.51	0.219		6	3.87	0.424
12	11.90	0.104		12	15.45	0.117
18	14.30	0.353		18	16.73	0.403
24	21.70	0.300		24	24.25	0.334

***-1% level; **-5% level;*-10% level

Table 8 Parameter Estimates, Threshold Autoregressive Model, 1988.01-2004.12

	Global Estimates		Regime MEFATBIL3<=5.43		Regime MEFATBIL3>5.43	
Variable	Estimate	St Error	Estimate	St Error	Estimate	St Error
Constant	2.481	0.645	15.208	2.671	2.114	0.557
P1	1.205	0.125	0.149	0.188	1.308	0.100
P2	-0.408	0.132	-0.374	0.128	-0.481	0.095
SY	-0.097	0.106	0.165	0.393	-0.073	0.098
CY	-0.142	0.080	0.177	0.464	-0.125	0.081
Observations	204		20		184	
Degrees of Freedom	199		15		179	
Sum of Squared Errors	165.575		10.548		124.089	
Residual Variance	0.832		0.703		0.693	
R-squared	0.782		0.328		0.833	
Sum of Squared Errors	165.575			134.638		
Residual Variance	0.832			0.694		
Joint R-Squared	0.782			0.822		
Heteroskedasticity Test (p-value)	0.002			0.054		

Note: The estimation result is

$$p_t = (15.21 + 0.15 p_{t-1} - 0.37 p_{t-2} + 0.17SY + 0.18CY)1(MEFATBIL_{t-3} \le 5.43)$$
$$+(0.56 + 1.31 p_{t-1} - 0.48 p_{t-2} - 0.07SY - 0.13CY)1(MEFATBIL_{t-3} > 5.43) + \varepsilon_t$$

Table 9a. Parameter Estimates, Milk-Equivalent Dairy Stocks, 2000.01-2004.12[a]

Parameter	*Estimate*		*St.d.*
MU	7965.900	***	678.344
AR1,1	1.071	***	0.047
AR1,2	-0.120	**	0.047
M1	1203.200	***	128.059
M2	1874.800	***	163.412
M3	2034.800	***	186.924
M4	2620.800	***	202.924
M5	3328.700	***	213.047
M6	3761.600	***	218.792
M7	3895.900	***	212.980
M8	2961.600	***	202.820
M9	2102.300	***	186.923
M10	1075.600	***	163.419
M11	-154.248		128.029
Constant Estimate	387.488		
Variance Estimate	82563.940		
Std Error Estimate	287.339		
AIC	850.861		
SBC	879.947		
Number of Residuals	59		
White Noise Residual Check			
Lag	ChiSq	Pr>Chisq	
6	0.7	0.951	
12	8.1	0.619	
18	12.47	0.711	
24	24.51	0.321	

***-1% level; **-5% level;*-10% level

[a] MEFAT is the dependent variable.

Table 9b. Forecasting Milk-Equivalent Dairy Stocks, 2004.12-2006.12

Obs	*Forecast*	*St.d.*	*95% Confidence Upper Limits*	*95% Confidence Lower Limits*
60	7745.03	287.34	7181.86	8308.21
61	8889.53	421.13	8064.13	9714.94
62	9533.36	534.96	8484.87	10581.86
63	9661.32	641.19	8404.62	10918.02
64	10202.42	744.69	8742.86	11661.98
65	10906.37	848.08	9244.16	12568.57
66	11338.02	937.80	9499.96	13176.09
67	11478.00	1016.41	9485.88	13470.13
68	10553.24	1085.23	8426.23	12680.25
69	9707.94	1144.98	7463.82	11952.06
70	8701.60	1196.01	6357.46	11045.75
71	7494.15	1238.50	5066.74	9921.56
72	7672.49	1273.49	5176.49	10168.49
73	8900.82	1301.93	6349.08	11452.55
74	9598.16	1324.65	7001.90	12194.42
75	9784.09	1342.45	7152.94	12415.24
76	10395.52	1356.10	7737.61	13053.42
77	11127.91	1366.35	8449.91	13805.90
78	11584.11	1373.85	8891.42	14276.80
79	11740.33	1379.17	9037.20	14443.46
80	10826.53	1382.82	8116.25	13536.82
81	9986.02	1385.22	7271.05	12701.00
82	8976.37	1386.70	6258.48	11694.25
83	7761.94	1387.56	5042.38	10481.50
84	7929.86	1388.00	5209.44	10650.28

Table 10. Forecasts of Milk Prices from the AR(2) Model with Seasonal Cycle and One-Month Lagged Inverse Stocks, 2005.01-2006.12

Obs	*p*	*Forecast*	*St.d.*	*95% Confidence Upper Limits*	*95% Confidence Lower Limits*
61	14.14	15.75	1.09	13.62	17.88
62	14.70	14.74	1.81	11.19	18.29
63	14.08	14.24	2.30	9.73	18.75
64	14.61	14.18	2.59	9.09	19.26
65	13.77	14.05	2.75	8.66	19.44
66	13.92	13.82	2.83	8.27	19.36
67		13.46	2.86	7.86	19.07
68		12.92	2.87	7.29	18.55
69		12.47	2.88	6.83	18.10
70		11.94	2.88	6.31	17.58
71		11.64	2.88	6.00	17.28
72		11.84	2.88	6.21	17.48
73		11.63	2.88	5.99	17.26
74		11.26	2.88	5.63	16.90
75		11.58	2.88	5.94	17.21
76		12.27	2.88	6.64	17.91
77		12.77	2.88	7.14	18.41
78		13.02	2.88	7.38	18.65
79		13.00	2.88	7.36	18.64
80		12.67	2.88	7.03	18.31
81		12.33	2.88	6.70	17.97
82		11.86	2.88	6.22	17.50
83		11.56	2.88	5.93	17.20
84		11.73	2.88	6.10	17.37

Table 11. An Example Information Set For December 17, 2004

Month	*Jan 05*	*Feb 05*	*Mar 05*	*Apr 05*	*May 05*	*Jun 05*	*Jul 05*	*Aug 05*	*Sep 05*	*Oct 05*	*Nov 05*	*Dec 05*
Futures Price	13.43	13.10	12.71	12.79	12.8	12.92	12.94	13.45	13.68	12.95	12.5	12.25
Forecast Price	15.75	14.74	14.24	14.18	14.05	13.82	13.46	12.92	12.47	11.94	11.64	11.84
Unconditional Mean (1988-2004)	11.81	11.29	11.47	11.84	11.92	11.99	12.30	12.7	13.09	12.74	12.25	12.24
Unconditional Mean (2000-04)	10.66	10.60	11.04	12.28	12.86	12.40	12.42	12.61	13.12	12.78	11.62	11.78
Forecast St.d.	1.09	1.81	2.30	2.59	2.75	2.83	2.86	2.87	2.88	2.88	2.88	2.88
Unconditional St.d. (1988-2004)	1.64	1.12	1.36	2.29	2.57	2.11	1.81	1.89	1.85	1.62	2.06	2.21
Unconditional St.d. (2000-04)	0.99	1.10	2.13	4.27	4.66	3.73	2.66	2.63	2.62	2.22	2.58	2.69
Put Premium	0.41	0.50	0.61	0.65	0.66	0.71	0.76	0.83	0.89	0.88	0.86	0.87
Price Percentile of the Futures Price (1988-2004 Gamma Fit)	85	95	83	70	67	70	65	67	65	57	57	52
Price Percentile of the Futures Price (1995-2004 Gamma Fit)	81	91	75	63	61	63	56	55	51	49	57	51
Price Percentile of the Futures Price (2000-04 Gamma Fit)	99	99	82	60	53	60	60	65	61	55	67	60

Table 12. Average Returns and Standard Deviations of Returns, Futures Hedges, 2000.01-2004.12

Maturity Month	*No Hedging*		*Sell Futures 4 Months Ahead*		*Sell Futures 6 Months Ahead*		*Sell Futures 8 Months Ahead*		*Sell Futures 10 Months Ahead*	
	Ave	St.d.	Ave	St.d.	Ave	St.d.	Ave	St.d.	Ave	St.d.
January	10.66	0.99	11.57	0.78	**11.74**	0.46	11.57	0.36	11.59	**0.33**
February	10.60	1.10	10.99	0.75	**11.61**	0.71	11.50	0.53	11.34	**0.42**
March	11.04	2.13	11.20	0.77	11.56	0.65	**11.64**	**0.41**	11.44	0.45
April	**12.28**	4.27	11.27	0.69	11.12	0.68	11.50	0.48	11.41	**0.32**
May	**12.86**	4.66	11.42	0.58	11.40	0.67	11.51	0.60	11.44	**0.41**
June	**12.40**	3.73	11.71	1.18	11.87	0.74	11.56	0.69	11.64	**0.35**
July	12.42	2.66	12.77	1.75	**12.63**	**0.51**	12.39	0.67	12.33	0.53
August	12.61	2.63	13.33	2.09	**12.78**	0.94	12.70	0.82	12.28	**0.84**
September	13.12	2.62	13.22	1.23	**13.15**	1.40	13.13	0.63	12.86	**0.85**
October	12.78	2.22	12.88	0.89	**13.09**	1.06	12.76	0.73	12.60	**0.63**
November	11.62	2.58	12.35	0.73	**12.42**	0.66	12.21	0.62	12.19	**0.40**
December	11.78	2.69	11.84	0.97	11.82	0.30	**12.03**	0.30	11.99	**0.17**
Ave Rtn /Year	12.01	2.74	12.05	1.29	**12.10**	0.97	12.04	0.77	11.94	**0.69**

Note: The numbers in Bold are the maximum returns or the minimum standard errors by row. They may not be statistically significant different from the others since the number of observations is small.

Table 13. Average Returns and Standard Deviations of Returns, Buying Put Options, 2000.01-2004.12.

Maturity Month	*No Hedging* total obs=60		*Buy Put Options 4 Months Ahead* total obs=60		*Buy Put Options 6 Months Ahead* total obs=60		*Buy Put Options 8 Months Ahead* total obs=55		*Buy Put Options 10 Months Ahead* total obs=15	
	Ave	St.d.	Ave	St.d.	Ave	St.d.	Ave	St.d.	Ave	St.d.
January	10.66	0.99	10.91	0.80	**11.03**	0.58	10.88	0.49	10.98	0.35
February	10.60	1.10	10.57	0.71	**10.93**	0.67	10.87	0.45	--	--
March	11.04	2.13	11.42	1.40	**11.59**	1.28	11.45	1.26	--	--
April	12.28	4.27	**12.70**	3.67	12.45	3.68	12.61	3.56	--	--
May	12.86	4.66	**13.28**	4.05	13.24	3.90	13.22	3.84	--	--
June	12.40	3.73	12.72	3.04	**13.19**	2.56	--	--	--	--
July	12.42	2.66	12.81	1.64	**12.97**	1.47	12.85	1.47	--	--
August	12.61	2.63	**13.48**	1.50	13.14	1.17	13.10	1.11	--	--
September	13.12	2.62	13.24	1.30	13.46	1.24	**13.47**	1.27	--	--
October	12.78	2.22	12.73	1.13	12.87	0.88	**12.94**	0.96	--	--
November	11.62	2.58	12.24	1.32	12.36	1.10	**12.26**	1.14	12.16	1.37
December	11.78	2.69	11.98	2.14	11.90	1.91	12.03	1.77	**12.04**	1.96
Ave Rtn /Year	12.01	2.74	12.34	2.16	**12.43**	2.00	--	--	--	--

Note: The numbers in Bold are the maximum returns by row. They may not be statistically significant different from the others since the number of observations is small.

a. --put option contract is missing in the month, for at least one of the five years.

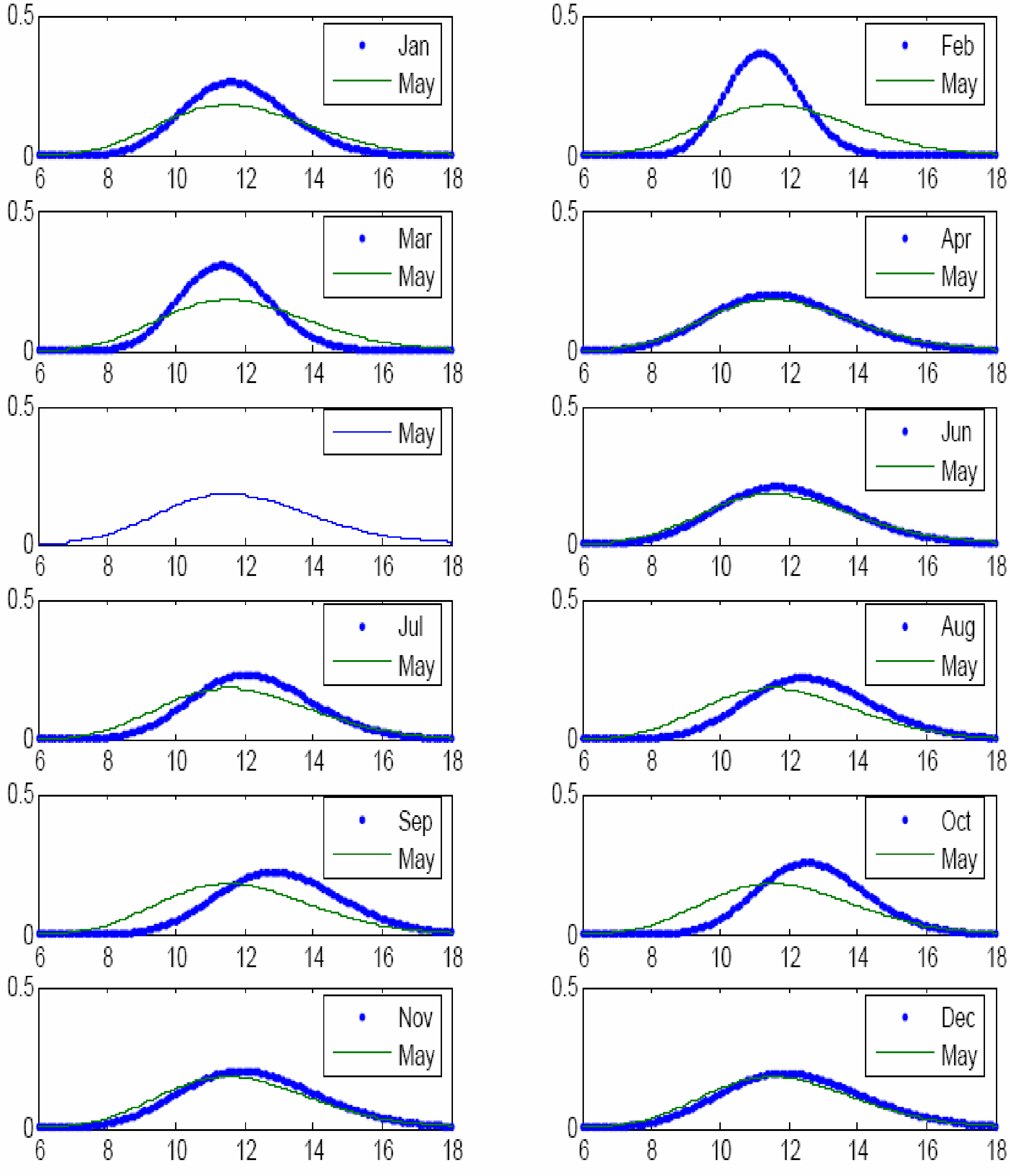

Figure 1. Class III Price Distributions by Month (Gamma Distribution), 1988-2004

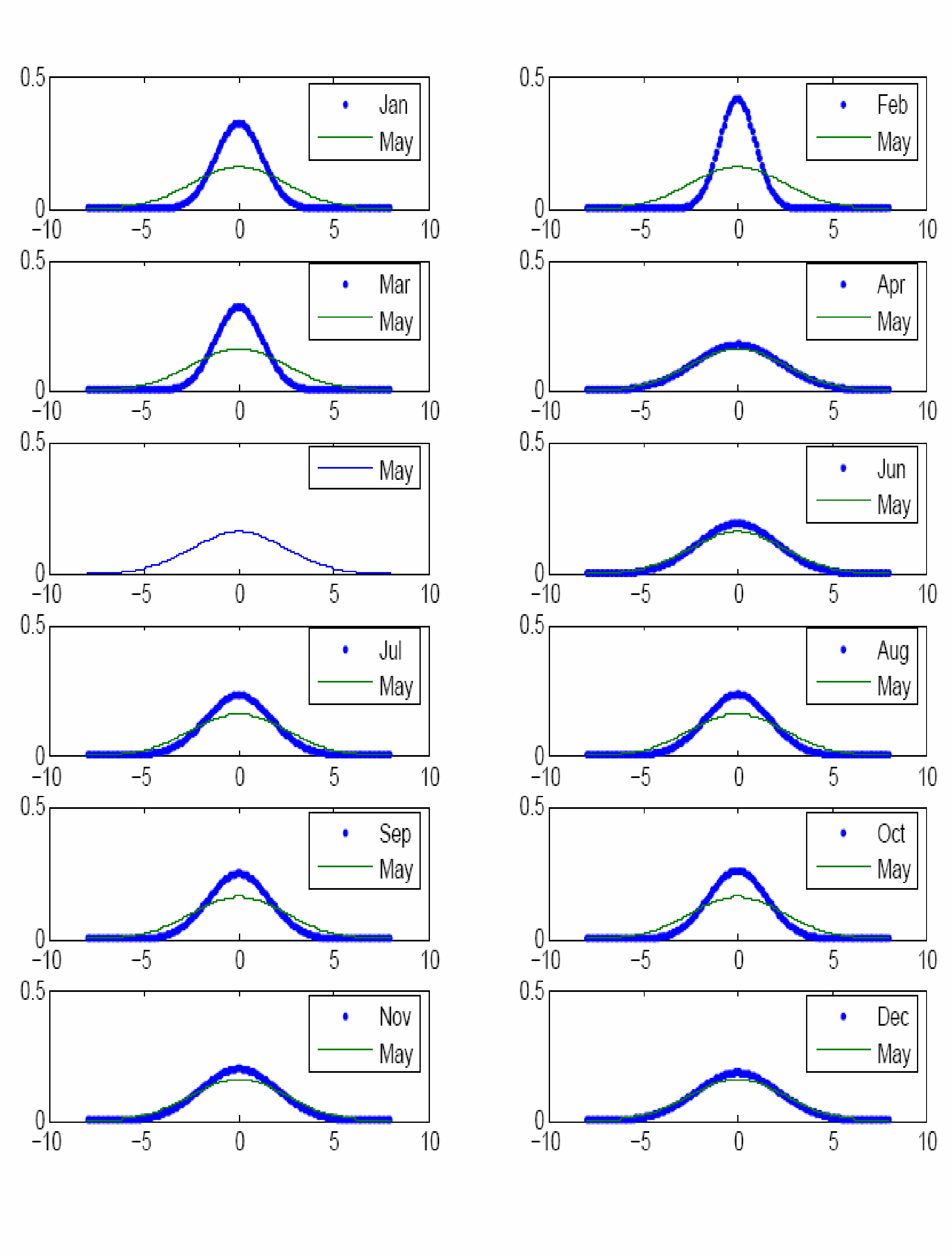

Figure 2. De-meaned Class III Price Distributions by Month (Normal Distribution), 1988-2004

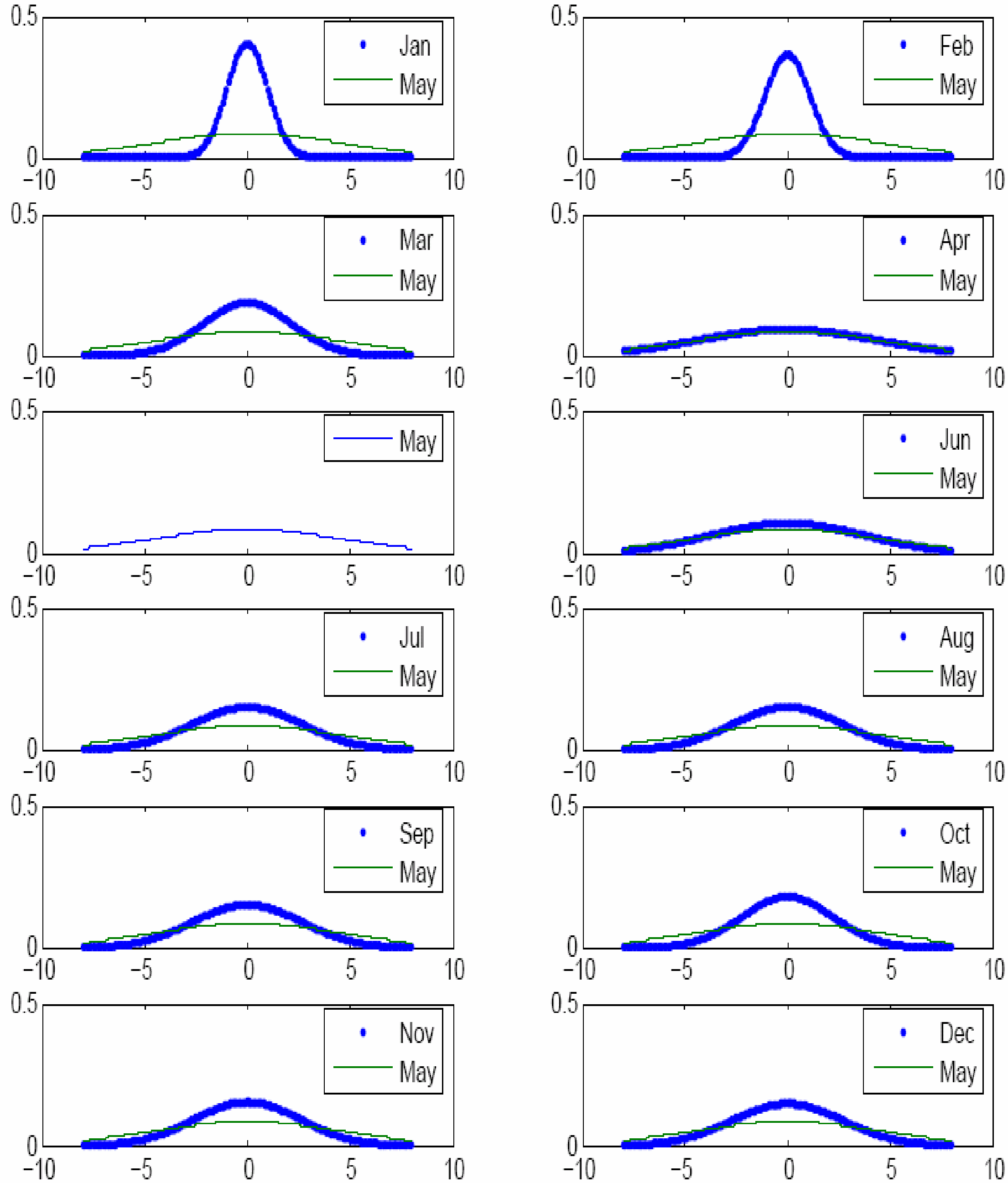

Figure 3. De-meaned Class III Price Distributions by Month (Normal Distribution) for Regime III, 2000-2004

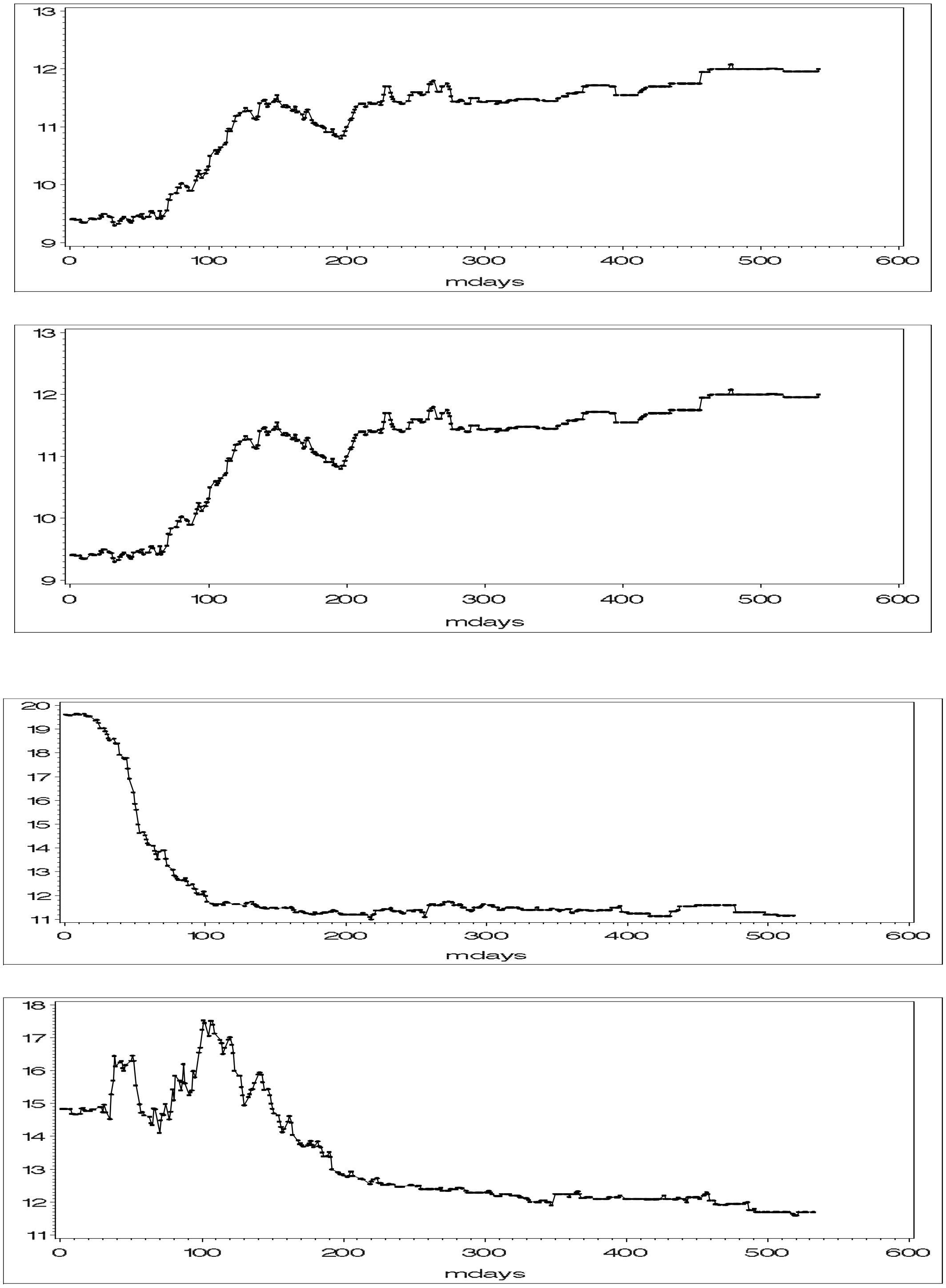

Figure 4. Prices of Class III Milk Futures by Time-to-Maturity[1], Four Examples: April 2003, July 2003, April 2004 and July 2004.

See note 1

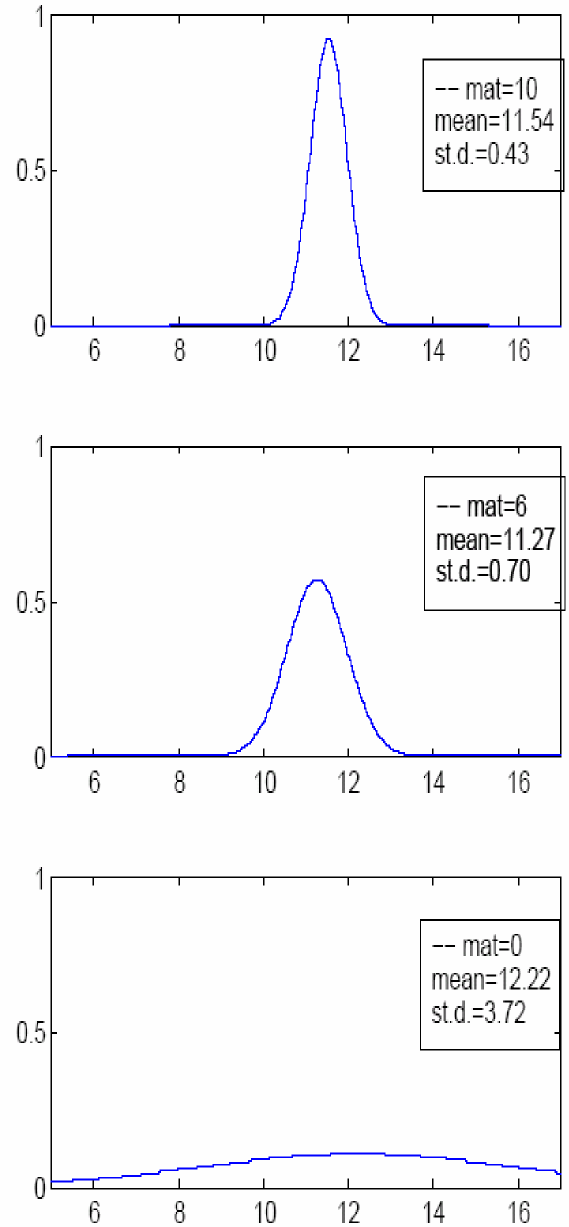

Figure 5. April Futures Contract Price Distributions, 2000-04, by Time-to-Maturity[1]

See end note 1

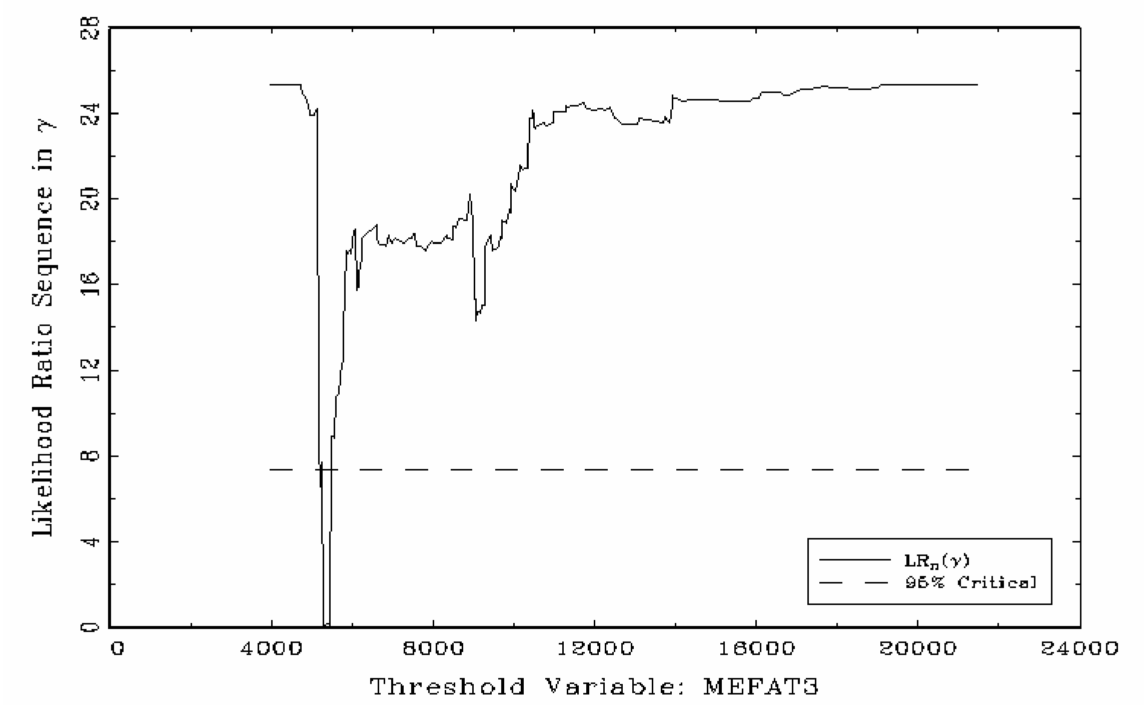

Figure 6. Likelihood Ratio of Threshold Test Using Three-Month Lagged Dairy Stock (MEFAT3) and the Simplified Model Specification.

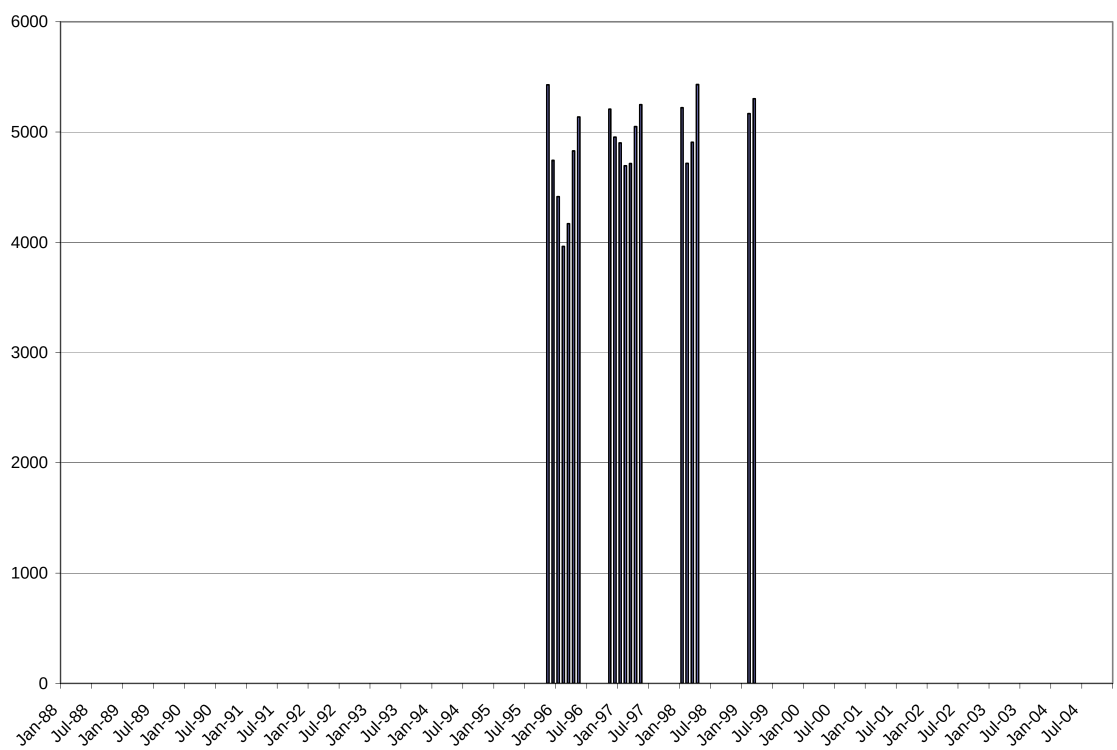

Figure 7. Stock Levels of the Observations in the Lower Stock Regime (MEFAT3 Less Than 5.43 Billion Pounds).

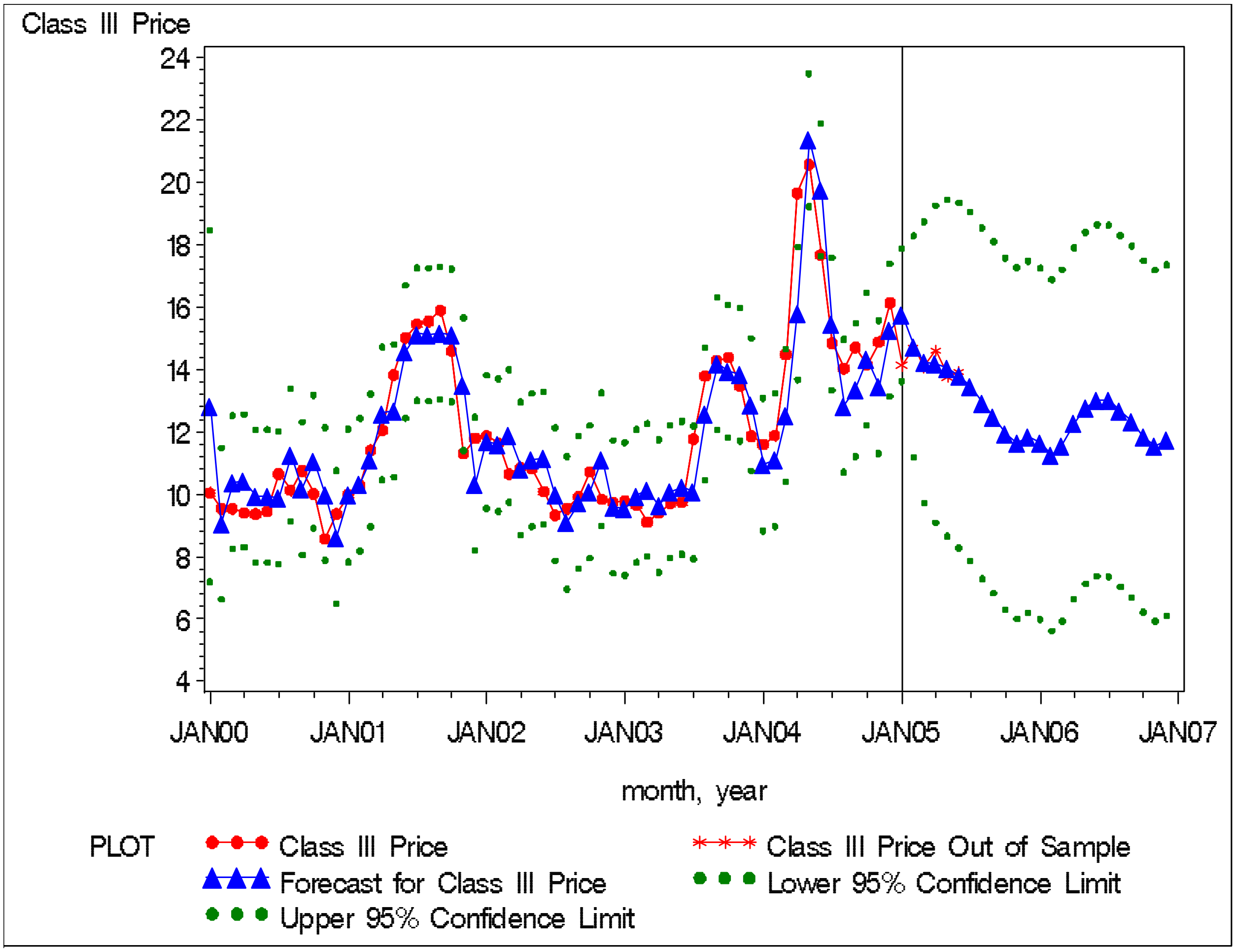

Figure 8. Class III Prices and the Forecasts for 2005.01-2006.12 Using Simplified 2000.01-2004.12 Model.

Appendix I

Tong (1978) and Tong and Lim (1980) proposed Threshold Autoregressive (TAR) model, which assumes that the regime that occurs at time t can be determined by an observable variable q_t, relative to a threshold value γ.

Hansen (1997, 2000) developed a distribution theory for least-squares estimator of the TAR models. The asymptotic distribution of the likelihood ratio statistic for testing hypothesis is asymptotically free of nuisance parameters. His method can be used to approximate a general nonlinear autoregressive structure by a threshold autoregression with a small number of regimes.

The threshold regression model takes the form

$$y_i = \theta_1' x_i + \varepsilon_i \qquad q_i \le \gamma \tag{AI-1}$$

$$y_i = \theta_2' x_i + \varepsilon_i \qquad q_i > \gamma \tag{AI-2}$$

where q_i is the threshold variable, and is used to split the sample into two groups and ε_i is a regression error. The distribution of the threshold estimate is nonstandard. Since it is based on an asymptotic distribution theory, a confidence interval of the test statistics can be constructed to tell whether the splitting is significant or not. Monte Carlo simulations can be used to assess the accuracy of the asymptotic approximations. Once threshold level γ is found, simple regressions can yield consistent estimators within each group.