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Department of Applied Economics and Management
Cornell University, Ithaca, New York 14853-7801 USA

HOW WORKERS GET POOR *BECAUSE* CAPITALISTS GET RICH: A GENERAL EQUILIBRIUM MODEL OF LABOR SUPPLY, COMMUNITY, AND THE CLASS DISTRIBUTION OF INCOME

Indraneel Dasgupta
Ravi Kanbur

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How Workers Get Poor *Because* Capitalists Get Rich: A General Equilibrium Model of Labor Supply, Community, and the Class Distribution of Income [•]

by

Indraneel Dasgupta

University of Nottingham

indraneel.dasgupta@nottingham.ac.uk

and

Ravi Kanbur

Cornell University

sk145@cornell.edu

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Abstract

We develop an integrated, general equilibrium, model of how the presence of vertical ties of 'community' between sections of workers and sections of capitalists can critically affect the distribution of income between capitalists as a class and workers as a class, as well as between workers belonging to different communities. We show that an exogenous increase in the incomes of capitalists sets in motion community and market processes that subsequently (a) further increase capitalists' incomes, (b) can reduce workers' earnings as well as welfare, and (c) systematically influence earnings differentials between workers belonging to different communities.

Keywords: Community, Class, Public Goods, Income Distribution.

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1. Introduction

Inequality in earnings has increased significantly in many countries over the last two decades or so.[1] One aspect of this is the increase in the earnings gap between workers and those who own capital assets. Another aspect is the increase in the earnings gap between workers belonging to different religious, ethnic or language groups.[2] Both aspects have been documented and debated at length. Our paper contributes to this literature by providing one explanation of how an initial increase in the incomes of capital owners, regardless of its cause, can feed back into the economy to: (a) reduce the earnings of workers of all communities, (b) increase the earnings gap between workers belonging to different communities even when there is no segmentation or discrimination in the labor market, and (c) increase the incomes of capital owners even further.

Two strands of thought motivate our analysis—the first emanating from a social observation, and the second from a question. The social observation is that "vertical" ties of community cut across "horizontal" class difference between poor and rich individuals. These community ties can be of different types—ethnic, religious, clan, etc—but they exert a pull and affect behavior over and above class position. This is the observation. The question is as follows: can the poor become poorer *because* the rich become richer? Or, to put the question differently, if the rich become richer for whatever reason, could this event, simply by itself, generate forces that would subsequently reduce the welfare of the poor? Both the observation and the question need elaboration. We start with the question.

The claim that prior wealth gains by the rich can somehow be *causally responsible* for subsequently *aggravating* poverty may appear perplexing in the light of standard economic theory. Typically, rich individuals are those who own significant amounts of non-labor factors of production, in particular, land or physical capital, and derive their incomes mainly from such ownership. Thus, by and large, rich individuals are *capitalists*, whether industrial or agrarian. Conversely, poor individuals are mostly *workers*, i.e., those who do not own significant amounts of non-labor assets, and are primarily dependent on the labor market for their incomes. Wealth

[1] For recent surveys, see Kanbur (2000) and Kanbur and Lustig (2000).

[2] There are numerous analyses of ethnic inequality in the US. An early attempt at rigorous analysis of inter-ethnic inequality in a developing country context was by Anand (1983).

gains by capitalists would imply either (a) returns from their existing holdings of land or capital assets have gone up for some reason, or (b) they have somehow come into possession of more land or capital stock, or both. Reasons typically cited for possibility (a) include population growth, biased technological progress, state subsidies funded by additional taxes on workers, and demand driven shifts in relative prices in favor of land-intensive or capital-intensive goods, which reduce the wage rates, and thus, the earnings of the workers. However, in this case, it is some extraneous demographic, technological, politico-economic or demand side factor that is causally responsible for *both* income losses by workers and income gains by capitalists: the latter does not cause the former. Thus, workers become poorer *while*, but *not because*, capitalists become richer.[3] If possibility (b) obtains, then, since total land or capital stock in the economy rises, labor demand should increase, increasing wage rates and/or employment, thereby making workers *better off*. Thus, claims regarding the existence of causal mechanisms connecting prior income gains by the rich (capitalists) with subsequent income losses by the poor (workers) are often discounted as dubious. We attempt to rehabilitate such claims by advancing one causal mechanism in this paper.

In standard analyses of the class basis of income distribution, and of distributive conflicts between capitalists and workers,[4] the two classes are linked with one another only through the market (employment) relationship.[5] The argument we develop builds however on the observation that in many, if not most, societies some sections of capitalists and some sections of workers are also vertically linked through social ties of 'community', whereas other sections of workers are excluded from this relationship, by virtue of their belonging to a different community.

Most ethnic, religious, racial, caste or geographic communities are characterized by internal differentiation and heterogeneity with respect to ownership of productive non-labor

[3] It is in this sense that Proudhon's oft-quoted dictum about property being theft under capitalism is a condemnation of wealth ownership on moral, but not functional grounds. In his argument, it is the legal framework of private ownership of capital which allows capitalists to acquire wealth which 'rightfully' belongs to workers, and thereby impoverishes workers, but such wealth, once acquired, does not generate *further* impoverishment of the workers. See Proudhon (1994).

[4] See, for example, Lancaster (1973), Przeworski and Wallerstein (1982), Alesina and Rodrik (1994) and Somanathan (2001).

[5] An exception is Roemer (1998), who explicitly considers the role played by ties of religion in the determination of the level of redistribution in a political-economic equilibrium.

assets, and, consequently, income levels. At the same time, what seems to define such communities as communities is the presence of something, (a) to which all members have common access, but from which non-members are excluded, and (b) in the process of accessing which members also acquire social access to each other, despite differences in their income and wealth status. Thus, one way of formalizing the notion of *belonging* to a community is by positing access to a community-specific local public good.[6] For example, belonging to a particular religious community can be interpreted to imply (a) common access to that community's places of worship and religious activities, and (b) access to social interaction with other members of the community *via* common participation in worship and collective religious activities. Similarly, the right to participate in ethno-linguistic festivals and rituals can be considered constitutive of the ethnic identity of an individual, and participation in such activities may provide a platform, perhaps the only platform, where rich and poor members of the ethnic group come together to interact socially.[7] At a slightly different level, the presence of local or neighborhood public goods (such as parks, roads, libraries, concert halls, auditoria, museums, sports clubs, safety etc.), and the social interaction between rich and poor which is brought about as an indirect consequence of accessing and using such public goods, can be considered as defining that which constitutes a *community* out of a collection of individuals living in geographical proximity to each other.

The productivity of a worker can be expected to increase with the size of the public good(s) which defines the community to which he belongs. This is for direct as well as indirect reasons. Most obviously, better infrastructure, in the form of a larger quantum, or improved quality, of local public goods of the kind mentioned above, can be expected to improve her physical and psychological health, analytical abilities, knowledge and awareness of the world, reduce the stress and exertion related to commuting etc., all of which would improve her ability to work more intensely, or allow her to maintain high levels of efficiency for longer periods. An expansion in the magnitude of the community specific public good would also expand the scope

[6] Such a public (or rather, 'club') good-based interpretation of community identity and membership is developed in detail by Dasgupta and Kanbur (2001).

[7] There is a large anthropological literature on community identity. Economists have recently become interested in this notion. See Akerlof and Kranton (2000). The link of public goods provision and ethnic divisions has been explored in Alesina, Baqir and Easterly (1999). There is also a psychological literature on group identity, as exemplified by Wetherell (1996).

for social interaction with other poor individuals, as well as rich individuals. This can lead to more information pooling, improvements in inter-personal social skills, learning of better work, health and recreation practices, finding job environments which provide better fits for one's individual idiosyncratic characteristics and abilities, etc.[8] Thus, workers' productivity may go up because of effects working through this indirect social channel as well.[9]

The intuitive form of the argument we develop and formalize is the following. Consider a society comprised of two, say ethnic, communities, A and B. Suppose that, for some reason, capitalists belonging to community A acquire more wealth. They may be expected to spend a part of their additional wealth on public goods specific to their community. This, however, for reasons already discussed, will improve the productivity of workers of community A. Consequently, labor supply of workers of community A, measured in efficiency units, will go up, reducing the wage rate per efficiency unit of labor supply. Additional labor supply, will, however, increase returns to capital ownership earned by all capitalists (irrespective of community affiliation). This second-order effect, while improving the welfare of every capitalist in society, will also increase labor supplies of both communities, thereby further depressing the wage rate. The overall fall in the wage rate will reduce the welfare of *every* worker in society, if aggregate labor demand is sufficiently inelastic. Indeed, even if labor demand is elastic, workers from one community can suffer a reduction in their welfare if their productivity gains are much lower than those of workers from the other community. Regardless of the elasticity of labor demand, the earnings gap between workers belonging to different communities can go up with an exogenous increase in the incomes of capitalists, simply because the effect of such an increase on the productivity of workers in community A can be different from those in community B.

The outline of the paper is as follows. Section 2 sets up the basic framework. Section 3 establishes the general equilibrium in the model. Section 4 conducts comparative statics with

[8] These "social network" benefits to individuals are well discussed and documented. See, for example, Loury (2002), Ligon, Thomas and Worrall (2002) and Fachamps and Minten (1995).

[9] That socialization with people from diverse economic backgrounds, especially in one's youth, may have a positive impact on one's own productivity is an idea that underlies, at least in part, deliberate attempts by many governments to encourage class heterogeneity within student populations in public educational institutions, and to subsidize housing for the poor in affluent cities and neighborhoods. The understanding that individuals from certain communities, particularly those born into ethnic, religious or racial minority groups, may have less opportunity for such beneficial social interaction within civil society often drives measures to make state institutions more 'inclusive', in terms of ethnic, religious, racial or caste composition, in their recruitment practices. The same understanding, somewhat paradoxically, often also motivates and justifies coercive state measures designed to force minority groups to 'integrate', i.e. adopt ethnic, religious, caste, cultural or language practices of the majority group.

respect to an exogenous increase in capitalists' incomes and establishes the basic proposition of the paper. Section 5 concludes.

2. The Framework

The picture we wish to paint is that of a society of communities. Each community consists of two classes of agents: capitalists (those who own capital stock) and workers (those who own only their labor power). Production takes place according to a constant returns to scale aggregate production function, labor and capital markets are competitive, and there is no discrimination on the basis of community affiliation in the labor market. What defines community is access to a public good. All agents consume a private good, a community specific public good, and leisure. They face a common wage rate per efficiency unit in the labor market, and are endowed with one time unit of labor. The number of efficiency units of labor that are generated from one time unit, however, depends on, and increases with, the amount of the public good specific to the community to which the agent belongs. All agents choose their labor supply and allocation of their total income between the private good and the public good of their own community in Cournot fashion.

For a formalization of this picture, consider a society consisting of n individuals, $n \geq 4$. The set of individuals is $N = \{1,...,n\}$. There is an exogenously given amount of capital (or land) stock in the economy, K. The society contains two classes, rich (R) and poor (P). The rich are capitalists (or landlords): ownership of the total capital stock, K, is shared equally among all R agents in society. The poor are workers: no P agent owns any capital. The numbers of rich and poor individuals are, respectively, n_R and n_P. Each class contains at least 2 agents, and, obviously, $n_R + n_P = n$.

Society is divided into two *communities*, A and B. Each community consists of some R individuals, and some P individuals. The number of P individuals belonging to community A is n_{PA}, while the number of P individuals belonging to community B is n_{PB}. Thus, $n_P = n_{PA} + n_{PB}$. Similarly, $n_{RA} + n_{RB} = n_R$.[10]

[10] We can allow capital ownership of R individuals to vary across communities without affecting the results.

We first model the production side of our economy.

A. Production and labor demand:

Our society produces a single good, by means of competitive capital and labor markets, according to the constant returns to scale aggregate production function:

$$Q = Q(K, l),$$

where K is the total (exogenously given) amount of capital stock in the economy, and l is the total amount of labor supplied, measured in efficiency units (to be determined endogenously as an equilibrium outcome); $Q_K, Q_l > 0, Q_{KK}, Q_{ll} < 0, Q_{Kl} > 0$, $Q_{Kll} \leq 0$. Thus, if l^* is the amount of labor supplied in equilibrium, then each R agent earns the amount $\left[\frac{K}{n_R} Q_K\left(K, l^*\right)\right]$ as rental income, while the wage rate per efficiency unit of labor is: $\left[Q_l\left(K, l^*\right)\right]$. The inverse demand function for labor is given by:

$$w = d\left(K, l^D\right) \equiv Q_l\left(K, l^D\right),$$

so that, given capital stock K, if firms employ l^D efficiency units of labor, the wage prevailing in the labor market must be $d\left(K, l^D\right)$, $d_{l^D} < 0$, $d_K > 0$. We assume that the wage function is bounded from above by some $\overline{w}$, i.e., $Q_l(K,0) = \overline{w}$, where $\overline{w} \in \Re_{++}$.

Our primary interest lies in isolating and investigating the consequences of an increase in earnings of R individuals, irrespective of the origins of this increase. We thus assume that, in addition to her rental income, each R agent belonging to community k has some non-negative endowment of the single good in our model, D_k, so that her total income is $\left[D_k + \frac{K}{n_R} Q_K\left(K, l\right)\right]$. To fix ideas, one can think of the economy receiving an amount $\left[n_{RA} D_A + n_{RB} D_B\right]$ of the good from outside the country as foreign inflow, or "manna from heaven", which is divided between R agents of the two communities in some fashion, with all R agents within a community receiving identical amounts. No poor agent in society receives such

an exogenous payment.[11] We shall analyze the comparative static properties of our model by looking at its equilibrium response to increases in D_k.

All agents live for one period, and all capital stock is used up in the process of production, so that total consumption is simply total output plus total foreign inflows. We thus abstract from dynamic investment considerations.

B. Consumption and labor supply:

Each individual consumes leisure, a private good and a public good that is specific to the community she belongs to, and is endowed with one unit of labor time. Thus, individuals purchase the single commodity that is produced in the economy, and can put it to alternative uses: some uses are characterized by rivalry in consumption, other methods of use entail non-rivalrous consumption within the community. For any individual $i \in N$, belonging to community k, $k \in \{A, B\}$, amounts of leisure, the private good and the public good consumed are, respectively, l_i, x_i and y^k. Thus, individuals belonging to community A only have (costless) access[12] to the public good specific to their community, and similarly for individuals belonging to community B.[13]

For an agent i, belonging to community k, preferences are given by:

$$u = g\left(x_i, y^k\right) - L_i,$$

where $L_i = (1 - l_i)$ is the amount of labor, in time units, supplied by the agent, g is increasing, twice continuously differentiable and strictly quasi-concave in its arguments.[14]

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[11] This is only for convenience of exposition. We can generalize the treatment by allowing the poor to receive some foreign inflow as well.

[12] The assumption of costless access for community members is for notational simplicity. We can allow the possibility that community members have to pay a fixed amount, a 'membership fee', without changing our results.

[13] The public goods specific to the two communities can be considered identical in character, as, for example, might be the case with civic amenities in two cities, but residents of one city would choose not to access the amenities of another city because travel or relocation costs are two high. Alternatively, in cases of, say, religious or ethnic communities, individuals born into community A would not derive any utility from the public good, say temples, definitive of the other community.

[14] The assumption of utility functions being identical across communities is for notational simplicity. We can work with the more general formulation that, for an individual i belonging to community k, preferences are given by $g^k\left(x_i, y^k\right) - \rho_k L_i$, with $\rho_k > 0$, thereby allowing preferences to vary across communities, without affecting our results.

Our first major assumption regarding preferences is the following.

A1. (i) For all $y_k \in \Re_{++}$, $\underset{x_i \to 0}{Lt} \frac{g_x\left(x_i, y^k\right)}{g_y\left(x_i, y^k\right)} = \infty$, and for all $x_i \in \Re_{++}$, $\underset{y^k \to 0}{Lt} \frac{g_x\left(x_i, y^k\right)}{g_y\left(x_i, y^k\right)} = 0$.

(ii) $g_{xx}, g_{yy} < 0, g_{xy} \geq 0$.

Remark 2.1. A1(i) ensures that all agents must consume positive amounts of both private and public goods in an equilibrium. A1(ii) implies $\frac{g_x\left(x_i, y^k\right)}{g_y\left(x_i, y^k\right)}$ is increasing in y^k and decreasing in x_i. Together, A1(i) and A2(ii) ensure that, for an agent with the utility function g, both the private and public goods are normal goods in the standard sense.

The productivity of an hour's labor provided by an individual depends on the amount of the public good that she can consume. More specifically, when an agent *i*, belonging to community *k*, provides L_i amount of market labor in time units, the amount of market labor provided by her in efficiency units is given by:

$$e_i = f\left(y^k\right) L_i, \tag{2.1}$$

where f is increasing, twice continuously differentiable and strictly convex, with $\left[f(0) = 1\right]$, $\left[f'(0) \in \left(0, \frac{1}{\overline{w}}\right)\right]$ and $\left[\underset{y^k \to \infty}{Lt} f\left(y^k\right) = \boldsymbol{d}\right]$, such that $\boldsymbol{d} \in (1, \infty)$. Thus, no agent can have a labor endowment in efficiency units more than $\boldsymbol{d}$, which is finite and greater than unity. The maximum possible marginal gain in labor efficiency, $f'(0)$, is also finite, and such marginal gain decreases with an increase in the community's stock of the public good. It follows that society's labor supply can never be more than $\boldsymbol{d}n$. Define $\underline{w} = d(n\boldsymbol{d})$. Thus, the wage rate in the labor market must lie within the interval $\left[\underline{w}, \overline{w}\right]$. We shall assume that $\underline{w} > 0$.

Agents are differentiated, because of their community affiliations, in terms of their consumption of the (community-specific) public good. Consequently they may, possibly, vary in terms of their endowments of labor, when measured in efficiency units, even though each of them owns one time unit of labor. However, agents are integrated in the labor market, in that, every agent in society, regardless of community affiliation, faces a wage rate of w per *efficiency*

unit in the labor market, $w \in [\underline{w}, \overline{w}]$.[15] Thus, the wage rate per *time* unit of labor, faced by a member of community *k*, is simply $wf(y^k)$. Let the non-labor income of an individual belonging to class j and community k be I_{jk}. Since P agents do not own any capital stock, nor do they receive any foreign flows, $I_{PA} = I_{PB} = 0$; while $I_{RA}, I_{RB} \geq 0$.

Individuals take the market wage rate per efficiency unit, w, as given, and simultaneously choose their labor supply, as well as the allocation of their total incomes between the private good and the public goods of their respective communities.

Consider an agent *i* who belongs to class *j* and community *k*. Given any amount of the public good contributed by all other members of her community, y^k_{-i}, her own optimal consumption bundle is given by the solution to:

$$\underset{x_i, y^k, L_i}{Max} \; g(x_i, y^k) - L_i \tag{P1}$$

subject to the budget constraint:

$$x_i + y^k = I_{jk} + wf(y^k)L_i + y^k_{-i}, \tag{2.2}$$

and the additional constraint:

$$y^k \geq y^k_{-i}. \tag{2.3}$$

The net price effectively paid by an agent belonging to community *k*, for an additional unit of the public good of her community, is thus $[1 - wf'(y^k)L_i]$. Since $L_i \in [0,1]$, and, by assumption, $\left[f'(0) \in \left(0, \frac{1}{\overline{w}}\right)\right]$ and $f'' < 0$, this net price is always positive, but not more than 1. Let the minimum net price possible be $\underline{p}$. Clearly, $\underline{p} = (1 - \overline{w} f'(0))$, which implies $\underline{p} \in (0,1)$.

3. Equilibrium

A general equilibrium in our model must have three features. First, the labor market must be in equilibrium. Second, the capital market must be in equilibrium. Third, given the equilibrium wage rate, say w^*, and capitalists' equilibrium rental incomes

[15] In other words, while community identity and membership for individuals is not an act of choice, but something given, by birth, upbringing or some other historical factor, competitive employers do not discriminate on the basis of community identity in their hiring practices.

$I_{Rk}{}^{*} - D_k = \frac{K}{n_R} Q_K\left(K, d^{-1}\left(w^*\right)\right)$, $k \in \{A, B\}$, the total amount of labor supply in efficiency units generated by community k, in the Nash equilibrium of the game defined by (P1), (2.2) and (2.3) and corresponding to $\left\langle I_{Rk}{}^{*}, w^* \right\rangle$, say L_k^*, must be such that [$L_A^* + L_B^* = d^{-1}\left(w^*\right)$]. In this section, we address the questions of existence, uniqueness and characteristics of such a general equilibrium.

In Section 1, we identified the poor with workers. Our first step is to ensure that, in equilibrium, P individuals, regardless of their community affiliation, all turn out to be agents who provide market labor. To this end, we assume the following.

A2. $g_x\left(\overline{w}\boldsymbol{d}, 0\right) \geq \frac{1}{\underline{w}}$.

The following implication is then immediate.

Lemma 3.1. *Given A1 and A2, and given any* $w \in \left[\underline{w}, \overline{w}\right]$, *it must be the case that, for every P individual in society, the amount of labor, in time units, supplied in any Nash equilibrium corresponding to w is 1.*

Thus, all P agents, regardless of their community affiliations, will necessarily sell their entire endowments of labor power in any Nash equilibrium.[16]

Remark 3.2. It follows from Lemma 3.1 that, given A1 and A2, the amount of labor supplied in efficiency units in any labor market equilibrium must be at least n_P. Hence, if both capital and labor markets are in equilibrium, then, for any R agent of community k, $I_{Rk} - D_k \geq \left(\frac{K}{n_R}\right) Q_K\left(K, n_P\right)$.

We now take up the issue of labor supply by R agents. As mentioned in Section 1, by rich individuals we understand individuals whose primary source of income is non-labor earnings. In line with this characterization, we wish to ensure that, in equilibrium, R individuals do in fact turn out to be the idle rich, i.e., those who do not participate in the labor market, preferring, instead, to live off the rental income from their asset holdings, and foreign flows.

Consider the problem:

[16] Allowing P agents to sell only part of their labor endowment complicates the exposition, but does not add anything by way of insight.

$\underset{x_i, y^k}{Max}\, g(x_i, y^k)$ subject to:

$$x_i + py^k = r\,.$$

Let $V(r,p)$ be the indirect utility function corresponding to g, derived by solving this problem, in the standard way. Obviously, $V_r(r,p) > 0$. We assume the following.

A3. (i) For every $p \in \lfloor \underline{p}, 1 \rfloor$, (a) $V_{rr}(r,p) < 0$, and (b) $\underset{r\to\infty}{Lt}\, V_r(r,p) = 0, \underset{r\to 0}{Lt}\, V_r(r,p) = \infty$.

(ii) For every $k \in \{A, B\}$,

$$\left(\frac{K}{n_R}\right) Q_K(K, n_P) + D_k > max\left\{r \in \Re \mid \overline{w}\, dV_r(r,p) = 1 \;\; for\ some \;\; p \in \lfloor \underline{p}, 1 \rfloor\right\}.$$

Assumptions A1, A2 and A3, together, will suffice to ensure that, in equilibrium, our R agents will indeed turn out to be the idle rich, or rentier-capitalist coupon clippers.

Lemma 3.3. *Given A1 and A3,* $k \in \{A, B\}$, *any* $w \in [\underline{w}, \overline{w}]$, *any* $n_{Pk} \geq 0$, *and any* $I_{Rk} \geq D_k + \left[\frac{K}{n_R}\left(Q_K(K, n_P)\right)\right]$, *no R agent in community k will provide market labor in any Nash equilibrium corresponding to* $\langle I_{Rk}, w \rangle$.

Proof: See the Appendix.

We now proceed to address the question of contributions to the public goods. Consider a counterfactual community consisting only of R individuals belonging to community k, n_{Rk} in number, and suppose (a) $w \in [\underline{w}, \overline{w}]$, and (b) $I_{Rk} \geq D_k + \left[\frac{K}{n_R}\left(Q_K(K, n_P)\right)\right]$. By Lemma 3.3, these individuals will consume their entire endowments of leisure, and thus, spend only their non-labor earnings, I_{Rk} each, on the private and public goods. Let the amount of the public good of community k generated in the corresponding Nash equilibrium be given by $\alpha(I_{Rk}, n_{Rk})$.[17] It is easy to check that the function α is increasing in its arguments. We shall assume that $\alpha_{II} \leq 0$.

In light of Lemma 3.3, it then follows that, given A1 and A3, the Nash equilibrium level of the public good in our original, two-class, community k must be at least $\alpha(I_{Rk}, n_{Rk})$. It will be

[17] A1 suffices to guarantee the existence and uniqueness of such a Nash equilibrium. See Remark 2.1 and Bergstrom, Blume and Varian (1986).

exactly this amount if only R agents are contributory; it will be more than $\alpha(I_{Rk}, n_{Rk})$ if P agents are contributory as well. Our next step is to rule out the latter case.

A4. $\lim_{y^k \to \infty} \frac{g_x(\overline{w}\delta, y^k)}{g_y(\overline{w}\delta, y^k)} = \infty$.

A1 and A4 together yield:

there exists $\tilde{y} \in \Re_{++}$ such that: $\left[\frac{g_x(\overline{w}\delta, \tilde{y})}{g_y(\overline{w}\delta, \tilde{y})} = \frac{1}{1-\overline{w}\delta}\right]$.

Given A1(ii) (Remark 2.1), and any $w \in [\underline{w}, \overline{w}]$, it immediately follows that:

for all $y^k \geq \tilde{y}$, $\left[\frac{g_x(wf(y^k), y^k)}{g_y(wf(y^k), y^k)} > \frac{1}{1-wf(y^k)}\right]$. (3.1)

(3.1) essentially implies that, if capitalists of community k spend a sufficiently large amount on the public good defining their community, then workers of their community will spend nothing on the public good, preferring, instead, to completely free-ride on the capitalists. Our next assumption is simply that capitalists indeed spend such a large amount.

A5. For all $k \in \{A, B\}$, $\alpha\left(D_k + \frac{K}{n_R}(Q_K(K, n_P)), n_{Rk}\right) \geq \tilde{y}$.

Noting that A1 and Lemma 3.3 together imply that, given $I_{Rk} \geq D_k + \left[\frac{K}{n_R}(Q_K(K, n_P))\right]$, in a Nash equilibrium, $y^k \geq \alpha(I_{Rk}, n_{Rk})$, and using (3.1), we have the following result.

Lemma 3.4. *Given A1-A5, any* $k \in \{A, B\}$, *any* $I_{Rk} \geq D_k + \left[\frac{K}{n_R}(Q_K(K, n_P))\right]$, *and any* $w \in [\underline{w}, \overline{w}]$, *in every Nash equilibrium corresponding to* $\langle I_{Rk}, w \rangle$,

(i) no P agent in k will contribute to her community's public good,

and

(ii) $y^k = \alpha(I_{Rk}, n_{Rk})$.

Remark 3.5. In light of the preceding discussion, it is easy to see that, given A1-A5, and given any $w \in [\underline{w}, \overline{w}]$, and any $I_{Rk} \geq D_k + \left[\frac{K}{n_R}(Q_K(K, n_P))\right]$, both existence and uniqueness of the Nash equilibrium must be satisfied for community k.

In our model, rental incomes of capitalists of a community determine the level of public good in that community. The latter, in turn, determines the productivity of workers in the community, and thus, aggregate labor supply (in efficiency units) in the society. Total labor supply, in turn, determines the marginal product of capital, and thereby, rental incomes of capitalists. We now turn to the problem of equilibrium determination of such rental income.

Consider the function:

$$Z(t, D_A, D_B) = \frac{K}{n_R}\left(Q_K\left(K, f\left(\alpha(t + D_A, n_{RA})\right)n_{PA} + f\left(\alpha(t + D_B, n_{RB})\right)n_{PB}\right)\right). \tag{3.2}$$

We have, from (3.2), (since $Q_{Kl}, \alpha_I, f' > 0$):

$$Z_t = \frac{K}{n_R}\left(Q_{Kl}\right)\left(f'\left(\alpha(t + D_A, n_{RA})\right)\alpha_I(t + D_A, n_{RA})n_{PA} + f'\left(\alpha(t + D_B, n_{RB})\right)\alpha_I(t + D_B, n_{RB})n_{PB}\right) > 0, \tag{3.3}$$

and,

$$\text{for every } k \in \{A, B\},\ Z_{D_k} = \frac{K}{n_R}\left(Q_{Kl}\right)\left(f'\left(\alpha(t + D_k, n_{Rk})\right)\alpha_I(t + D_k, n_{Rk})n_{Pk}\right) > 0. \tag{3.4}$$

Since, by assumption, $Q_{Kll} \le 0, f'' < 0, \alpha_{II} \le 0,$, it follows from (3.3) that:

$$Z_{tt}, Z_{tD_k} < 0. \tag{3.5}$$

Note now that $Z(0,0,0) = \frac{K}{n_R}\left(Q_K\left(K, n_P\right)\right) > 0$. Let $\hat{t} = \left[\frac{K}{n_R}\left(Q_K\left(K, n_P\right)\right)\right]$. Then, since $D_A, D_B \ge 0$, (3.3)-(3.4) together imply:

$$Z(\hat{t}, D_A, D_B) > \hat{t}. \tag{3.6}$$

We assume the following.

A6. $Z_t(\hat{t}, 0, 0) < 1$.

In light of (3.3) and (3.6), then, the next result is immediate.

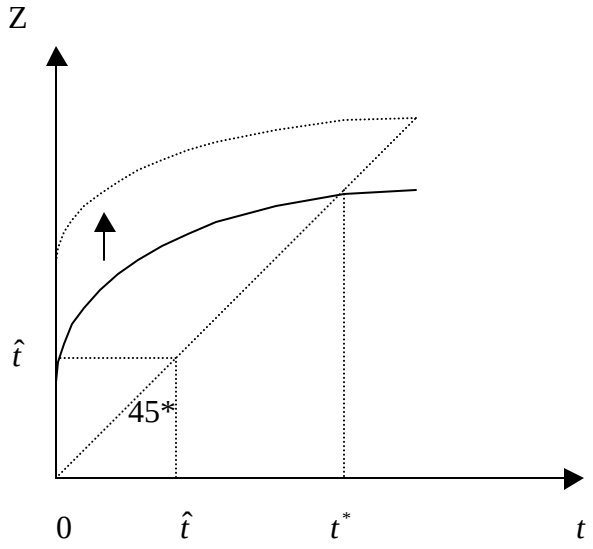
Lemma 3.6. *Let* $Z(t, D_A, D_B)$ *be defined by (3.2), and let* $\hat{t} = \left[\frac{K}{n_R}\left(Q_K\left(K, n_P\right)\right)\right]$. *Given A6 and* (D_A, D_B),

(i) there exists a unique $t^* \in \Re_+$ *such that: (a)* $Z(t^*, D_A, D_B) = t^*$, *(b) for all* $t \in [0, t^*)$, $Z(t, D_A, D_B) > t$, *and (c) for all* $t > t^*$, $Z(t, D_A, D_B) < t$;

(ii) furthermore, $t^* > \hat{t}$, *and, for every* $k \in \{A, B\}$, $t^*_{D_k} > 0$.

Lemma 3.6 is illustrated in Figure 1 below. The arrow shows how the Z schedule moves up with an increase in D_k.

Figure 1



Our last assumption is required to ensure that, given the amounts of foreign flows received by R agents, D_A, D_B, the corresponding value of $t^*(D_A, D_B)$ can be achieved as rental income, given the capital stock, K, in the economy.

A7. $t^*(D_A, D_B) < \dfrac{K}{n_R} Q_K(K, \delta n_P)$.

We are now ready to address the issue of general equilibrium in our economy. Suppose A1-A7 are all satisfied. Since $Q_{Kl} > 0$, in light of Lemma 3.6, A6-A7 imply:

$$[n_P < f(\alpha(t^* + D_A, n_{RA}))n_{PA} + f(\alpha(t^* + D_B, n_{RB}))n_{PB} < \delta n_P].$$

Then, $[w^* = d(f(\alpha(t^* + D_A, n_{RA}))n_{PA} + f(\alpha(t^* + D_B, n_{RB}))n_{PB})]$ is well-defined. Consider w^*. Given A1-A5, by Lemmas 3.1, 3.3 and 3.4, if the rental income of every R individual is t^*, then

labor supply from community k, in the Nash equilibrium corresponding to $\langle t^* + D_k, w^* \rangle$, is $f\left(\alpha\left(t^* + D_k, n_{Rk}\right)\right)n_{Pk}$; thus, total labor supply is $\left(f\left(\alpha\left(t^* + D_A, n_{RA}\right)\right)n_{PA} + f\left(\alpha\left(t^* + D_B, n_{RB}\right)\right)n_{PB}\right)$. Hence, w^* constitutes an equilibrium wage rate in the labor market if the rental income of every R individual is t^*. Given the employment at this equilibrium, $\left(f\left(\alpha\left(t^* + D_A, n_{RA}\right)\right)n_{PA} + f\left(\alpha\left(t^* + D_B, n_{RB}\right)\right)n_{PB}\right)$, capital market equilibrium indeed yields a rental income of t^* per R individual in community k. Hence, a general equilibrium exists in our economy. Lemma 3.6 (i) also implies that this general equilibrium must be unique (and stable).

We now combine Lemma 3.1, Lemma 3.3, Lemma 3.4 and Lemma 3.6 to present the following proposition.

Proposition 3.7. *Given A1-A7, there exists a unique* $w^* \in \left[\underline{w}, \overline{w}\right]$ *such that labor supply generated by community A in the Nash equilibrium corresponding to* $\langle t^* + D_A, w^* \rangle$, *and that generated by community B in the Nash equilibrium corresponding to* $\langle t^* + D_B, w^* \rangle$, *together add up to exactly* $d^{-1}\left(w^*\right)$, *where* $t^* = \frac{K}{n_R} Q_K\left(K, d^{-1}\left(w^*\right)\right)$. *Furthermore, these community-specific Nash equilibria are such that:*

(i) $d^{-1}\left(w^*\right) = \left\lfloor f\left(y_A^*\right)n_{PA} + f\left(y_B^*\right)n_{PB} \right\rfloor$, *where* y_A^*, y_B^* *are the amounts of the public goods specific to communities A and B, respectively, generated in the Nash equilibria of the respective communities;*

(ii) *no R agent enters the labor market,*

(iii) *every P agent belonging to community k supplies* $f\left(y_k^*\right)$ *efficiency units of labor,*

(iv) *no P agent contributes to her community's public good,*

(v) *for all* $k \in \{A, B\}$, $y_k^* = \alpha\left(t^* + D_k, n_{Rk}\right)$,

and

(vi) w^* *decreases in* D_A, D_B.

We can now proceed to address the issue of comparative statics.

4. Comparative Statics

Consider an exogenous increase in D_k, i.e., the amount of foreign flow to capitalists, provided in the form of the single commodity in our economy. Since there is no investment in our model, capitalists will consume all such additional income, the increment in demand being supplied from abroad. Consequently, there will be no change in the demand for labor schedule. It follows that, in the standard framework, where workers' labor supply does not depend on capitalists' consumption, workers' earnings will remain invariant. Thus, additional income for capitalists will lead to a Pareto improvement. What happens in our framework?

Proposition 3.7 (vi) implies that, if foreign flow incomes of capitalists increase even in just one community, then the equilibrium wage rate per efficiency unit of labor must fall. Since the labor demand schedule remains unchanged, this implies that aggregate labor supply, in efficiency units, must rise. Consequently, rental incomes of *all* capitalists must rise as well. Thus, a prior increase in the incomes of capitalists even in one community sets in motion community and market processes that generate additional gains for every capitalist, regardless of community affiliation. Capitalists gain as a class, by virtue of their ownership of capital assets. What happens to the wage income, and welfare, of an individual worker?

Let aggregate employment in the initial equilibrium be l^*, total labor supplied by community k be l_k^*, and the wage rate w^*. Then, by Proposition 3.7((ii) and (iii)), the equilibrium wage income of a P individual, belonging to community k, is given by:

$$M_{Pk}^* = w^* f\left(y_k^*\right) = \frac{l_k^* w^*}{n_{Pk}} . \tag{4.1}$$

Given any community k, let –k denote the other community. Then, (4.1) yields:

$$\text{for every } j \in \{A,B\},\ \frac{\partial M_{Pk}^*}{\partial D_j} = \frac{w^*}{n_{Pk}}\left[\frac{\partial l_k^*}{\partial D_j}\right]\left[1 - \frac{q_k^*}{h^* \Delta_k^*}\right],$$

$$\text{where: } h^* = -\frac{w^*}{l^* d'\left(l^*\right)},\ q_k^* = \left[\frac{l_k^*}{l^*}\right] \text{ and } \Delta_k^* = \left[\frac{\frac{\partial l_k^*}{\partial D_j}}{\frac{\partial l^*}{\partial D_j}}\right] . \tag{4.2}$$

Note now that Proposition 3.7((ii), (iii) and (v)) together yield $\left[\frac{\partial I_k^*}{\partial D_j}\right]$, $\left[\frac{\partial I_{-k}^*}{\partial D_j}\right]>0$. Using (4.2), we thus have the following.

Lemma 4.1. *For every* $j \in \{A,B\}$, $\frac{\partial M_{Pk}^*}{\partial D_j} < 0$ *if and only if* $[\boldsymbol{h}^* \Delta_k^* < \boldsymbol{q}_k^*]$.

Since $\Delta_k^* \in (0,1)$, by Lemma 4.1, a small increase in foreign income of every capitalist in at least one community must necessarily reduce the income of every worker in community k if the elasticity of labor demand at the initial equilibrium is less than the share of community k in total labor supply. If the elasticity of labor supply at the initial equilibrium is less than the share of either community in total labor supply, i.e., if $\boldsymbol{h}^* < \min\{\boldsymbol{q}_A^*, \boldsymbol{q}_B^*\}$, then a small increase in foreign income of every capitalist in at least one community must necessarily reduce the incomes of *all* workers in society. More generally, if labor demand is inelastic, then such an increase must *necessarily* reduce the earnings of workers belonging to *at least one* community. This is possible (though not necessary) even when labor demand is elastic. Intuitively, this will happen if the productivity of workers in one community goes up much less than the productivity of their counterparts in the other community (Δ_k^* is low). This in turn is likely when (a) capitalists of the other community, -k, receive the wealth increase, and/or (b) community k already has a large stock of the public good (f"<0). [18]

Since P individuals are non-contributory to the public good, it must be the case that, if any given increase in supply of the public good is matched by a reduction in wage income of the same amount, then the welfare of these individuals must fall. Thus, from (4.2), we have a sufficient condition for a reduction in welfare of P agents belonging to community k:

$$\left[1 + w^* f'(y_k^*)\left[1 - \frac{\boldsymbol{q}_k^*}{\boldsymbol{h}^* \Delta_k^*}\right]\right] < 0. \qquad (4.3)$$

It is evident that (4.3) will indeed hold if either the elasticity of labor demand, or the share of community k in additional labor supply, is sufficiently low.

We summarize our results in the form of the following Proposition.

[18] In this case, however, workers in at least one community must see their incomes rise.

Proposition 4.2. *Suppose A1-A7 are satisfied. Let the share of community k in total labor supply in efficiency units at the initial labor market equilibrium be* $\boldsymbol{q}_k^*$*, and let the elasticity of labor demand at that equilibrium be* $\boldsymbol{h}^*$*. Consider a small increase in foreign income of every R agent in at least one community, and let* Δ_k^* *denote the share of community k in the additional labor supply generated in the equilibrium subsequent to this increase. There exists* $\hat{\boldsymbol{h}} \in (0, \boldsymbol{q}_k^*)$ *such that, if* $\boldsymbol{h}^* \Delta_k^* < \hat{\boldsymbol{h}}$*, then the welfare of every P agent in community k must be lower in the new equilibrium.*

Lastly, how does an increase in capitalists' incomes affect inequality between workers belonging to different communities? Consider the special case where $n_{RA} = n_{RB} = \tilde{n}$. From (4.1), we have:

$$\frac{\partial M_{PA}^*}{\partial D_B} = f(y_A^*)\frac{\partial w^*}{\partial D_B} + w^* f'(y_A^*)\boldsymbol{a}_I(t^* + D_A, \tilde{n})t_{D_B}^*. \tag{4.4}$$

$$\frac{\partial M_{PB}^*}{\partial D_B} = f(y_B^*)\frac{\partial w^*}{\partial D_B} + w^* f'(y_B^*)\boldsymbol{a}_I(t^* + D_B, \tilde{n})(1 + t_{D_B}^*). \tag{4.5}$$

$$\frac{\partial M_{PA}^*}{\partial D_B} + \frac{\partial M_{PA}^*}{\partial D_A} = f(y_A^*)\left(\frac{\partial w^*}{\partial D_B} + \frac{\partial w^*}{\partial D_A}\right) + w^* f'(y_A^*)\boldsymbol{a}_I(t^* + D_A, \tilde{n})(t_{D_B}^* + t_{D_A}^* + 1). \tag{4.6}$$

First note that, if, in the initial equilibrium, A workers earn more than B workers, then it must be that $D_A > D_B$. Since, by assumption, $f'' < 0, \boldsymbol{a}_{II} \leq 0$, it follows that, if $M_{PA}^* > M_{PB}^*$, then an increase in foreign incomes of R agents in B alone increases the incomes of P agents in B more than their counterparts in A, and thereby reduces income inequality among workers. Paradoxically, if initially P agents in A earn significantly less than their counterparts in B, then the exogenous increase in the incomes of R agents in B can increase the incomes of P agents in A more than those in B, and thereby, once again, reduce earnings inequality across communities among the poor. On the other hand, in intermediate situations where initially P agents in A earn somewhat less than those in B, such an increase will aggravate inequality among the poor. If all R agents receive identical increases in foreign income, then those P agents who were initially

better off must gain less, or lose more; thus, there will be an unambiguous reduction in income inequality amongst workers across communities.[19]

5. Conclusion

In this paper, we have shown how a prior increase in the incomes of capitalists, or, more generally, rich individuals, sets in motion community and competitive market processes which subsequently (a) further increase capitalists' incomes, (b) can reduce workers' earnings as well as welfare, and (c) systematically influence earnings differentials between workers belonging to different communities. In doing so, we have presented an integrated, general equilibrium, analysis of how the presence of vertical ties of 'community' between sections of workers and sections of capitalists can critically affect distribution of income between capitalists as a class and workers as a class, as well as between workers belonging to different communities.

We have modeled 'community' in terms of a group-specific public good, produced according to the standard, additive, technology. This however is purely for convenience of exposition. More general specifications of public good technology, such as the CES functional form (Cornes 1993) and 'impure' public goods, where agents derive utility not only from the total amount of the public good, but from the size of their own contributions as well, are compatible with our analysis. In assuming a one-period framework, we have abstracted from issues surrounding investment. In a more general dynamic setting, one would expect an increase in incomes of capitalists to be partially reflected in increased investment, and thereby, additional labor demand.

Our central point, however, is that when community intersects with class, the standard analysis of distribution is modified significantly, and in interesting ways. We look forward to further explorations of this intersection in the economics literature.

[19] These claims will hold in the more general case, with different numbers of R agents in the two communities, as well so long as $M^{*}_{Pk} > M^{*}_{P(-k)}$ implies $\boldsymbol{a}'\left(t^{*} + D_{k}, n_{Rk}\right) \leq \boldsymbol{a}'\left(t^{*} + D_{-k}, n_{R(-k)}\right)$.

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Appendix

To prove Lemma 3.3, we first note the following.

Lemma N1. *Given A1, any* $k \in \{A, B\}$, *any* $I_{Rk} \geq 0$ *and any* $w \in [\underline{w}, \overline{w}]$, *every R agent must contribute a positive amount to the public good of her own community in any Nash equilibrium corresponding to* $\langle I_{Rk}, w \rangle$.

Proof of Lemma N1.

Suppose a Nash equilibrium exists corresponding to $\langle I_{Rk}, w \rangle$. It is easy to check that at least one class of agents must always contribute to the public good of the community in a Nash equilibrium. Hence, to establish the claim above, we only need to show that, if P agents are contributory in the Nash equilibrium, then R agents must be contributory in the Nash equilibrium as well. Suppose not. Let x_{Rk}, x_{Pk} be the private consumptions of R and P individuals, respectively, in community k. Then, since $\frac{g_x}{g_y}$ is decreasing in x_i by A1(ii) (Remark 2.1), it must be the case that, in the Nash equilibrium, $x_{Rk} < x_{Pk}$. In an equilibrium, however, for every individual, $wfu_x \leq 1$. If $x_{Rk} < x_{Pk}$ in equilibrium, then, given $u_{xx} < 0$ (A1(ii)), it must be the case that $wf(y^k)u_x(x_{Pk}, y^k) < 1$ in equilibrium. But this implies that P agents provide 0 market labor. As $I_{Pk} = 0$, this in turn implies that P agents cannot contribute to the community's public good, a contradiction which establishes our claim.

Proof of Lemma 3.3.

Let the amount of the public good of community k, generated in the Nash equilibrium be y^*, the amount of private good consumed by an R individual belonging to k, *i*, be x_i^* and the amount of labor supplied by this individual be L_i^*. Since A1 holds by assumption, by Lemma N1, R individuals in k must contribute to the public good of k. Then the Nash equilibrium is given as the solution to the problem (P1) subject to (2.2) alone. Given L_i^*, $\langle x_i^*, y^* \rangle$ is defined as the solution to the following pair of equations:

$$\frac{g_x\left(x_i^*, y^*\right)}{g_y\left(x_i^*, y^*\right)} = \frac{1}{1 - wf'\left(y^*\right)L_i^*}, \tag{N1}$$

and

$$x_i^* + y^* = I_{Rk} + wf\left(y^*\right)L_i^* + y_{-i}^*. \tag{N2}$$

Let $p^* = \left(1 - wf'\left(y^*\right)L_i^*\right)$. The budget constraint (N2) above can then be rewritten as:

$$x_i^* + p^* y^* = I_{Rk} + y_{-i}^* + wL_i^* y^* \left(\frac{f\left(y^*\right)}{y^*} - f'\left(y^*\right) \right) = r^*. \tag{N3}$$

It follows that $\left\langle x_i^*, y^* \right\rangle$ constitutes the solution to:

$$\underset{x_i, y^k}{Max} \; g\left(x_i, y^k\right) \text{ subject to: } x_i + p^* y^k = r^*,$$

which implies:

$$g_x\left(x_i^*, y^*\right) = V_r\left(r^*, p^*\right). \tag{N4}$$

Note now that, since $f'' < 0$, (N3) implies $r^* > I_{Rk}$. Then, since $p^* \in \left[\underline{p}, 1\right]$, using A3 and (N4), we have:

$$g_x\left(x_i^*, y^*\right) < \frac{1}{\overline{w}\boldsymbol{d}}.$$

Since $wf\left(y^*\right) < \overline{w}\boldsymbol{d}$, we thus get:

$$g_x\left(x_i^*, y^*\right) < \frac{1}{wf\left(y^*\right)}. \tag{N5}$$

(N5) however implies $L_i^* = 0$. ◊