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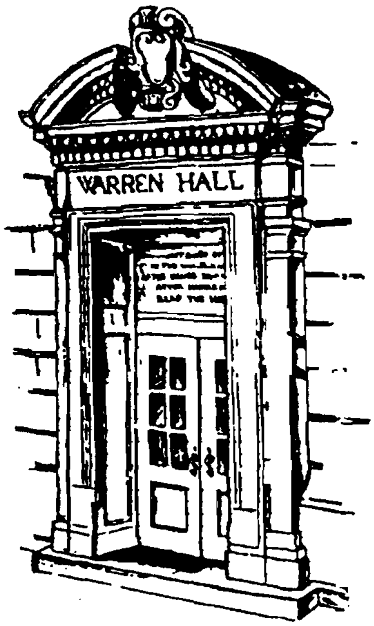

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DEMAND SYSTEMS FOR ENERGY FORECASTING: PRACTICAL CONSIDERATIONS FOR ESTIMATING A GENERALIZED LOGIT MODEL

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and
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Abstract

Generalized Logit models of demand systems for energy and other factors have been shown to work well in comparison with other popular models, such as the Almost Ideal Demand System and the TransLog model. The main reason is that the derived price elasticities are robust when expenditure shares are small, as they are for electricity and fuels. A number of different versions of the Generalized Logit model have been applied in the literature, and the primary objective of the paper is to determine which one is the best. Using annual data for energy demand in the USA at the state level, the final model selected is similar to a simple form that was originally proposed by Considine. A second objective of the paper is to demonstrate that the estimated elasticities are sensitive to the units specified for prices, and to show how price scales should be estimated as part of the model.

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Demand Systems for Energy Forecasting: Practical Considerations for Estimating a Generalized Logit Model

Weifeng Weng and Tim Mount

I. Introduction

Generalized Logit models (GL) have been used in a number of different application to estimated demand systems for energy. Recently, Rothman, Hong and Mount have shown that a GL model of consumer demand performed much better than the popular Almost Ideal Demand System, proposed by Deaton and Muellbauer (DM), or the TransLog model, proposed by Christensen, Jorgenson and Lau (TL). This analysis was based on United Nations data for a cross-section of 53 different countries. Although all three models gave similar estimates of price elasticities at the mean of the sample, the economic consistency of the DM and TL models tended to breakdown when expenditure shares differed from the mean values. For a nine commodity system, the estimated demand equations were consistent with theory for only 9% and 26% of the countries using the DM and TL models, respectively. In contrast, the GL model gave consistent demand equations for 96% of the countries. The simplest explanation of why the GL model performed better is that the price elasticities in the DM and TL models are sensitive to situations in which some expenditure shares are close to zero. This issue is discussed more fully in Section II.

Reasons for using a GL model are not limited to judging its relative performance with other models. The structure of the GL model also enhances the types of analysis that can be conducted by making it possible to consider extreme situations which are not observed directly in the sample. For example, Dumagan and Mount show how a GL model of a demand system, which includes electricity, natural gas and oil, can be used to represent an all-electric customer who is not affected by changes in the prices of natural gas or oil. In this case, the issue is how price elasticities behave when some expenditure shares are zero.

The basic tradeoff between using a DM model of consumer demand (or a TL model of factor demand) and a GL model is that the structure of the GL model is more difficult to estimate. Hong has shown that a Generalized Barnett model, which was much harder to estimate than a GL model, also performs better than the DM and TL models using the data from 53 countries, but characteristics of the corresponding price elasticities were very difficult to interpret. Fortunately, the expressions for the price elasticities derived from a GL model are simple functions of the parameters and easy to interpret.

The main complication of estimating a GL model compared to the DM model, for example, is that weighting functions for prices must be specified to approximate the symmetry restrictions derived from economic theory. A variety of different parameterizations of the weighting function have been specified for GL models in the literature. The primary objective of this paper is to specify a general form of weighting function, and to determine which specific parameterization is supported best by the data. It should be noted that one parameterization gives price elasticities which are almost identical to the DM and TL models. A secondary objective of the paper is to introduce a new issue concerning how to scale the price variables. This issue is shown to affect the economic properties of the estimated elasticities in a significant way. The empirical results are presented in Section III and IV using annual data for energy demand in the USA at the state level. Separate GL demand systems are estimated for the Residential, Commercial and Industrial sectors.

II. Economic Properties of the Generalized Logit Model

This section describes the basic structure of a linear regression model which can be applied to both consumer demand and factor demand systems. Using this general form, the structures of the TL model of factor demand and the DM model of consumer demand are compared to the corresponding GL models. This comparison is used to identify reasons for preferring the properties of the price elasticities in a GL model. Since a factor demand system is simpler in structure than a consumer demand system, factor demand is discussed first and consumer demand is treated as an extension of the factor demand system.

1) Model Specification for Factor Demand

The general form of a demand system for n input factors can be written as a series of (n-1) linear regression equations:

$$y_i = \alpha_{i0} + \alpha_{i1}x_{i1} + \ldots + \alpha_{in-1}x_{in-1} + \beta_{i1}z_{i1} + \ldots + \beta_{im}z_{im} + e_i$$

$$= \alpha_{i0} + \sum_{j=1}^{n-1}\alpha_{ij}x_{ij} + \sum_{k=1}^{m}\beta_{ik}z_{ik} + e_i \qquad i = 1, 2, \ldots, n\text{-}1 \qquad [1]$$

where y_i is the dependent variable, x_{ij} a price variable, z_{ik} is a non-price variable (e.g. dummy variables for different locations), and e_i is a residual. Important restrictions on the price coefficients (α_{ij}) can be derived from economic theory. These restrictions tend to increase the efficiency of the estimation (i.e. reduce the standard errors of estimated coefficients) as well as ensure that the demand responses are consistent with theory.

a. The TransLog Demand System

One of the most widely used models of factor demand is the TL model developed by Christensen, Jorgenson and Lau. If C is the total cost of all input factors, p_i is the price of factor i, and q_i is the quantity of factor i, then the dependent variable in [1] is the expenditure share of factor i, and the price variables are the logarithms of price ratios:

$$y_i = w_i = p_i q_i / C; \qquad 1 \geq w_i \geq 0; \qquad \sum_{i=1}^{n} w_i = 1$$

$$x_{ij} = \log\left(p_j / p_n\right) \qquad j = 1, 2, \ldots, n-1$$

The main restrictions from economic theory imply symmetry of the price coefficients in the demand system ($\alpha_{ij} = \alpha_{ji}$ for all i and j). Using price ratios in x_{ij} ensures that the expenditure shares are not affected by pure inflation (the same proportional change in all prices). Economic theory also implies that the Hicksian price effects should be consistent with conditions for concavity of the cost and utility functions.

The standard (Hicksian) price elasticities (holding production fixed) have a relatively simple form, but they are functions of the expenditure shares (note that $\alpha_{in} = -\sum_{j=1}^{n-1} \alpha_{ij}$).

Cross-price

$$E_{ij} = \alpha_{ij}/w_i + w_j , \qquad \text{for all } i \neq j \qquad [2a]$$

Own-price

$$E_{ii} = \alpha_{ii}/w_i + w_i - 1 \qquad [2b]$$

The TL model is widely used, but there is a practical problem associated with it when an expenditure share w_i is close to zero. The value of the price elasticities are very sensitive to small changes of w_i when $w_i \rightarrow 0$, and if $\alpha_{ii} > 0$ (price inelastic), the own-price elasticity will violate economic logic by becoming positive. Since expenditure shares on fuels and electricity are often quite small, the TL model for energy demand is vulnerable to this problem. Thus, it is desirable to find an alternative model which is more suitable for situations when expenditure shares are small. The GL model is one way to solve the problem.

b. The Generalized Logit Demand System

The GL model is a simple modification of the standard regression equation in [1]:

$$y_i = \alpha_{i0} + \sum_{j=1, j\neq i}^{n} \alpha_{ij} x_{ij} - \sum_{j=1}^{n-1} \alpha_{nj} x_{nj} + \sum_{k=1}^{m} \beta_{ik} z_{ik} + e_i \qquad [3]$$

where

$$y_i = \log(w_i / w_n) \qquad i = 1, 2, \ldots, n\text{-}1,$$

$$x_{ij} = \theta_{ij}\log(p_j / p_i) \qquad \text{for all } j \neq i,$$

where θ_{ij} is a known function of w_i and w_j (discussed below). In both the TL and GL models, the restrictions $\alpha_{ij} = \alpha_{ji}$ are implied by economic theory.

Even though the form of the regression equations in the GL model is more complicated than it is for the TL model, the expressions for the Hicksian price elasticities for the GL are simple.

Cross-price

$$E_{ij} = \alpha_{ij}\theta_{ij} + w_j \quad \text{for all } i \neq j \qquad [4a]$$

Own-price

$$E_{ii} = -\sum_{k=1,k\neq i}^{n} \alpha_{ik}\theta_{ik} + w_i - 1 \qquad [4b]$$

Unlike the TL elasticities, the GL elasticities are not sensitive to small expenditure shares ($w_i \to 0$) if the form of θ_{ij} is specified appropriately.

2) Model specification for Consumer Demand

In models of factor demand, the logarithm of production can be included as an explanatory variable (one of the z_{ik}) if returns to the scale of production are not constant. In consumer demand, the equivalent assumption to constant returns to scale is that all income elasticities are unity. In most applications, this simplification is not realistic for consumer demand. When income elasticities are allowed to differ from unity, a problem arises in specifying a demand system that is consistent with economic theory because the Hicksian price elasticities are defined holding utility constant. Unlike the level of production in models of factor demand, utility is not observable and cannot be included as an explanatory variable. As an alternative, the observed level of income is included, and Marshallian price elasticities (holding income constant) are generally reported for consumer demand. Nevertheless, the important symmetry restrictions derived from economic theory are still defined in terms of the Hicksian price elasticities, and there is a simple relationship linking the Hicksian to the Marshallian elasticities:

$$E_{ij} = E_{ij}^m + w_j E_{iI}^m \qquad \text{for all i and j} \qquad [5]$$

where E_{ij}^m is the Marshallian price elasticity, and E_{iI}^m is the Marshallian income elasticity.

The model of consumer demand corresponding to the form of the TL model of factor demand is the DM model proposed by Deaton and Muellbauer. In the DM model, the logarithm of real (deflated) income is included (z_{i1} = log(I/QPI), where I is nominal income and QPI is a quadratic price index of $\log(p_i)$, i =1, 2, ..., n). In the GL Model of consumer demand, the logarithm of real income is also included but the deflator is the standard Stone Price Index (see appendix). The dependent variables, the price variables and the symmetry restrictions ($\alpha_{ij} = \alpha_{ji}$) have the same forms as the TL and GL models of factor demand, defined above. Under these specifications, the income elasticities have the following simple forms:

DM Model

$$E_{iI} = 1 + \beta_{i1}/w_i \qquad [6a]$$

GL Model

$$E_{iI} = 1 + \beta_{i1} - \sum_{k=1}^{n} \beta_{k1} w_k \qquad [6b]$$

The Hicksian price elasticities for consumer demand, corresponding to [3] and [4], can be written as follows (the Marshallian price elasticities can be derived using [5] and [6], but the expressions are relatively complicated):

DM Model

Cross-Price

$$E_{ij} = \alpha_{ij}/w_i + w_j + \beta_{i1}\beta_{j1}\log(I/QPI)/w_i \qquad \text{for } i \neq j \qquad [7a]$$

Own-Price

$$E_{ii} = \alpha_{ii}/w_i + w_i + \beta_{i1}^2 \log\left(I / QPI\right) / w_i - 1 \qquad [7b]$$

GL Model

Cross-Price

$$E_{ij} = \alpha_{ij}\theta_{ij} + w_j \qquad \text{for } i \neq j; \qquad [8a]$$

Own-Price

$$E_{ii} = -\sum_{j=1, j\neq i}^{n} \alpha_{ij}\theta_{ij} + w_i - 1 \qquad [8b]$$

The forms of the elasticities for the GL Model are identical for factor demand [4] and consumer demand [8], but the elasticities in the DM Model of consumer demand [7] have an additional term compared to the TL Model of factor demand [2] (this extra term is zero if real income is normalized to one at the point of evaluation). More importantly, the price elasticities for the DM Model still exhibit the undesirable property of being sensitive to small expenditure shares ($w_i \rightarrow 0$).

III. Functional Form of the Cross-Price Weight in the GL Model

The functional form of θ_{ik} in a GL model [3] is critical in determining the properties of the price elasticities. All elasticity expressions in [4] and [8] have been derived conditionally on the value of θ_{ij}. Using this simplification, any form of θ_{ij} must satisfy the property $w_i\theta_{ij} = w_j\theta_{ji}$ (for $i \neq j$) to ensure that the symmetry conditions required by economic theory are met.

When $w_j \rightarrow 0$, it is desirable to have the cross-price elasticity $E_{ij} \rightarrow 0$ because it implies that the demand for commodity i is unresponsive to changes in prices for commodities that are not purchased. The following forms of weighting scheme have been used in previous studies:

(i) $\theta_{ij} = w_j$. With this form, the cross-price elasticities in [4] and [8] are simple linear functions of w_j (Considine).

(ii) $\theta_{ij} = \dfrac{w_j^{1-\gamma}}{w_i^{\gamma}}$ where $0 \leq \gamma \leq 1$ is a parameter (Dumagan). This is a general form of the weight in (i), but the problem is that as $w_i \rightarrow 0$, $\theta_{ij} \rightarrow \infty$. If $\gamma = 1$, the cross-price elasticities in [4] have a similar form to the TL elasticities for factor demand [3], and therefore, this form of θ_{ij} can be used to test the appropriateness of the TL model.

(iii) $\theta_{ij} = w_i^{-\gamma}\, w_j^{1-\gamma}$ and $\gamma \leq 0$. This form is similar to the weight in (ii) but since $\gamma \leq 0$, the problem associated with $w_i \rightarrow 0$ is eliminated, and $\theta_{ij} = 0$ if $w_i = 0$ or $w_j = 0$.

(iv) $\theta_{ij} = w_i^{-\gamma}\, w_j^{1-\gamma}\,(1 - w_i - w_j)$ and $\gamma \leq 0$. The term $(1 - w_i - w_j)$ is added to (iii) to deal with complementarities (when any α_{ij} has a negative value large enough to make $E_{ij} < 0$). It ensures that all pairs of commodities must be substitutes, as theory requires, if any two commodities dominate expenditures ($w_i + w_j \rightarrow 1$). This form of θ_{ij} has been shown to perform well in comparison to other models (see Rothman, Hong and Mount).

(v) $\theta_{ij} = \dfrac{w_j}{(w_i + \delta)^{\gamma}(w_j + \delta)^{\gamma}}$ and $\delta > 0$. This form is closely related to (ii), but because $\delta > 0$, it does not explode when $w_i \rightarrow 0$. Forms (i), (ii) and (iii) can be viewed as special cases of (v). To ensure that θ_{ij} increases monotonically with w_j, the restriction $\gamma \leq 1 + \delta$ must hold.

For standard data sets which are characterized by substitution among commodities, forms (i), (ii) and (iii) are possible choices for θ_{ij}, and all three can be approximated by (v) when $\delta \rightarrow 0$. All three cases exhibit the desirable property $E_{ij} \rightarrow 0$ when $w_j \rightarrow 0$. However, the behavior of E_{ij} when $w_i \rightarrow 0$ is determined by the sign of γ, and this has implications for the economic logic of the model. One would expect that the elasticity for changing price j (for a given w_j) would be larger if w_i was small. Consequently, GL models with $\gamma > 0$ should be preferred. These results are illustrated in **Figure 1**, and the dashed lines ($\gamma = .2$) have the desirable properties in both **1a** and **1b**, but the dotted line ($\gamma = -.2$) in **1a** is counter intuitive. In the next section, the value of γ is estimated, and as a result, it will be possible to determine whether the data support the TL or DM model ($\gamma = 1$), Dumagan's GL model ($\gamma = .5$), Considine's GL model ($\gamma = 0$), or Rothman, Hong and Mount's GL model ($\gamma = -1$).

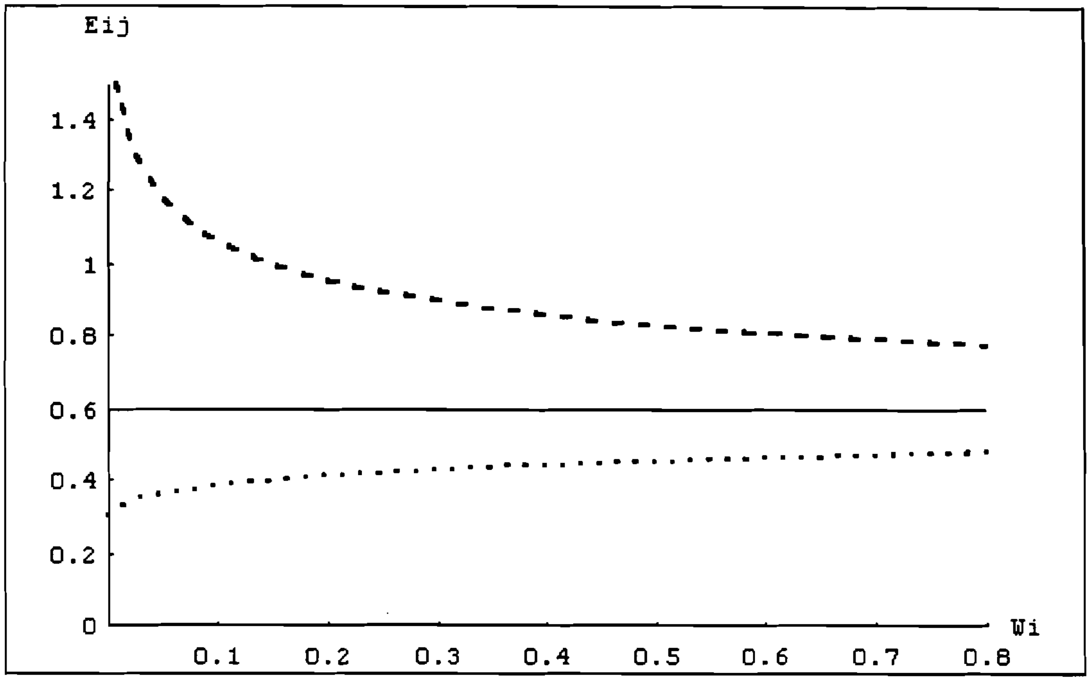

Figure 1a Hicksian cross-price elasticity E_{ij} (holding $w_j = .2$ constant)

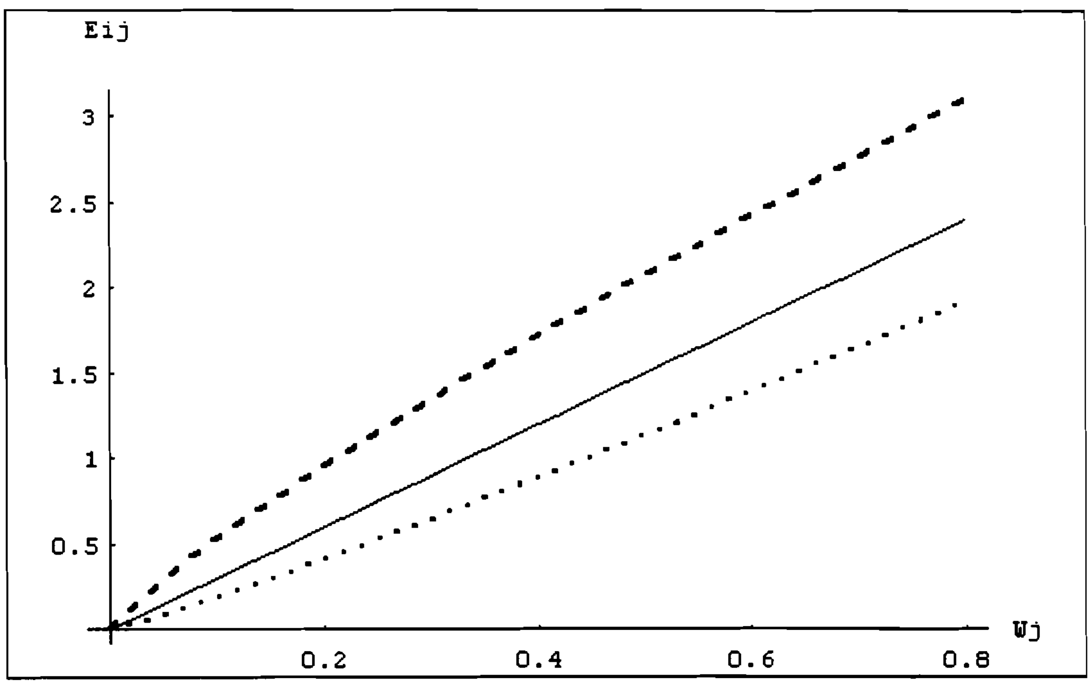

Figure 1b Hicksian cross-price elasticity E_{ij} (holding $w_i = .2$ constant)

$\gamma = -.2 \cdots\cdot$, $\gamma = 0$ ——, $\gamma = .2$ - - -

IV. Estimation and Price Scaling

In most previous applications of the GL Model, the form of the cross-price weight θ_{ij} is fully specified, including the values of the parameters γ and δ. The models estimated by Dumagan are exceptions. One objective of this paper is to allow the data to determine the best form of θ_{ij}. Before this can be done, however, another important question must be discussed. Why are the results sensitive to the units of prices in the GL model but not in TL or DM model? The reason is that in the TL or DM model changing the scale of any price results in offsetting changes of the intercepts. For the GL model, the presence of the cross-price weights makes the model more complicated. If $c_j > 0$ is a scalar (e.g. for normalizing price j to a given year), then $\alpha_{ij}x_{ij} = \alpha_{ij}\log((c_jp_j)/p_n) = \alpha_{ij}\log(c_j) + \alpha_{ij}\log(p_j/p_n)$ for the TL and DM models, and $\alpha_{ij}x_{ij} = \alpha_{ij}\theta_{ij}\log((c_jp_j)/p_i) = \alpha_{ij}\theta_{ij}\log(c_j) + \alpha_{ij}\theta_{ij}\log(p_j/p_i)$ for the GL model. The estimated price coefficients (α_{ij}) are unaffected in the TL and DM models by the choice of the price scalar (c_j), but these parameters should be estimated in the GL model because θ_{ij} is a variable. The alternative is to adopt a specific way to normalize the prices in a GL model, and the results are then conditional on that choice.

GL models with and without price scaling are estimated by using a range of values of γ and a specified value of $\delta = .005$. By varying γ from -1 to 1, the goodness of fit and the economic validity of the model can be determined and the effects of scaling prices assessed. First, any set of estimated price elasticities should be logical and consistent with economic theory (the Eigen values of the Hicksian price effects should be non-positive). For estimation, the best value of γ is selected by finding the smallest determinant of the variance-covariance matrix of residuals across equations, which corresponds to the best fit of the model. A finer grid of γ values is used close to the best fit (γ = 1, 0.5, 0.1, 0.05, 0.01, 0, -0.01, -0.05, -0.1, -0.5, -1).

The data used for estimation are a pooled cross-section of 48 states and an annual time-series from 1970 to 1992 (Residential) and 1978 to 1992 (Commercial & Industrial) using data from the Energy Information Administration and the Bureau of Economic Analysis (see Weng and Mount). A separate GL demand system is fitted for the Residential, Commercial and Industrial sectors, and the factors included are:

Residential	**Commercial**	**Industrial**
Electricity (EL)	Electricity (EL)	Electricity (EL)
Natural Gas (NG)	Natural Gas (NG)	Natural Gas (NG)
Oil (OL)	Oil (OL)	Oil (OL)
Other (OT)	Capital (CE)	Coal (CL)
	Labor (LB)	Capital (CE)
		Labor (LB)

Figure 2 gives a summary of the estimated own price elasticities in the Residential sector for different values of γ. The first observation is that the price elasticities, particularly for electricity in the model without price scaling, are sensitive to the value of γ. Demand is generally more price responsive at the extreme values of γ and less responsive for values close to zero. The second observation is that price scaling matters. In the model without price scaling, two of the cases violate economic logic and give price elasticities for electricity that are positive. In addition, the value of gamma with the best fit (γ = 0.01) is close to the invalid models (γ =.05 and .1). The model with price scaling is consistent with economic theory for all values of γ, and the corresponding price elasticities are much more robust to different values of γ. Thus, the model with price scaling is preferred, and this conclusion is also reached in the Commercial and Industrial sectors.

For $\delta = .005$, the best fit is obtained at $\gamma = .01$, $.075$ and $.05$ for the Residential, Commercial, Industrial sectors, respectively. In order to get an economically valid model for the Industrial sector, weak separability among fossil fuels was imposed. This type of simplification is easy to impose using the restrictions $\alpha_{ik} = \alpha_{jk}$ for all i, j that belong to the fossil-fuel group and all k that do not belong to that group.

The estimated values of γ in the cross-price weights for the three sectors are all positive and close to zero. The positive signs are consistent with the desired properties of the cross-price elasticities shown in **Figure 1**. The small values provide support for Considine's simple weighting scheme with $\gamma = 0$ versus the TL and DM models ($\gamma = 1$) or Dumagan's ($\gamma = .5$) and Rothman, Hong and Mount's ($\gamma = -1$) GL model. For comparative purposes, Considine's form of GL model ($\gamma = \delta = 0$) was estimated with price scaling. For the Industrial and Commercial sectors, the fit with $\gamma = \delta = 0$ was worse (4% and 3% increase in the error, respectively), and the model for the Commercial sector violated the concavity requirements of economic theory. For the Residential sector, the fit with $\gamma = \delta = 0$ was slightly better (0.1% decrease in the error) and the estimated price elasticities were virtually identical. Estimated elasticities for the three sectors with price scaling, $\delta = .005$ and an estimated γ are presented in the final section.

V. Results and Conclusions

The matrices of estimated elasticities for the three sectors using the GL model are presented in **Tables 1-3**. Since the models include a dynamic adjustment process (see appendix), estimates of both the short-run and long-run elasticities are given (the restrictions derived from economic theory and the expressions in Section II refer to the short-run elasticities). The reported elasticities use the data for New York State in 1991 as the base point. The estimated regression models are summarized in the appendix.

In general, own price elasticities for all sources of energy in all three sectors are price inelastic in the short-run and in the long-run. Cross-price elasticities between sources of energy are very small ($|E_{ij}| < 0.1$) in the Residential and Industrial sectors, but generally exhibit strong substitutability ($E_{ij} > 0.1$) in the Commercial sector. Complementary relationships between energy and non-energy exist in the Residential sector. Strong substitutability between energy factors and capital exists in both the Commercial and Industrial sectors. In contrast, all but one of the relationships between energy and labor are complementary. Labor and capital are strong substitutes in both the Commercial and Industrial sectors. One surprise in the Residential sector is that the long-run income elasticity for oil is highly negative. This may reflect a general movement away from using oil for heating homes during the eighties.

One issue about the form of the GL models deserves further elaboration, and this relates to the relatively large number of negative estimates of the cross price coefficients (α_{ij}). Given the chosen form of the cross price weight (θ_{ij}), the cross price elasticities for the GL model in [4] and [8] can be written:

$$E_{ij} = w_j\left[\alpha_{ij} / \left(\left(w_i + \delta\right)^{\gamma}\left(w_j + \delta\right)^{\gamma}\right) + 1\right] \qquad [9]$$

Consequently, the sign of α_{ij} determines whether E_{ij} increases more than ($\alpha_{ij} > 0$) or less than ($\alpha_{ij} < 0$) proportionally with w_j. If α_{ij} is sufficiently negative, then E_{ij} is also negative and the relationship is complementary.

Since the effect of α_{ij} in [9] is largest, for any given w_j and $\gamma > 0$, when $w_i = 0$, the discussion will focus on how E_{ij} changes as w_j increases from 0 to 1 holding $w_i = 0$. If $\alpha_{ij} > -\delta^{2\gamma}$, E_{ij} is always positive (substitute), and if $\alpha_{ij} < -(1+\delta)^{\gamma}\delta^{\gamma}$, E_{ij} is always negative (complement). For $-(1+\delta)^{\gamma}\delta^{\gamma} < \alpha_{ij} < -\delta^{2\gamma}$, $E_{ij} < 0$ for small w_j and $E_{ij} > 0$, as economic theory requires, if $w_j \rightarrow 1$. Given these desirable properties, one could consider using $-\delta^{2\gamma}$ as a lower bound for α_{ij}, but the problem with this restriction is that the magnitude of $|E_{ij}|$ is too small to capture strong complementary relationships. Hence, the presence of $\alpha_{ij} < -(1+\delta)^{\gamma}\delta^{\gamma}$ must be accepted as a possibility, and the economic logic of the model would only hold for a limited range of w_i and w_j in this case (i.e. $w_i + w_j < c < 1$). This may not be a serious limitation in many applications if it is unlikely that any pair of factors will dominate expenditures. It is interesting to note that capital and labor are found to be substitutes in **Tables 2** and **3**. Since these two factors generally account for almost 90% of total expenditures, finding complementary relationships would have posed a potential problem. If complementaries are important in a particular application, as they are in the cross country study reported by Rothman, Hong and Mount, then modifying form (v) of θ_{ij} could be considered using the same rationale adopted to convert form (iii) to form (iv) in Section III.

In summary, this paper has focused on two practical issues related to estimating GL models of demand. The first issue is price scaling, which has not been discussed before in the literature. The results show that the estimated models are sensitive to price scaling, and that the estimated elasticities are more robust and consistent with economic theory when price scales are estimated. We conclude that price scaling should be adopted when estimating GL models.

The second issue in the paper considers the form of the cross price weights (θ_{ij}) in the GL model. In this paper, a general form is chosen that can approximate a range of models discussed in the literature, including the popular DM and TL models. The key parameter (γ) that determines the form of θ_{ij} is estimated using a grid search over the range -1 to 1. The estimated values are positive and close to zero in all three sectors. These results do not support the form of elasticity derived from a DM or TL model ($\gamma = 1$), and are closest to the GL model proposed by Considine ($\gamma = 0$). They are consistent with the economic expectation of how price elasticities should change when expenditure shares change ($\gamma > 0$). In this respect, the results provide more evidence that the GL model can provide a satisfactory way to represent demand systems for energy.

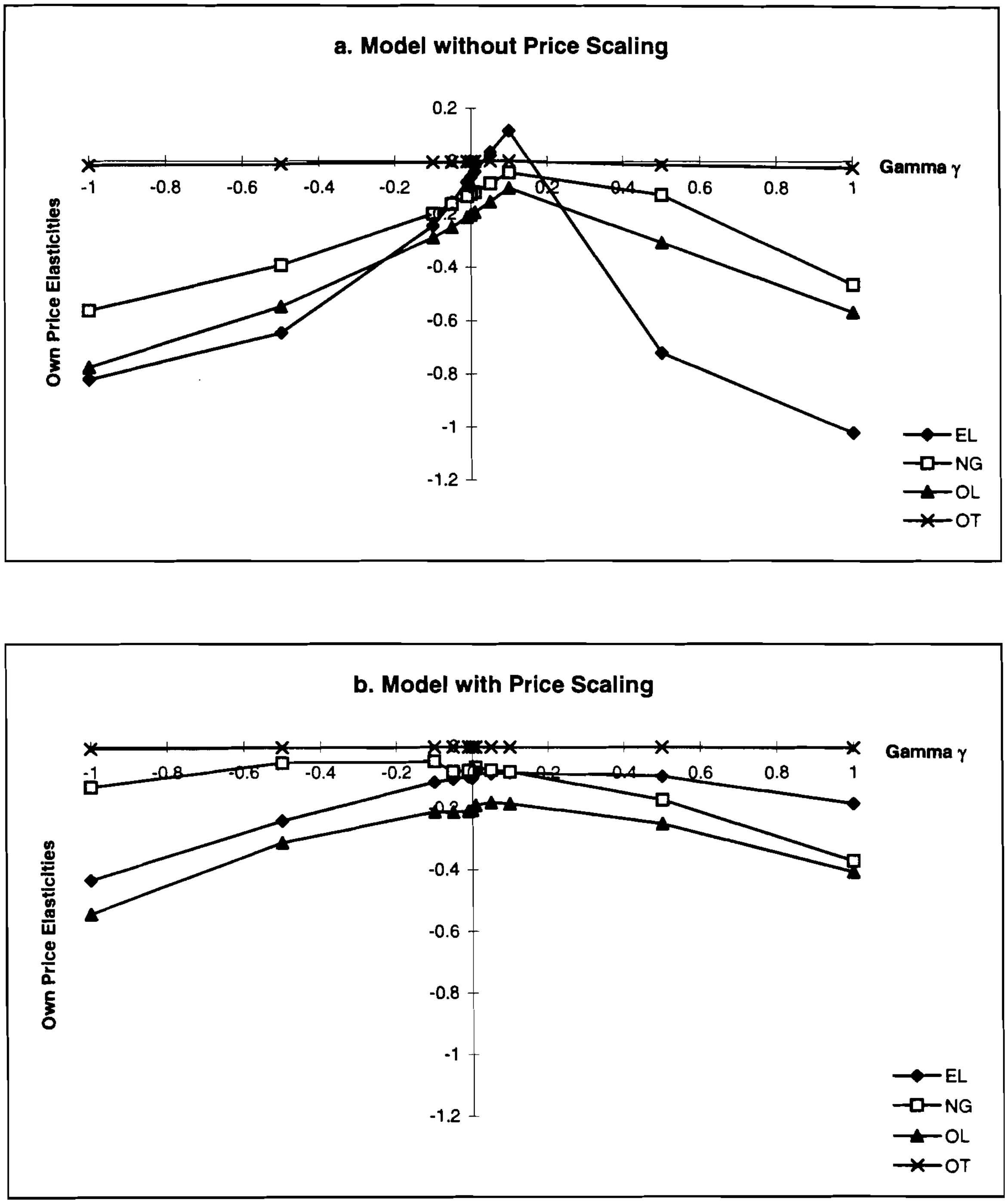

Figure 2: The Estimated Elasticities for Different Values of Gamma

Table 1: The Estimated Demand Elasticities for Residential Sector

Short Run Marshallian Income & Price Elasticities					
	Electricity	N gas	Oil	Other	Income
Electricity	-0.09767	0.03979	-0.00071	-0.84035	0.89894
N gas	0.07112	-0.07349	-0.00784	-0.80765	0.81787
Oil	0.00429	-0.00923	-0.19312	-0.13134	0.32939
Other	-0.01060	-0.00630	-0.00322	-0.98490	1.00502
Short Run Hicksian Elasticities					
	Electricity	N gas	Oil	Other	
Electricity	-0.08780	0.04539	0.00287	0.03954	
N gas	0.08010	-0.06840	-0.00459	-0.00711	
Oil	0.00791	-0.00718	-0.19181	0.19107	
Other	0.00044	-0.00005	0.00078	-0.00118	
Long Run Marshallian Income & Price Elasticities					
	Electricity	N gas	Oil	Other	Income
Electricity	-0.47167	0.17652	0.01189	-0.28263	0.56586
N gas	0.39810	-0.35114	-0.02373	-0.27262	0.24940
Oil	0.16085	0.06583	-0.89820	2.81973	-2.14868
Other	-0.00912	-0.00638	-0.00040	-1.00666	1.02245
Long Run Hicksian Elasticities					
	Electricity	N gas	Oil	Other	
Electricity	-0.46066	0.18275	0.01587	0.69618	
N gas	0.40908	-0.34490	-0.01975	0.70619	
Oil	0.17184	0.07205	-0.89414	3.79854	
Other	0.00187	-0.00015	0.00358	-0.02775	

Table 2: The Estimated Demand Elasticities for Commercial Sector

Short Run Price Elasticities	Electricity	N gas	Oil & Coal	Capital	Labor
Electricity	-0.04744	0.02598	0.01556	0.03275	-0.02685
N gas	0.13502	-0.18649	0.05598	0.12371	-0.12821
Oil & Coal	0.08633	0.05974	-0.19266	0.38053	-0.33394
Capital	0.00244	0.00177	0.00510	-0.34246	0.33315
Labor	-0.00060	-0.00055	-0.00135	0.10054	-0.09803
Long Run Price Elasticities	Electricity	N gas	Oil & Coal	Capital	Labor
Electricity	-0.04682	0.05601	0.02884	0.03356	-0.07164
N gas	0.40422	-0.41604	0.13057	0.24739	-0.36625
Oil & Coal	0.33236	0.18585	-0.40085	0.93375	-1.05142
Capital	-0.01774	-0.00083	0.00713	-0.83853	0.84996
Labor	0.00331	0.00004	-0.00174	0.24746	-0.24906

Table 3: The Estimated Demand Elasticities for Industrial Sector

Short Run Price Elasticities	Electricity	N gas	Oil	Coal	Capital	Labor
Electricity	-0.19054	-0.00010	-0.00018	-0.00003	0.10498	0.08586
N gas	-0.00038	-0.11256	0.00217	0.00211	0.18996	-0.08132
Oil	-0.00038	0.00124	-0.10676	-0.00276	0.18996	-0.08132
Coal	-0.00038	0.00731	-0.01665	-0.09894	0.18996	-0.08132
Capital	0.01047	0.00509	0.00890	0.00147	-0.59988	0.57395
Labor	0.00367	-0.00094	-0.00163	-0.00027	0.24613	-0.24697
Long Run Price Elasticities	Electricity	N gas	Oil	Coal	Capital	Labor
Electricity	-0.35704	-0.01227	-0.00974	-0.00234	0.35870	0.02261
N gas	0.00225	-0.35350	-0.00039	-0.00332	1.07140	-0.71659
Oil	0.00806	0.08300	-0.26201	-0.01121	0.09881	0.08325
Coal	0.00431	0.06738	-0.01212	-0.24444	0.44219	-0.25744
Capital	0.02815	0.02207	0.00655	0.00357	-1.44409	1.38371
Labor	0.00300	-0.00676	0.00291	-0.00035	0.58817	-0.58694

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APPENDIX

Regression Equations for the GL Model

Most empirical models of energy demand incorporate some form of dynamic response to price changes, implying that short run responses are generally smaller than the long run responses. This can be done in the GL model by adding the lagged value of $\log(q_i / q_n)$ as an explanatory variable. If the cross-price weights θ_{ij} remained constant, the long run elasticities could be derived analytically. However, cross price weights are functions of the expenditure shares, and the long run elasticities must be computed by simulation.

The Hicksian price elasticities can be derived from the share elasticities for factor demand . Since the logarithm of an expenditure share can be written as

$$\log w_i = \log p_i + \log q_i - \log C,$$

the long-run Hicksian own-price elasticities can be computed as

$$E_{iiLR} = \frac{(w_{iT} - w_{i0}) / w_{i0}}{(p_{iT} - p_{i0}) / p_{i0}} + w_{i0} - 1$$

and the long-run Hicksian cross-price elasticities can be computed as

$$E_{ijLR} = \frac{(w_{iT} - w_{i0}) / w_{i0}}{(p_{jT} - p_{j0}) / p_{j0}} + w_{j0} \qquad \text{for all } i \neq j.$$

where p_{i0} and p_{iT} are the initial and final price of factor i, and w_{i0} and w_{iT} are the corresponding expenditure shares. The long-run Marshallian price elasticities can be computed directly

$$E^m_{ijLR} = \frac{(q_{iT} - q_{i0}) / q_{i0}}{(p_{jT} - p_{j0}) / p_{j0}} \qquad \text{for any i and j;}$$

and the long-run Mashallian income elasticities can be computed as

$$E_{iILR}^{m} = \frac{(q_{iT} - q_{i0}) / q_{i0}}{(I_T - I_0) / I_0}.$$

The specific approach used to calculate the long-run elasticities reported in **Tables 1-3** is to 1) use the data for New York State in the year 1991 as the initial values (the intercepts of the estimated equations are determined through calibration to the initial values); 2) change (decrease or increase) one of the prices (or income) by 1% in 1992 and hold it at that level; 3) hold all other explanatory variables at their initial levels; 4) compute annual forecasts to 2010 (the forecasts in 2010 are the final values); and 5) use the average values of the elasticities computed by decreasing and increasing each price as the reported long-run elasticities.

For the regression models, s is the state, t is the year, HDD is heating degree days and CDD is cooling degree days. The distributed lag parameter for the lagged quantities is λ, c_i is the price scale for p_i, and the α_{ij} parameters correspond to [3]. The form of the cross-price weight θ_{ij} is:

$$\theta_{ij} = \frac{w_j}{(w_i + \delta)^{\gamma}(w_j + \delta)^{\gamma}}$$

The estimated Residential Model is given in (A1), where i=1 is electricity, 2 is natural gas, 3 is oil, n is other, non-energy goods.

$$\text{(A1)} \quad \log(w_{its} / w_{nts}) = (\alpha_{i0s} - \alpha_{n0s}) + \sum_{j=1,\ j\neq i}^{n}\left(\alpha_{ij}\theta_{ij(t-1)s}\log(p_{jts} / p_{its})\right) -$$

$$\sum_{j=1}^{n-1}\left(\alpha_{nj}\theta_{nj(t-1)s}\log(p_{jts} / p_{nts})\right) + \sum_{j=1, j\neq i}^{n}\left(\alpha_{ij}\theta_{ij(t-1)s}\log(c_j / c_i)\right) - \sum_{j=1}^{n-1}\left(\alpha_{nj}\theta_{nj(t-1)s}\log(c_j / c_n)\right)$$

$+ (\beta_i - \beta_n)\log(I_{ts} / SPI_{ts}) + \lambda(\log(q_{i(t-1)s} / q_{n(t-1)s})) + \gamma_{i1}\ HDD_{ts} + \gamma_{i2}CDD_{ts} +$

$(e_{its} - e_{nts})$, i = 1, 2, 3;

subject to $\alpha_{ij} = \alpha_{ji}$, $c_n = 0$, $\beta_n = 0$ and $\alpha_{n0s} = 0$. SPI_{ts} is a Stone Price Index using lagged expenditure shares defined as

$$\log SPI_{ts} = \sum_{j=1}^{n} w_{j(t-1)s} \log p_{jts} .$$

The estimated Commercial and Industrial Models correspond to (A2), where i = 1 is electricity, 2 is natural gas, 3 is oil in both sectors, 4 is capital, 5 is labor in the Commercial sector and 4 is coal, 5 is capital, 6 is labor in the Industrial sector.

$$\text{(A2)} \quad \log(w_{its} / w_{nts}) = (a_{i0s} - a_{n0s}) + \sum_{j=1, j\neq i}^{n}\left(\alpha_{ij}\theta_{ij(t-1)s}\log(p_{jts} / p_{its})\right) -$$

$$\sum_{j=1}^{n-1}\left(\alpha_{nj}\theta_{nj(t-1)s}\log(p_{jts} / p_{nts})\right) + \sum_{j=1, j\neq i}^{n}\left(\alpha_{ij}\theta_{ij(t-1)s}\log(c_j / c_i)\right) - \sum_{j=1}^{n-1}\left(\alpha_{nj}\theta_{nj(t-1)s}\log(c_j / c_n)\right)$$

$$+ \lambda(\log (q_{i(t-1)s} / q_{n(t-1)s})) + \gamma_{i1}HDD_{ts} + \gamma_{i2}\ CDD_{ts} + (e_{its} - e_{nts}),$$

i = 1, 2,..., n-1;

subject to $\alpha_{ij} = \alpha_{ji}$, $c_n = 0$, $\beta_n = 0$ and $\alpha_{n0s} = 0$.

Models for the Residential, Commercial and Industrial sectors have been estimated by iterated seemingly-unrelated-regression (ITSUR) using SAS. A summary of the estimated parameters and the fit of the equations are included in **Table A1** to **Table A3**. The relationship of the names of the parameters in the SAS output to those in (A1) and (A2) are as follows: RCij, CCij and ICij correspond to α_{ij} (R= Residential sector, C= Commercial sector, I = Industrial sector); RCiY corresponds to β_i ($\beta_4 = 0$ is used for normalization); RL11, CL11 and IL11 correspond to λ; WEi1 corresponds to γ_{i1}; WEi2 corresponds to γ_{i2}. RBi, CBi and IBi (for i=1, 2, ..., n-1) correspond to log c_i. In the Industrial model, IC13=IC14=IC12, IC35=IC45=IC25, and IC36=IC46=IC26 hold to reflect weak separability of the three fossil fuels.

Table A1: The Estimated Demand Models for Residential Sector(SAS Output)--
Generalized Logit model using form (v) with delta=0.005 and gamma=0.01

Nonlinear ISTUR Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE
ELEC	5.333	1051	2.112	0.0020	0.0448
NGAS	5.333	1051	9.202	0.0088	0.0936
OIL	4.333	1052	25.232	0.0240	0.1549

Nonlinear ISTUR Parameter Estimates

Para.name	Est. value	Std error	T ratio	Prob>ITI
RC12	5.77198	0.79794	7.23	0.0001
RC13	-0.25621	0.49223	-0.52	0.6028
RC14	-0.92057	0.01931	-47.66	0.0001
RC23	-1.96355	1.21077	-1.62	0.1052
RC24	-0.96289	0.02256	-42.69	0.0001
RC34	-0.76762	0.02484	-30.9	0.0001
RC1Y	-0.10608	0.01869	-5.67	0.0001
RC2Y	-0.18715	0.0411	-4.55	0.0001
RC3Y	-0.67563	0.06216	-10.87	0.0001
RB1	-0.25638	2.45514	-0.1	0.9169
RB2	1.08547	2.45441	0.44	0.6584
RB3	2.03102	2.99777	0.68	0.4982
RL11	0.78922	0.01497	52.71	0.0001
WE12	0.00017	9.82E-06	17.69	0.0001
WE21	0.00001	5.11E-06	2.24	0.0256

Table A2: The Estimated Demand Models for Commercial Sector(SAS Output)--Generalized Logit model using form (v) with delta=0.005 and gamma=0.075

Nonlinear ISTUR Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE
ELEC	5.75	554.3	1.915	0.0035	0.0588
NGAS	4.75	555.3	4.814	0.0087	0.0931
OIL	4.75	555.3	28.495	0.0513	0.2265
CAPITAL	3.75	556.3	5.126	0.0092	0.0960

Nonlinear ISTUR Parameter Estimates

Para.name	Est. value	Std error	T ratio	Prob>|T|
CC12	3.67407	1.216510	3.02	0.0026
CC13	2.15639	1.724310	1.25	0.2116
CC14	-0.57539	0.039190	-14.68	0.0001
CC15	-0.76135	0.024560	-30.99	0.0001
CC23	8.47007	3.028450	2.8	0.0053
CC24	-0.28369	0.060380	-4.7	0.0001
CC25	-0.79996	0.032350	-24.73	0.0001
CC34	0.42453	0.168230	2.52	0.0119
CC35	-0.98541	0.047150	-20.9	0.0001
CC45	-0.48788	0.017800	-27.4	0.0001
CB1	-1.65737	0.290490	-5.71	0.0001
CB2	-0.18124	0.281050	-0.64	0.5193
CB3	-0.33217	0.243760	-1.36	0.1735
CB4	3.42573	0.349550	9.8	0.0001
CL11	0.58897	0.014930	39.44	0.0001
WE11	0.00001	0.000005	1.3	0.1952
WE12	0.00008	0.000018	4.45	0.0001
WE21	0.00002	0.000007	2.56	0.0109
WE31	0.00002	0.000018	0.9	0.3692

Table A3: The Estimated Demand Models for Industrial Sector(SAS Output)--
Generalized Logit model using form (v) with delta=0.005 and gamma=0.05

Nonlinear ISTUR Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE
ELEC	3.55	402.5	1.642	0.0041	0.0639
NGAS	5.3	400.7	6.390	0.0160	0.1263
OIL	4.3	401.7	4.475	0.0111	0.1056
COAL	4.3	401.7	15.465	0.0385	0.1962
CE	2.55	403.5	8.244	0.0204	0.1430

Nonlinear ISTUR Parameter Estimates

Para.name	Est. value	Std error	T ratio	Prob>ITI
IC12	-0.68700	0.06332	-10.85	0.0001
IC15	-0.50055	0.08993	-5.57	0.0001
IC16	-0.71993	0.03327	-21.64	0.0001
IC23	-0.55101	0.10780	-5.11	0.0001
IC24	-0.02649	0.41055	-0.06	0.9486
IC25	-0.25129	0.08946	-2.81	0.0052
IC26	-0.88416	0.03233	-27.35	0.0001
IC34	-1.43896	0.46956	-3.06	0.0023
IC56	-0.12467	0.04377	-2.85	0.0046
IB1	-1.97709	0.89683	-2.2	0.0281
IB2	2.77073	0.46545	5.95	0.0001
IB3	-0.32021	0.55037	-0.58	0.561
IB4	1.80407	0.49588	3.64	0.0003
IB5	5.28239	0.97854	5.4	0.0001
CL11	0.59277	0.01677	35.35	0.0001
WE12	0.00001	0.00002	0.37	0.7082
WE21	0.00001	0.00002	0.56	0.5755
WE22	-0.00007	0.00004	-1.69	0.0924
WE31	0.00010	0.00002	6.17	0.0001
WE41	0.00002	0.00003	0.77	0.4394

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