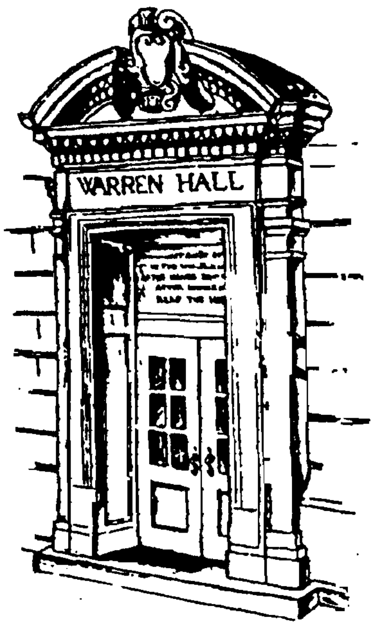
WP 96-13

October 1996



Working Paper

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PRODUCTIVITY OF DAIRY FARMS MEASURED BY NON-PARAMETRIC MALMQUIST INDICES

by

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Abstract

The productivity of individual dairy farms, decomposed into efficiency and technological change components, was measured annually from 1985 through 1993 from distance functions estimated using nonparametric programming methods. Technology is measured regressively only if it is regressive to all previous periods rather than just the immediate previous period. Average productivity increased 2.8 percent each year, with about half of the gain due to gains in efficiency, and the other half due to technological improvements. Twenty-five percent of the farms failed to increase productivity sufficiently over the period to offset the decreased ratio of output to input prices.

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* Professor, Department of Agricultural, Resource, and Managerial Economics, Cornell University, Ithaca, New York. Paper presented at the Annual meeting of the American Agricultural Economics Association in San Antonio, Texas, July 29-31, 1996. I wish to thank Richard Boisvert, Rolf Färe, Wayne Knoblauch, Ed LaDue, and Bill Tomek for their suggestions. This research was completed under Cornell Project 121-413.

The Productivity of Dairy Farms Measured by Non-Parametric Malmquist Indices

The ratio of the index of prices received for milk to the index of prices paid by dairy farmers decreased 13 percent in New York from 1985 through 1993. With input prices rising faster than output price, it is necessary for dairy farms to increase their productivity in order to remain profitable. The number of dairy farms in New York with 30 or more cows decreased from 11,500 in 1985 to 9,100 in 1993.[1] There may have been a number of reasons for this 21 percent reduction in dairy farm numbers, but a failure to increase productivity, during a time of unfavorable price changes, may be one reason for their demise.

The purposes of this paper are to measure the productivity changes of a group of New York dairy farms during this period of time, and to determine how many were able to increase their productivity sufficiently to offset the unfavorable change in the output/input price ratio. However, since productivity consists of technical improvement, as well as gains in efficiency within a given technology set, both technical and efficiency change are measured by decomposing the Malmquist productivity index into these two separate components. The Malmquist index is based upon the distance function, can be measured by a primal approach, and thus does not require the assumption of cost minimization or profit maximization behavior necessary for many other total productivity indices (Chambers, 1988). These indexes are measured using nonparametric programming methods (Färe, Grosskopf, Norris, and Zhang, 1994).[2]

The data set used first measured a significant number of farms making regressive technological progress. It is demonstrated how these results may be due to data point migration between periods. Färe *et al.*, 1994, measure technological change relative to an adjacent time period only. By measuring technological change relative to all previously displayed netput vectors, technological regression is measured only when technology in a given period is regressive to all previous periods. The results are much fewer technological regressive incidents.

Malmquist Productivity Indices

Productivity measurement consists of measuring the change in the ratio of outputs to inputs used in a production process. Since a number of inputs are used, and joint output may be involved, a number of procedures have been developed to aggregate inputs and outputs and to measure changes. Recently, the Malmquist index, originally formulated by Malmquist, 1953, has been further developed within the nonparametric or Data Envelopment Analysis (DEA) framework by Färe *et al.*, 1994.

An output distance function can be defined at time t as (Cornes, 1992):

(1) $$D_o^t\left(x^t, y^t\right) = \min\left\{\theta : \left(x^t, y^t / \theta\right) \in s^t\right\} = \left(\max\left\{\theta : \left(x^t, \theta y^t\right) \in s^t\right\}\right)^{-1}.$$

This essentially shows how much output(s) y can be increased given a quantity of input(s) x, such that x and θy remain in the production set. An input distance function can similarly be defined and under constant returns its value is the reciprocal to the output distance function. An output rather than an input distance function is used here since farmers more likely try to increase their outputs given their use of inputs, rather than try to decrease inputs given their outputs.

To construct the Malmquist index, it is necessary to define distance functions with respect to two different time periods as:

(2) $$D_o^t(x^{t+1}, y^{t+1}) = \left(\max\left\{\theta : \left(x^{t+1}, \theta y^{t+1}\right) \in s^t\right\}\right)^{-1}$$

and

(3) $$D_o^{t+1}(x^t, y^t) = \left(\max\left\{\theta : \left(x^t, \theta y^t\right) \in s^{t+1}\right\}\right)^{-1}.$$

The distance function specified by equation (2) measures the maximal proportional change in output required to make (x^{t+1}, y^{t+1}) feasible in relation to the technology at time t. Similarly, the distance function specified by equation (3) measures the maximal proportional change in output required to make (x^t, y^t) feasible in relation to the technology at time t+1.

Efficiency change between year t and t+1 is measured as:

$$E_o^{t+1}\left(y^{t+1},x^{t+1},y^t,x^t\right) = \frac{D_o^{t+1}(x^{t+1},y^{t+1})}{D_o^t(x^t,y^t)},$$ where the numerator is the distance function, equation (1), measured at time t+1.

Technical change between year t and t+1 is measured as:

$$T_o^{t+1}\left(y^{t+1},x^{t+1},y^t,x^t\right) = \left[\left(\frac{D_o^t(x^{t+1},y^{t+1})}{D_o^{t+1}(x^t,y^t)}\right) x \left(\frac{D_o^t(x^t,y^t)}{D_o^{t+1}(x^{t+1},y^{t+1})}\right)\right]^{\frac{1}{2}}.$$

The Malmquist productivity change is the product of efficiency change and technical change,

$M_o^{t+1}(.) = E_o^{t+1}(.) \bullet T_o^{t+1}(.)$.

A graphic illustration of the measures is shown in Figure 1. That figure shows unit isoquants for period t, Q(t), and period t+1, Q(t+1). These are the frontier, or best practice isoquants. Also shown are the use of inputs by a single firm to produce a unit of output in period t (y^t) and period t+1 (y^{t+1}). The firm is inefficient in period t, as measured by the radial distance Oa/Ob. It is inefficient in period t+1 by the amount Oc/Od. The relative change in inefficiency between period t and t+1 is then measured as:

$$E_o^{t+1} = \frac{Oc / Od}{Oa / Ob}.$$

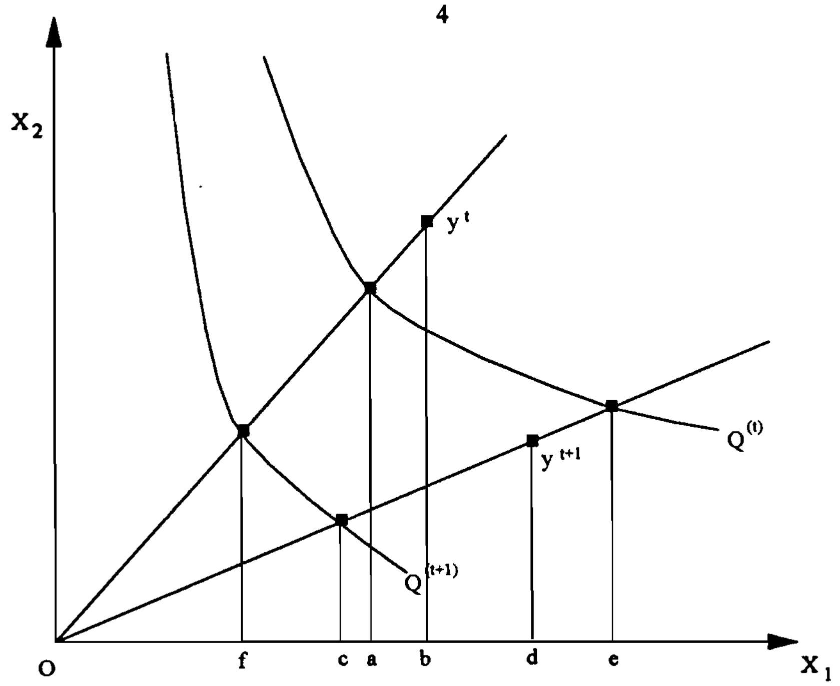

Figure 1. Unit Isoquants to Measure Efficiency, Technological, and Productivity Indices

The change in technology is measured by the inputs used in period t+1 relative to the isoquant in period t, Oe/Od, divided by the inputs used in period t relative to the isoquant in period t+1, that value multiplied by the ratio of the distance function for periods t and t+1. The result is the movement of the unit isoquant Q(t) relative to the unit isoquant Q(t+1), measured as the geometric means of the contraction of the two radial lines that pass through y^t and y^{t+1},

$$T_0^{t+1} = \left[\left(\frac{(Oe / Od)}{(Of / Ob)}\frac{(Oa / Ob)}{(Oc / Od)}\right)\right]^{\frac{1}{2}} = \left[\frac{Oe}{Oc} \bullet \frac{Oa}{Of}\right]^{\frac{1}{2}}.$$

Although each distance function used to measure T_0^{t+1} entails a proportional (radial) expansion or contraction of the output vector y, the index T_0^{t+1} reduces to the geometric mean of two separate radial lines that may not coincide. As such, the measured technological change is not necessarily Hicksian neutral, since the shift in the isoquants illustrated in Figure 1 may not be parallel.

Measuring Malmquist Indices

These distance functions are reciprocals to the output-based Farrell measure of technical efficiency and can be calculated for each firm using nonparametric programming techniques (Färe *et al.*, 1994). The linear programming model to calculate output distance function (1) for each of the K firms for each time period t is:

$$(5) \quad \left(D_o^t(x^{k',t}, y^{k',t})\right)^{-1} = \max \theta^k$$

subject to

$$(5.a) \quad \sum_{k=1}^{K} z^{k,t} y_m^{k,t} \geq \theta^{k'} y_m^{k',t} \qquad m = 1,\ldots,M$$

$$\sum_{k=1}^{K} z^{k,t} x_n^{k,t} \leq x_n^{k',t} \qquad n = 1,\ldots,N$$

$$(5.b) \quad z^{k,t} \geq 0 \qquad k = 1,\ldots,K$$

where z is the intensity vector, y is output, x is input, θ is the inverse of the efficiency score, M is the number of outputs, N is the number of inputs, and K is the number of firms. The technology specified here is nonparametric but assumes constant returns to scale and strong disposability of inputs and outputs. Variable returns can be specified but are not used here because there was insufficient variability in the size of the dairy farms used as data. The nonparametric computation of $D_o^{t+1}\left(x^{k',t+1}, y^{k',t+1}\right)$ is exactly like (5), where t+1 is substituted for t.

The two distance functions specified in equations (2) and (3) require firm data from adjacent periods. The first is computed for firm k as

$$(6) \quad \left(D_o^t(x^{k',t+1}, y^{k',t+1})\right)^{-1} = \max \theta^k$$

subject to

$$\sum_{k=1}^{K} z^{k,t} y_m^{k,t} \geq \theta^{k'} y_m^{k',t+1} \qquad m = 1,\ldots,M$$

$$n = 1,\ldots,N$$

$$\sum_{k=1}^{K} z^{k,t} x_n^{k,t} \leq x_n^{k',t+1} \qquad k = 1,\ldots,K$$

$$z^{k,t} \geq 0$$

The second is specified as in (6), but the t and t+1 superscripts are transposed.

Data

The New York Dairy Farm Business Summary (DFBS) program allows dairy farmers, at the end of a year, to enter their farm production and financial information into a software package that permits an analysis of their businesses (Putnam, Knoblauch and Smith, 1995). This helps them determine strengths and weaknesses of their business and ascertain where changes might be appropriate and useful. The data are transmitted to Cornell University where they are combined with information from other participants to generate benchmarks for comparisons. Over the 9 year period of 1985 through 1993, 70 dairy farms participated each and every year (Smith, Knoblauch, and Putnam, 1994). These data are used here.[3]

Various expenditures and receipts are collected on an accrual basis. Most items are in dollars, with little information collected on quantities or prices except for milk production and labor usage. These items are listed in Table 1 under the column DFBS Items Aggregated. In order to effectively apply nonparametric programming to measure the Malmquist indices, it is necessary to aggregate these items into a smaller set. Leibenstein and Maital, 1992, note that, given enough inputs, all (or most) firms are rated efficient. This is a direct result of the dimensionality of the

Table 1. Data Categories

Variable	Price Index	DFBS Items Aggregated	1993 Average (in 1993 dollars)
Labor input	None	Months operator(s)	22.0
		Months hired	34.7
		Months family unpaid	2.4
Purchased feed input	Purchased feed	Dairy grain and concentrate	$133,726
		Non-dairy feed	48
	All hay	Dairy roughage	2,097
Energy input	Fuel and energy	Fuel (less gas tax refund)	10,022
		Electricity	11,658
Crop input	Fertilizer	Fertilizer and lime	10,856
	Seed	Seed and plants	7,055
	Chemicals	Spray, other crop expenses	7,385
	Machinery	Machinery depreciation (tax)	26,510
		Interest on machinery (4%)	8,761
		Machinery repairs / parts	25,154
		Machinery hire expenses	5,548
		Auto expense (farm share)	833
Livestock input	Purchased animals	Replacement livestock purchases	4,840
		Expansion livestock	16,470
		Cattle lease	144
		Interest on livestock (4%)	10,473
		Other livestock expense	23,675
	Farm services and rent	Breeding fees	5,894
		Veterinarian and medicine	12,902
		Milk marketing expenses	20,050
		Telephone	959
		Insurance	5,548
		Miscellaneous	9,447
Real estate input	Real estate	Cash rent	7,795
		Building depreciation (tax)	20,014
		Interest on real estate (4%)	21,825
	Building and fencing supplies	Building and fence repair	7,824
	Property taxes	Real estate taxes	10,357
Milk output	None	Milk production	36,837 (cwt.)
Other output	CPI	Government payments	$7,220
		Custom machine work	917
		Miscellaneous receipts	6,657
	Slaughter cows	Dairy cattle sales	50,382
		Other livestock sales	388
	Slaughter calves	Dairy calves sales	9,271
	All hay	Crop sales	9,290

input/output space relative to the number of observations (firms). Thomas and Tauer, 1994, show using New York Dairy Farm Business Summary data that defining eight inputs results in 38 percent of the firms measured as efficient; fourteen inputs results in 78 percent of the firms measured as efficient. Six inputs and two outputs are defined here.

Receipts and expenditures, except for milk and labor, were first converted into quantities by dividing by annual price indices (1984=100). This converts expenditures and receipts into 1984 dollars, assuming that all farms paid and received the same prices for each item in any given year. To the degree that some individual farm expenditures were greater because of higher prices paid for a quality input (feed for instance), dividing by the same price for all farms converts these inputs into a quality-adjusted input, reflected as a larger quantity of a constant-quality input. The deflated expenditures and receipts were then aggregated into the six input, two output categories listed in column 1 of Table 1.

Results with No Restrictions on Regressive Technology

The distance functions were computed using linear programming. For each firm each year, three distance functions as specified by equations (1), (2), and (3) were estimated. With 70 farms and 9 years this results in 1890 linear programming models. The scalar values from those distance functions were then used to compute the change in efficiency, technology, and productivity for each firm between years. The results for each firm are summarized in Table 2, which shows the average (geometric) change in efficiency, technology and productivity for each of the 70 farms. Also shown is the average efficiency of each farm over the nine-year period. Many farms were efficient some years but not other years so that their average efficiency was below one. Yet eleven of the farms were technically efficient each and every year.

Of the 70 farms, 42 increased their efficiency over the nine-year period (averages greater than one), while 28 decreased their efficiency. Of the group, 53 experienced technological progression, while 17 farms experienced regressive technology, or a shift downward in the production function. Productivity is the product of efficiency and technology, and of the 70 farms, 46 increased their productivity over the nine-year period while 24 decreased their productivity. The fact that 24 farms had a productivity decrease over the nine-year period is troublesome. Yet, the fact that 17 farms on average experienced regressive technological change is even more troublesome.

Results Adjusting for Apparent Regressive Technology

A possible explanation for measured regressive technology is illustrated by Figure 2, where technology on the y^{t+1} ray is progressive, while technology on the y^t ray is regressive. Why might technological change be measured as mostly regressive along the y^t ray? My hypothesis is that it is due to the way the frontier isoquant is defined in each period. That procedure is by the data envelopment of the firms' input/output data at a specific time period. What if the frontier point on Q^t from Figure 2 during period t is defined by firm μ^t, but that firm then migrates to point μ^{t+1} during period t+1, leaving firm w^t defining the Q^{t+1} frontier point along ray y^t? The result is locally regressive technological change along ray y^t. The Malmquist index is formulated so that technological change is measured as the geometric mean of both the y^t and the y^{t+1} rays. As a result the technological change may be measured as regressive.

Färe and Grosskopf, 1996, demonstrate how the technological component of the Malmquist productivity index can be measured adjusting for bias changes. Their measurement technique for bias can also be used to determine if one of the rays is displaying regressive