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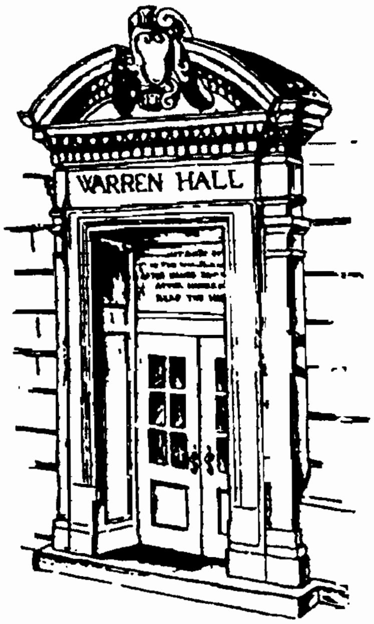

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On The Option Value of Old-Growth Timber: The Case of the Headwaters Forest

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ABSTRACT

The value of old-growth timber can be viewed as an option value. If old growth is preserved today, one preserves the option of cutting in the future. While waiting, the old growth provides a flow of amenity services and an expected capital gain. Amenity value is the nontimber value derived by visitors to an old-growth forest, potential visitors, or nonvisitors who believe remaining old-growth should be preserved for future generations.

This paper modifies a model developed by Reed (1993) and applies it to the Headwaters Forest, the last privately held stand of old-growth, coastal redwood. Estimates of seven bioeconomic parameters are needed to apply the model: β the expected drift in old-growth timber prices, σ_1^2 the variance rate for timber prices, α the expected drift in amenity value, σ_2^2 the variance rate for amenity value, ρ the correlation coefficient between the rate of change in timber price and amenity value, γ the rate of decay in the volume of merchantable timber and δ the rate of discount. The first five parameters were estimated from time-series data on the value of an old-growth, redwood log and from visitation rates to the Redwood National Park, fifty miles north of the Headwaters. The remaining two bioeconomic parameters, the discount rate, δ, and the rate of decay, γ, were selectively varied over a plausible range.

The optimal stopping rule is a barrier on the ratio of current timber value to current amenity value. If this barrier is denoted as C^*, it is optimal to cut the Headwaters the first time that $C(t) = V(t)/A(t) = C^*$, where $V(t)$ is current timber value and $A(t)$ is current amenity value. Amenity value is difficult to measure. Current timber value, V_0, is more readily estimated, and if one can calculate C^*, then current amenity value must exceed $A^* = V_0/C^*$ to justify continued preservation.

The analysis revealed that preservation was unequivocally optimal for a discount rate less than 4.6 percent per annum, the rate of increase in amenity value. For rates of discount greater than 4.6 percent it may be optimal to cut the Headwaters. If the current value of standing old-growth timber in the Headwaters is V_0 = \$550 million, then, with a discount rate of 6 percent and a decay rate of zero, current annual amenity value of the Headwaters must exceed \$3.623 million to justify continued preservation (based on a calculated $C^* = 151.789$). If the discount rate increases to 10 percent and the decay rate increases to 1.5 percent, the critical value drops to $C^* = 26.396$, and current amenity value would have to exceed A^* = \$20.836 million. Continued preservation is harder to justify at higher rates of discount or decay.

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I. Introduction

Resource economics would appear to be a fertile field for the application of stopping rule theory. From its initial development in finance, stopping rule theory has been applied to the problems of opening and closing a mine (Brennan and Schwartz 1985), making specialized (resource) investments (McDonald and Siegel 1986), cutting trees that exhibit stochastic growth (Clarke and Reed 1989 and Reed and Clarke 1990), developing land (Clarke and Reed 1990), controlling the accumulation of a nondegradable pollution stock (Conrad 1992) and preserving old-growth forest (Reed 1993).

Stopping rule theory is applicable to decisions which, at any instant, are binary (Dixit and Pindyck 1994). One must choose between continuation of a prevailing regime or state or taking an action which might irreversibly alter the state of a firm, individual, or society. For example, Dixit (1989) considers output prices that might cause a firm to enter or exit an industry. While a change in the firm's status is reversible, the firm faces start-up (entry) and shut-down (exit) costs which, in part, determine "price triggers" that would make an idle firm active or an active firm idle.

In other cases the change in status might be irreversible. Some investments may be so specialized that, once made, capital can never be retrieved for use in another activity. The decision to build nuclear reactors commits a society to the long-term (irreversible) storage of radioactive waste. The cutting of remaining stands of old-growth forest is akin to exercising an option which precludes cutting at a later date and also results in an irreversible loss of amenity value. Amenity value is the flow of benefits to actual visitors, the option value of potential visitors, and the existence value of nonvisitors who view preservation as a desirable bequest to future generations. Stopping rule theory, which employs stochastic dynamic programming, provides the proper methodology for estimating option value (Weisbrod 1964) and quasi-option value (Arrow and Fisher 1974, Henry 1974).

The next section poses a stopping rule problem which seeks to determine when, if ever, it would be optimal to cut a stand of old-growth forest. Section III provides some background on the Headwaters Forest - the last remaining stand of old-growth coast redwood (*Sequoia sempervirens*) held as private property. In Section IV mean drift and variance rates for timber and amenity value are estimated from time-series data on the value of an old-growth redwood log and the visitation rate to the Redwoods National Park, 50 miles north of the Headwaters. Section V presents a numerical analysis of the Headwaters, identifying a critical barrier on the ratio of current timber value to amenity value which would make cutting optimal. If the current value of standing timber in the Headwaters is $550 million (a figure which would have reputedly enticed a voluntary sale from the current owners) it is possible to calculate a lower bound on current amenity value which would justify continued preservation. A summary and conclusions are offered in the sixth section.

II. Preservation of Old-Growth Forest as a Stopping Rule Problem

Consider an old-growth forest where the volume of merchantable timber is in a process of slow decay. This is the result of a natural process where ancient trees ultimately topple and decay, releasing nutrients to the forest soil. Suppose this process takes place at rate γ. If the volume of merchantable timber today is denoted by X_0, then the volume at instant t in the future will be $X(t) = X_0 e^{-\gamma t}$.

The commercial value of the old-growth forest in the future is equal to the merchantable volume times a per unit price, $P = P(t)$. Specifically, let the value of the old-growth forest be denoted by $V(t) = P(t)X(t)$, or suppressing the time index, $V = PX$. While the process of decay is slow and steady, the per unit price of timber is volatile, making P, and thus future value, uncertain. Suppose that the future price of old-growth timber evolves according to geometric Brownian motion (GBM) so that

$$dP = \beta P\, dt + \sigma_1 P\, dz_1 \qquad (1)$$

where β is called the mean drift rate, σ_1^2 is the variance rate, and $z_1(t)$ is a standard Wiener process. Price is an Itô variable. This is not an unreasonable assumption; many commodity and stock prices appear to follow a process of "random walk with drift," which leads to equation (1) when time is treated as a continuous variable. Pindyck and Rubinfeld (1991, p.465), for example, find that lumber prices from 1870 to 1987 are consistent with a random walk hypothesis.

With future timber prices evolving as an Itô variable, the value of merchantable timber becomes a function of an Itô variable, specifically, $V = PX_0\, e^{-\gamma t}$. One can use Itô's Lemma to show that

$$dV = (\beta - \gamma)V\, dt + \sigma_1 V\, dz_1 \qquad (2)$$

The mean drift rate for the value of the old-growth forest is thus equal to the mean drift rate in price less the decay rate in volume, while the variance rate is the same for both processes.

Suppose that amenity value, $A = A(t)$, also follows a process of geometric Brownian motion given by

$$dA = \alpha A\, dt + \sigma_2 A\, dz_2 \tag{3}$$

where α is the mean drift rate in amenity value, σ_2^2 is the variance rate and $z_2(t)$ is another standard Wiener process. The assumption that amenity value follows a process of geometric Brownian motion is perhaps more tenuous. If the amenity services of an old-growth forest depend, at least in part, on visitation rates, which trend with total population, but vary with changes in recreational taste or other recreational opportunities, then equation (3) might be a reasonable model for the evolution of amenity value.

Assuming that timber and amenity values evolve according to equations (2) and (3), when, if ever, would it be optimal to harvest the old-growth forest? A decision to cut the forest is tantamount to "killing an option," thereby foregoing the possibility of even higher timber values in the future, and stopping the flow of amenity values forever. Reed (1993) has shown that the optimal stopping rule is to cut if and only if the *current* ratio of timber to amenity value, $V(t)/A(t)$, reaches a critical value, C^*. In the appendix to this paper it is shown that

$$C^* = \frac{1}{(\delta - \alpha)}\left(\frac{1 + \theta}{\theta}\right) \tag{4}$$

where δ is the instantaneous rate of discount, α is the mean drift in amenity value and θ is the positive root of a characteristic equation, given by

$$\theta = -\,[1/2 + (\beta - \gamma - \alpha)/\sigma^2] + \sqrt{[1/2 + (\beta - \gamma - \alpha)/\sigma^2]^2 + 2(\delta + \gamma - \beta)/\sigma^2} \tag{5}$$

and σ^2 is the variance of $\{\ln[V(t)/A(t)]\}$ given by

$$\sigma^2 = \sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2 \tag{6}$$

where ρ is the correlation between the rate of change in timber price and amenity value.

Intuitively, the critical value C^* represents the ratio of timber to amenity value where it is no longer optimal to wait. C^* will depend on seven "bioeconomic" parameters α, β, γ, δ, σ_1, σ_2, and ρ. Using Itô's Lemma, and noting that $C(t) = V(t)A(t)^{-1}$ it can be shown that

$$dC = (\beta - \gamma - \alpha + \sigma_2^2 - \rho\sigma_1\sigma_2)C\,dt + (\sigma_1\,dz_1 - \sigma_2\,dz_2)C \tag{7}$$

Thus, the ratio of current timber value to current amenity value also follows geometric Brownian motion. The evolution of C(t) depends on all of the above parameters *except* δ. Figure 1 shows C^* and three sample paths (or realizations). The time to go from $C(0) = V(0)/A(0)$ to C^* is a random variable. Sample path #1 reaches C^* earlier that sample path #2 ($t_1 < t_2$), while sample path #3 has failed to reach C^* during the depicted horizon.

In the present model, it will *never* be optimal to cut the old-growth forest if $\delta + \gamma - \beta < 0$ or if $\delta - \alpha < 0$. The first condition implies that the remaining timber is appreciating in value at an expected rate greater than the sum of the rates of discount and decay. Preservation is optimal on a strictly financial basis. The second condition implies that the expected rate of increase in amenity value exceeds the rate of discount, making preservation optimal based on the rapid appreciation of the forest as a "natural asset."

III. The Headwaters Forest

The Headwaters Forest is a 3,000 acre tract of predominantly ancient coast redwood. The parcel is part of a forest inventory of 195,000 acres owned by the Pacific Lumber Company (Palco). Logs from these forested lands are milled at Palco's plant in the town of Scotia, located in Humboldt County, in northern California.

Palco was regarded as a conservatively-managed forest company which had been in existence for over 100 years when it was acquired during a hostile takeover by Maxxam Inc., a holding company founded by Charles Hurwitz, a Texas financier. The acquisition was financed largely with junk bonds. At the time of acquisition Hurwitz was also a principal in a savings and loan, the United Savings Association of Texas (USAT), which subsequently became insolvent, ultimately requiring $1.6 billion from federal taxpayers to make good on its debt to depositors.

Strapped for cash, and confronted by high interest rates, Hurwitz accelerated the rate of harvest from Palco's forest inventory. In addition to the 3,000 acres in the Headwaters, Palco's inventory included about 40,000 acres of second-growth redwood, 40,000 acres of residual old-growth species from previous selective logging, and 7,000 acres of low-elevation, old-growth Douglas fir. See King and Mardon (1990).

Residents of the State of California had an opportunity to acquire the Headwaters, but voted down two initiatives in November of 1990. Passage of either *The California Environmental Protection Act* (alias "Big Green") or *The Forest and Wildlife Protection and Bond Act* (alias "Forests Forever"), would have required the State to purchase the Headwaters Forest.

In late 1994, the *Headwaters Forest Act*, which had been introduced in the U.S. House by California representatives, Dan Hamburg and Pete Stark, passed by a vote of 288 to 133. The Act was not considered by the U.S. Senate before adjournment. Hamburg was not reelected, and the Republican majority in the U.S. Congress makes reintroduction unlikely. The legislation would have authorized the federal government to purchase 44,000 acres from Palco, including the Headwaters Forest and second- and third-growth redwood stands which would act as an ecological buffer zone.

Hurwitz did not want to sell all 44,000 acres, but has indicated that he would have been willing to sell 4,500 acres, including the 3,000 acre Headwaters Forest and a smaller, 1,500 acre, buffer zone. The asking price for this 4,500 acre tract was between $500 and $600 million [Gannon (1994) and Skow (1994)]. When Hurwitz purchased Palco in 1985 he paid $900 million for all 195,000 acres and the mill in Scotia. Several environmental groups contend that the government should pay nothing for the Headwaters Forest. Given Hurwitz's equity in, and the subsequent $1.6 billion bailout of USAT, they feel that the Headwaters Forest should be given to the federal government as compensation to U.S. taxpayers.

Maxxam officials reject such logic. They see no relationship between USAT and the Headwaters. They regard the trees on Palco's land as private property and feel that sufficient ancient redwood have been protected in the

Redwood National Park, lying 50 miles north of the Headwaters Forest. That park contains almost 39,000 acres of old-growth redwood. Humboldt State Park contains another 17,000 acres of old-growth, coast redwood.

The analysis of the next section takes the following perspective. Suppose a "social forester," was seeking to determine when, if ever, to cut the Headwaters Forest. To apply the model of the previous section one would need to estimate the mean drift and variance rates for timber and amenity value, the correlation between their rates of change, the rate of decay in merchantable timber and the appropriate discount rate. Time-series data on amenity value is the main obstacle to applying the model. This might be overcome if one can argue that another, observable, time-series is perfectly correlated with unobservable amenity value.

IV. Model Estimation

Table 1 contains data on the real average annual price for a 16 foot, old-growth, redwood log (assumed to contain over 850 board feet of timber) for the years 1977 through 1995. This time-series was taken from the California State Board of Equalization's *Harvest Value Schedule* for Timber Area One, which includes Humboldt and Del Norte counties. These prices, are used in calculating the yield taxes owed the State of California by firms harvesting timber. They were deflated by the Producer Price Index. 1977 was the first year that the State of California collected a timber yield tax. Prior to 1977 counties collected a property tax based on the assessed value of land and standing timber. It is assumed that if this time series does not measure the real market price of an old-growth redwood log, that it is perfectly correlated with the real market price.

Table 1 also contains time-series data on the annual visitation rate to the Redwood National Park. The Redwood National Park was created in 1968 and is actually comprised of three state parks totaling 34,790 acres and 75,541 acres of federally owned land. Groves of ancient coast redwood cover 38,982 acres. It is assumed that the visitation rates at the Redwood National Park are perfectly correlated with the unknown amenity value of the Headwaters Forest. This is a more tenuous assumption. While it seems plausible that visitation rates are proportional to use value, they may not be proportional to option and existence values. Alternatives to this assumption will be discussed in Section VI.

Are the redwood prices and visitation rates in Table 1 consistent with geometric Brownian motion? Table 2 shows the results of unrestricted and restricted regressions which can be used to test the joint null hypothesis of a zero time trend and unit root. Let $p_t = \ln P_t$ and $a_t = \ln A_t$. Then, if the data are consistent with p_t and a_t being Brownian motion, they are, from

Itô's Lemma, consistent with P_t and A_t being geometric Brownian motion. In the case of p_t, we come close to rejecting the null hypothesis that $\beta = 0$ and $\rho = 1$ at the 5% level. For a_t the presumption of Brownian motion is more comfortably maintained.

The mean and standard deviation of the series $\ln(P_{t+1}/P_t)$ and $\ln(A_{t+1}/A_t)$, t = 1977 to 1994, will provide maximum likelihood estimates of β and σ_1 and α and σ_2, respectively (Reed and Clarke 1990). These estimates, along with an estimate of the correlation coefficient, ρ, are given in Table 3. Plots of P_t and A_t for t = 1977 to 1995 are shown in Figure 2.

The remaining parameters, γ and δ, were not estimated, but varied over a range of plausible values. The inflation-free, risk-free discount rate was varied from $\delta = 0.02$ to $\delta = 0.08$, in increments of 0.02. The rate of decay in merchantable timber, γ, was varied between zero and 0.015 in increments of 0.005.

V. Results

The critical values, C^*, resulting from the estimated values for α, β, σ_1, σ_2 and ρ and the assigned values for γ and δ are given in Table 4. Also calculated in Table 4 is the minimum amenity value, A^*, which current amenity value must exceed to justify continued preservation if the current value of merchantable timber in the Headwaters Forest is V_0 = \$550 million.

Recall that this was reported to be what Charles Hurwitz would be willing to accept for sale of the Headwaters Forest along with a 1,500 acre buffer zone (Skow 1994, p.60). The critical amenity value was calculated as $A^* = (\$550 \text{ million})/C^*$.

The first thing to note in Table 4 is that for a discount rate of $\delta = 0.04$, $\delta - \alpha < 0$, thus preservation is optimal based on an estimated mean drift in amenity value of 4.6 percent. With an estimated rate of increase in real redwood prices of 3.9 percent, it is always the case that $(\delta + \gamma - \beta) > 0$ for $\delta \geq 0.04$.

For discount rates greater than $\alpha = 0.046$, the value of timber could drift high enough, relative to amenity value, to make cutting the Headwaters optimal. As the discount or the decay rate increases, C^* falls, increasing the minimum amenity value necessary to justify continued preservation. For example, when $\delta = 0.06$ and γ equals zero, $C^* = 151.789$ and current annual amenity value must exceed $A^* = \$3.623$ million per year to justify continued preservation. If the discount rate increases to $\delta = 0.10$, while the decay rate increases to $\gamma = 0.015$, C^* drops to 26.396, and the current amenity value must exceed \$20.836 million to justify continued preservation.

VI. Summary and Conclusions

The value of old-growth timber can be viewed as an option value. If old growth is preserved today, one preserves the option of cutting in the future. While waiting, the old growth provides a flow of amenity services and an expected capital gain, if timber prices are expected to increase over time. One can revisit the preservation decision at any future date. When evaluating continued preservation versus the cutting of old-growth timber, one must choose between the stumpage value that one could obtain by cutting today versus the discounted expected value of amenity services and capital gains.

In a model developed by Reed (1993), and modified for this paper, there are conditions where preservation is unequivocally optimal and conditions where cutting may be optimal. Preservation is unequivocally optimal if the sum of the discount rate plus the decay rate is less than the expected rate of increase in real timber prices or if the discount rate is less than the expected rate of increase in amenity value. These conditions were denoted as $\delta + \gamma - \beta < 0$ and $\delta - \alpha < 0$, respectively.

If preservation is *not* unequivocally optimal, there exists a critical value, C^*. If the current ratio of timber to amenity value reaches the critical value, then the old growth should be cut. If V(t) denotes current timber value, A(t) current amenity value, then it is optimal to cut when $V(t)/A(t) = C^*$ for the first time. The critical value will depend on the expected drift and variance rates for timber and amenity value, the correlation between the rate of change in timber price and amenity value, the discount rate, and in the model of this paper, the rate of decay in merchantable timber in the old-growth forest. These were referred to as the seven bioeconomic parameters, denoted as α, β, σ_1, σ_2, ρ, δ. and γ.

Amenity value, that is, the value experienced by visitors to the old-growth forest, as well as the option and existence value derived by nonvisitors, is very difficult to measure. The stumpage value from cutting today, V_0 is more easily measured. In the case where it might be optimal to cut, and where C^* can be calculated from the seven bioeconomic parameters, it is possible to calculate a critical annual amenity value, which if exceeded by current amenity value, would justify continued preservation. This is simply $A(t) > A^* = V_0/C^*$.

The model was applied to the Headwaters Forest, the last privately held stand of old-growth, coastal redwood. The drift and variance rates for timber and amenity value, and their correlation coefficient, were estimated from time-series data on the real price of a 16 foot, old-growth, redwood log, as estimated by the California State Board of Equalization, and from time-series data on visitation rates to the Redwood National Park. The discount rate, δ, and the rate of decay, γ, were varied over a plausible range, and values for C^* and A^* were calculated.

The mean drift in real redwood prices was 3.9 percent for the period 1977 through 1995. The mean drift in the visitation rate to the Redwood National Park was 4.6 percent. If the visitation rate to the Redwood National Park is perfectly correlated with the unknown amenity value of the preserved Headwaters, then the real, inflation-free rate of discount must exceed 4.6 percent if cutting is ever to be optimal.

If the current value of standing old-growth timber in the Headwaters is V_0 = \$550 million, then with a discount rate of 6 percent and a decay rate of zero, an amenity value of slightly more than \$3.623 million would justify continued preservation (based on a calculated $C^* = 151.789$). If the discount rate increases to 10 percent and the decay rate increases to 1.5 percent, the critical value drops to $C^* = 26.396$, and current annual amenity value would have to exceed A^* = \$20.836 million. Continued preservation is harder to justify at high rates of discount and decay.

What is the current amenity value of the Headwaters Forest? We know that in 1990 residents of the State of California voted down initiatives which would have required the State to purchase the Headwaters at fair market value. Given the other elements in these initiatives (including pesticide regulations) it is not clear that the negative vote was symptomatic of a low amenity value for the Headwaters. Preservation is likely to convey amenity value beyond the boarders of California, and therefore the federal legislation introduced in 1994 may be the appropriate level for government involvement. A well-designed contingent valuation survey might provide estimates of current amenity value which could be compared with the A^*s in Table 4.

Perhaps the most critical assumption in this paper is that the amenity value of the Headwaters is perfectly correlated with the visitation rate to the Redwood National Park, fifty miles north to the north. This seems like a reasonable first assumption, but other observable time series might be argued as being more closely correlated with the unknown amenity value of the Headwaters Forest.

Appendix

In the text it was assumed that timber and amenity value followed geometric Brownian motions as given by the equations

$$dV = (\beta - \gamma)V\,dt + \sigma_1 V\,dz_1 \tag{A.1}$$

$$dA = \alpha A\,dt + \sigma_2 A\,dz_2 \tag{A.2}$$

Let $v = \ln V$ and $a = \ln A$. Then from Itô's Lemma we know that

$$dv = (\beta - \gamma - \sigma_1^2/2)\,dt + \sigma_1\,dz_1 \tag{A.3}$$

$$da = (\alpha - \sigma_2^2/2)\,dt + \sigma_2\,dz_2 \tag{A.4}$$

It is also well known that a(t) is normally distributed with an expected value of $(\alpha - \sigma_2^2/2)t$ and a variance of $\sigma_2^2\,t$. Amenity value, A(t), is log normally distributed with an expected value given by

$$E\{A(t)\} = A(0)\,e^{\alpha t} \tag{A.5}$$

and a variance given by

Provided that $(\delta + \gamma - \beta) > 0$ and $(\delta - \alpha) > 0$ the expected present value integrals for timber and amenity value will converge, and it is possible to define the *intrinsic value* or *terminal payoff function*, as the value of timber less the expected present value of foregone amenity benefits. If the old-growth forest were cut at $t = T$, this terminal payoff function is given by

$$
\begin{aligned}
R[V(T),A(T)] &= V(T) - E\left\{\int_T^{\infty} A(t)\, e^{-\delta(t-T)}\, dt\right\} \\
&= V(T) - \int_T^{\infty} A(T)\, e^{\alpha(t-T)}\, e^{-\delta(t-T)}\, dt \\
&= V(T) - A(T)\int_T^{\infty} e^{-(\delta-\alpha)(t-T)}\, dt \\
&= V(T) - A(T)\left[-\frac{e^{-(\delta-\alpha)(t-T)}}{(\delta-\alpha)}\right]_T^{\infty} \\
&= V(T) - A(T)\left[0 + \frac{1}{(\delta-\alpha)}\right] \\
&= V(T) - \frac{A(T)}{(\delta-\alpha)}
\end{aligned}
$$

Or with $V(T) = e^{v(T)}$ and $A(T) = e^{a(T)}$ we can write this function as

$$R[v(T),a(T)] = e^{v(T)} - \frac{e^{a(T)}}{(\delta - \alpha)} \tag{A.7}$$

The *value function* for this stopping problem is denoted by W(v,a) and defined by the tautology W(v,a) = Max [R(v,a),W(v,a)]; that is, it is the maximum of either the terminal payoff or the value of preserving the option to cut at a later date.

Stopping rule problems are characterized by continuation regions, where the value function is W(v,a) > R(v,a), and stopping regions where W(v.a) $\leq$ R(v,a). On the boundary of the stopping region the value function equals the terminal payoff function. This is called the *continuity condition* or the *value-matching condition*.

In this and many other economic applications, the boundary between the continuation and stopping regions will be given by an unknown constant. In terms of v and a this constant will be a critical value $k^* = v - a$. Intuitively, this says that if v(t) ever exceeds a(t) by some amount k^*, the old growth should be cut. Given the relationship between v(t) and V(t) and a(t) and A(t), an equivalent stopping rule is to cut the old growth the first time that $V(t)/A(t) = C^* = e^{k^*}$. When a stopping-rule problem requires one to solve for both the value function and the boundary (in our case the critical constant k^*) it is called a *free-boundary problem*.

On the continuation region, where W(v.a) > R(v,a), the value function must satisfy the Hamilton-Jacoby-Bellman equation given by

$$\delta W = (\beta - \gamma - \sigma_1^2/2)W_v + (\alpha - \sigma_2^2/2)W_a + (\sigma_1^2/2)W_{vv} + (\sigma_2^2/2)W_{aa} + \rho\sigma_1\sigma_2 W_{va} \qquad \text{(A.8)}$$

where the subscripts on W indicate first - and second-order partial derivatives. It can be shown that the candidate function

$$W(v,a) = Be^{(1+\theta)v - \theta a - \theta k} \qquad \text{(A.9)}$$

satisfies the H-J-B equation provided

$$\theta^2/2 + [1/2 + (\beta - \gamma - \alpha)/\sigma^2]\theta - (\delta + \gamma - \beta)/\sigma^2 = 0 \qquad \text{(A.10)}$$

where B is a constant to be determined, k is the critical difference between v and a which induces cutting the old-growth forest, $\sigma^2 = \sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2$ is the variance of [v - a], and $E\{dz_1, dz_2\} = \rho dt$.

For W(v,a) > R(v,a), we must choose the positive root of (A.10). This can be confirmed by comparing W and R, and thus

$$\theta = -[1/2 + (\beta - \gamma - \alpha)/\sigma^2] + \sqrt{[1/2 + (\beta - \gamma - \alpha)/\sigma^2]^2 + 2(\delta + \gamma - \beta)/\sigma^2} \qquad \text{(A.11)}$$

To solve for the constant B and the critical value k^* we need an additional boundary condition. It turns out that there are two conditions in this two-dimensional problem. They are called the *smooth-pasting conditions* and require that the value function and the terminal payoff function have the same slope at the boundary, k^*. If this were not the case, either $W > R$ for $k > k^*$ or $W < R$ for $k < k^*$, either of which contradicts k^* being a critical boundary value. See Dixit and Pindyck (1994, pp. 130-132).

The smooth-pasting conditions require $W_v = R_v$ and $W_a = R_a$ when evaluated at the boundary where $k = v - a$. These conditions, along with the continuity condition, yield three equations, any two of which imply the third. They are listed in simplified form below.

$$Be^v = e^v - \frac{e^a}{(\delta - \alpha)} \qquad \text{(from } W = R\text{, when } k = v - a\text{)}$$

$$Be^v (1 + \theta) = e^v \qquad \text{(from } W_v = R_v\text{)}$$

$$Be^v \theta = \frac{e^a}{(\delta - \alpha)} \qquad \text{(from } W_a = R_a\text{)}$$

These conditions imply $B = 1/(1 + \theta)$ and $B\theta = e^{-k}/(\delta - \alpha)$ which can be solved for the critical value

$$C^* = e^{k^*} = \frac{1}{(\delta - \alpha)}\left(\frac{1 + \theta}{\theta}\right) \tag{A.12}$$

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Figure 1. The Boundary C^* and Three Sample Paths

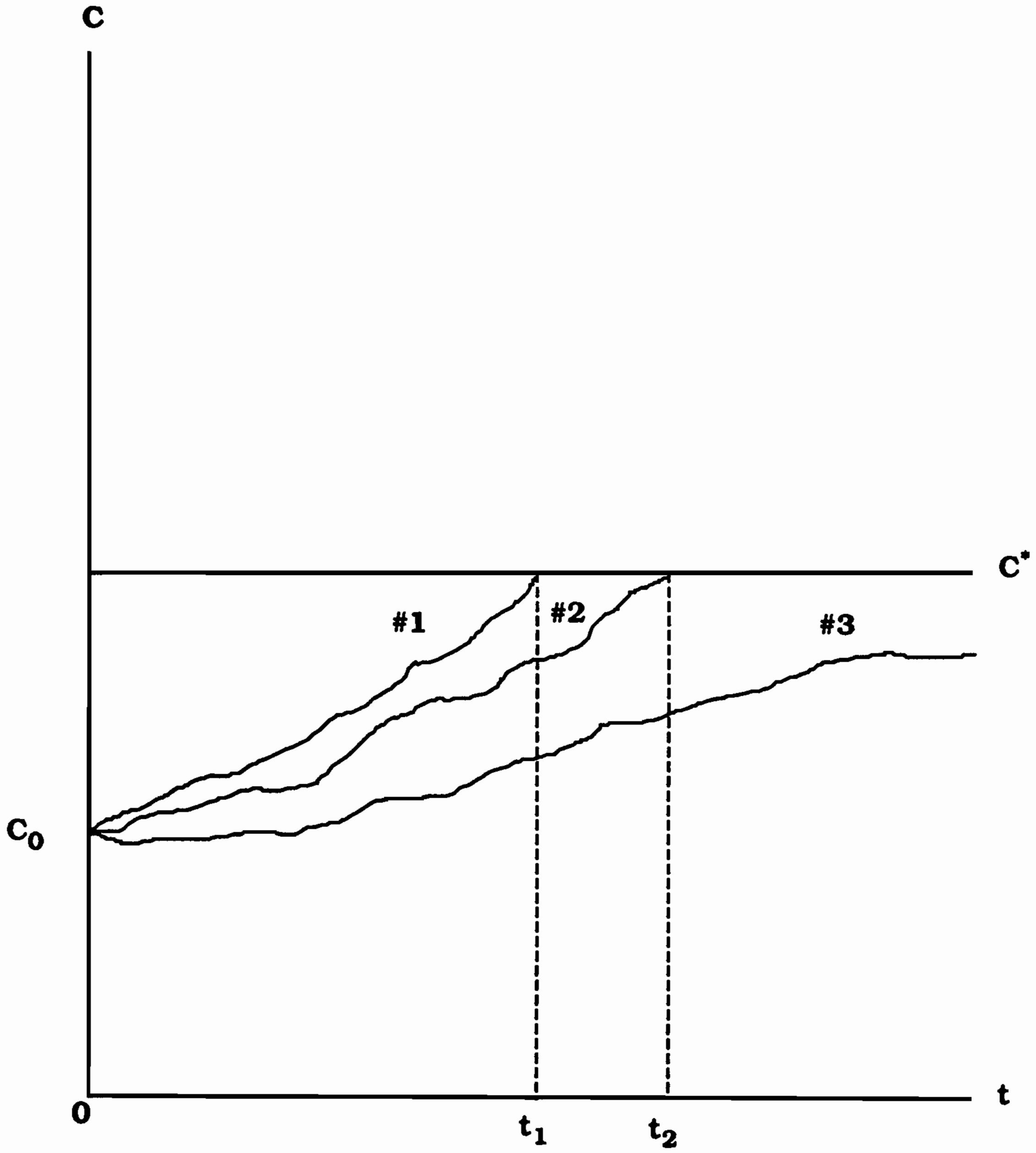

Figure 2. A Plot of the Real Redwood Price (x1000) and the Visitation Rate to the Redwood National Park

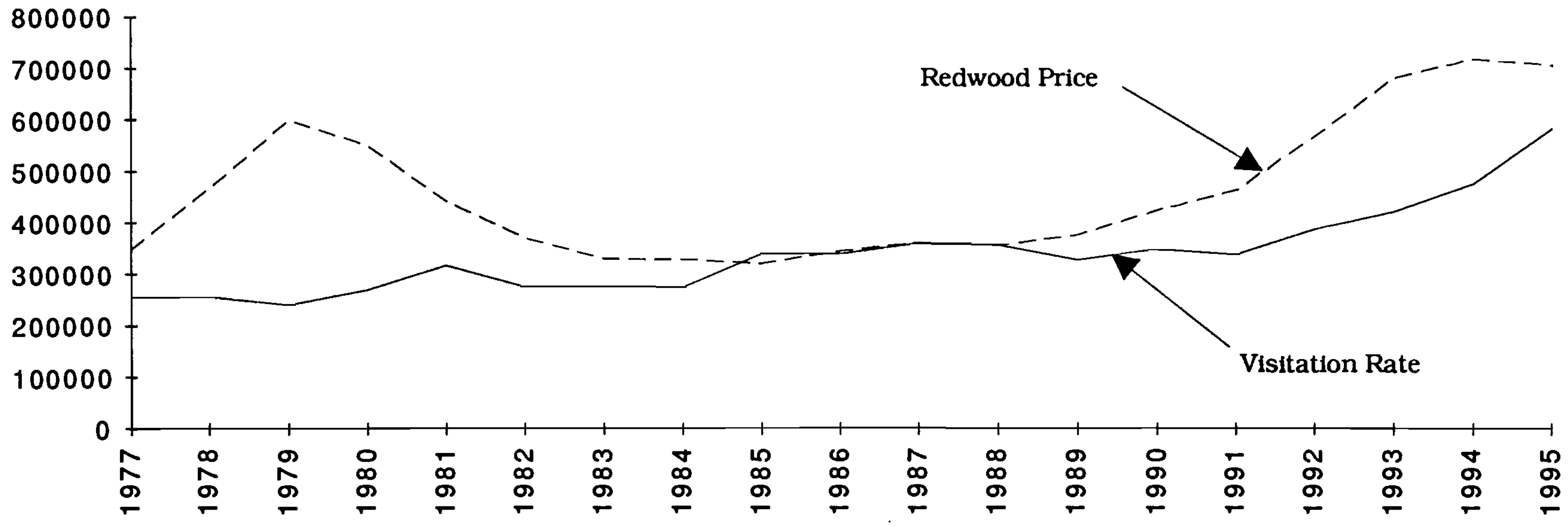

Table 1. The Price of Old-Growth Redwood and Visitation Rates at the Redwood National Park

Year	Price*	Visits†
1977	347.8	255,372
1978	465.6	256,705
1979	599.2	240,182
1980	551.1	269,717
1981	442.2	316,409
1982	370.0	274,718
1983	329.7	277,389
1984	327.9	273,392
1985	320.0	338,830
1986	344.0	339,275
1987	360.5	359,950
1988	356.5	356,230
1989	376.3	327,058
1990	423.7	348,458
1991	462.2	337,979
1992	568.2	387,781
1993	681.6	421,027
1994	717.1	475,033
1995	706.4	583,227

*The price is the real average annual price in dollars for an old-growth, 16 foot, redwood log of size quality #1 (over 850 board feet, net volume) from Timber Value Area 1. Source: California State Board of Equalization, *Harvest Value Schedules*. Prices were deflated by the Producer Price Index, 1982=100. The 1995 PPI of 127.4 is based on the index through April, 1995.

†The visitation rates are the estimated number of visitors. The 1995 figure is a simple projection based on cumulative visits through May, 1995. Source: John Wise, Redwood National Park and Tom Wade, Socioeconomics Research Group, National Park Service.

Table 2. Tests that $p_t = \ln P_t$ and $a_t = \ln A_t$ are consistent with Brownian Motion

A. $p_t = \ln P_t$

Unrestricted Regression: $p_t - p_{t-1} = \alpha + \beta\, t + (\rho - 1)\, p_{t-1} + \lambda\, (p_{t-1} - p_{t-2})$

Results:	α	β	$(\rho - 1)$	λ	SSR_u
	1.4411822	0.00984988	- 0.2542483	0.72655275	0.07395752
	(0.48856245)	(0.00391567)	(0.08180353)	(0.14846282)	

Restricted Regression: $p_t - p_{t-1} = \alpha + \lambda\, (p_{t-1} - p_{t-2})$

Results:	α	λ	SSR_r
	-0.0006692	0.59170838	0.14987221
	(0.02535093)	(0.17409561)	

T = 17 k = 4 q = 2 $F = \dfrac{(T - k)(SSR_r - SSR_u)}{qSSR_u} = 6.67$

H_0: $\beta = 0$, $\rho = 1$, Critical D-F at 5% is $F^* = 7.24$, Decision: Fail to Reject H_0

B. $a_t = \ln A_t$

Unrestricted Regression: $p_t - p_{t-1} = \alpha + \beta\, t + (\rho - 1)\, p_{t-1} + \lambda\, (p_{t-1} - p_{t-2})$

Results:	α	β	$(\rho - 1)$	λ	SSR_u
	7.36345305	0.02529017	- 0.5969419	0.18004034	0.12928559
	(5.32123048)	(0.014154)	(0.43045153)	(0.3617394)	

Restricted Regression: $p_t - p_{t-1} = \alpha + \lambda\, (p_{t-1} - p_{t-2})$

Results:	α	λ	SSR_r
	0.05123349	-0.0810753	0.16731727
	(0.02756849)	(0.27915735)	

T = 17 k = 4 q = 2 $F = \dfrac{(T - k)(SSR_r - SSR_u)}{qSSR_u} = 1.91$

H_0: $\beta = 0$, $\rho = 1$, Critical D-F is again $F^* = 7.24$, Decision: Fail to Reject H_0

Table 3. Estimates of Mean Drift and Standard Deviation for Redwood Timber Prices and Visitation Rates at the Redwood National Park, and the Estimated Correlation Coefficient for Prices and Visits*

$$\hat{\beta} = 0.039$$

$$\hat{\sigma}_1 = 0.140$$

$$\hat{\alpha} = 0.046$$

$$\hat{\sigma}_2 = 0.100$$

$$\hat{\rho} = -\ 0.101$$

*The estimates of mean drift and standard deviation are the sample mean and standard deviation of the series $\{\ln[P_{t+1}/P_t]\}$ and $\{\ln[A_{t+1}/A_t]\}$, where A_t is the visitation rate in year t, assumed to be perfectly correlated with the unknown amenity value of the Headwaters Forest. The correlataion coefficient between the series $\{\ln[P_{t+1}/P_t]\}$ and $\{\ln[A_{t+1}/A_t]\}$ provides an estimate of ρ.

Table 4. The Critical Ratio, C^*, and the Minimum Amenity Value, A^* (in 10^6 Dollars) that would Justify Continued Preservation of the Headwaters Forest if the Current Value of Old-Growth Timber were V_0 = $550 Million.

	δ			
γ	0.04	0.06	0.08	0.10
0.000	$\delta+\gamma-\beta>0$ $\delta-\alpha<0$	C^* = 151.789 A^* = 3.623	C^* = 51.505 A^* = 10.678	C^* = 29.566 A^* = 18.602
0.005	$\delta+\gamma-\beta>0$ $\delta-\alpha<0$	C^* = 133.920 A^* = 4.107	C^* = 48.273 A^* = 11.394	C^* = 28.307 A^* = 19.430
0.010	$\delta+\gamma-\beta>0$ $\delta-\alpha<0$	C^* = 122.189 A^* = 4.501	C^* = 45.770 A^* = 12.017	C^* = 27.266 A^* = 20.172
0.015	$\delta+\gamma-\beta>0$ $\delta-\alpha<0$	C^* = 113.967 A^* = 4.862	C^* = 43.791 A^* = 12.560	C^* = 26.396 A^* = 20.836

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