Working Papers in
AGRICULTURAL ECONOMICS
March 1993 **93-03**

A Convolutions Approach to Measuring the Differences in Simulated Distributions: Application to Dichotomous Choice Contingent Valuation

by
Gregory L. Poe
Eric K. Lossin
Michael P. Welsh

Department of Agricultural Economics
New York State College of Agriculture and Life Sciences
A Statutory College of the State University
Cornell University, Ithaca, New York, 14853-7801

Abstract

Resampling or simulation techniques are now frequently used in applied economic analyses, but previously developed significance tests for differences in empirical distributions have either invoked normality assumptions or used non-overlapping confidence interval criteria. This paper demonstrates that such methods will generally not be appropriate, and presents an exact empirical test, based on the method of convolutions, for assessing the statistical significance between approximate empirical distributions created by resampling techniques. Application of the proposed convolutions approach is illustrated in a case study using empirical distributions from dichotomous choice contingent valuation data.

The authors are, respectively, Assistant Professor, Department of Agricultural Economics, Cornell University, Research Assistant, Department of Agricultural Economics, University of Wisconsin - Madison, and Senior Project Manager, HBRS, Inc.
The comments of Richard Bishop and Bill Provencher are gratefully acknowledged, but, of course, any errors remain our responsibility. Funding for this paper was provided through the College of Agricultural and Life Sciences, University of Wisconsin-Madison and the College of Agriculture and Life Sciences, Cornell University. Data used in this paper was collected for Glen Canyon Environmental Studies sponsored by the U.S. Bureau of Reclamation.

A Convolutions Approach to Measuring the Differences in Simulated Distributions: Application to Dichotomous Choice Contingent Valuation

Introduction

Resampling or simulation techniques are increasingly applied to estimate standard deviations and confidence intervals for welfare measures (Kling; Kling and Sexton; Adamowicz, Fletcher and Graham-Tomasi; Creel and Loomis), elasticities and flexibilities (Dorfman, Kling and Sexton; Marquez), economies of size and scope (Eakin, McMillan and Buono; Schroeder), travel cost models (Loomis, Park and Creel), and contingent valuation (Park, Loomis and Creel; Duffield and Patterson; DesVousges et. al., 1992a, 1992b). While considerable effort has been focused on motivating, developing and comparing alternative methods of approximating distributions, very little attention has been given to developing formal statistical tests of the difference between approximate empirical distributions generated by these techniques. Such assessments are essential to applied economic and policy analyses in which comparison of point estimates are needed across policy alternatives, population and commodity groups, inputs, and levels of provision of non-marketed goods.

Using the dichotomous choice contingent valuation method (DC-CVM) as an example, this paper presents a statistical test, based on the method of convolutions, to evaluate the significance of the difference between approximate empirical distributions and illustrates how to apply this test to actual DC-CVM data. Because the convolution formula provides an exact measure of the statistical significance of the difference between empirical distributions, it is preferable to previous techniques that either impose restrictive assumptions of normality or adopt a non-overlapping confidence interval criterion. Moreover, the non-parametric nature of this test is a logical extension of the motivations for using empirical distributions in the first place.

The DC-CVM was chosen as a vehicle for demonstrating this approach because: 1) bootstrapping and other resampling techniques are widely being adopted to approximate distributions of DC-CVM benefit measures; 2) comparisons of benefit measures for different quality levels and scenarios is a fundamental objective of DC-CVM (Cummings, Brookshire and Schulze; Mitchell and Carson); 3) benefit comparisons are also essential to assessing the validity and reliability of the contingent valuation method (Bishop and Heberlein; Loomis); and 4) DC-CVM is an area in which statistically biased or otherwise inappropriate techniques for comparing approximated distributions have been used and reported by some researchers. It is essential to note, however, that the discussion that follows is not limited to the particular estimation approach applied in this example. The criticisms and suggested techniques developed in this paper with respect to DC-CVM are generalizable to any simulated distributions of economic parameters for which it is reasonable to ask "Is the difference between distributions significantly different from zero?".

The remainder of this paper is organized as follows. A critique of the methods currently being used to evaluate the significance of the difference between empirical distributions is provided in the following section. The third section presents the convolutions method. The mechanics of this technique are demonstrated using simple hypothetical distributions in the fourth section, and the convolutions technique is applied to DC-CVM data in the fifth section.

A Critique of Past Methods for Evaluating Differences in Simulated Distributions

Instead of providing a detailed review of resampling techniques currently being used in the economic literature, this paper assumes that two approximate empirical distributions of point estimates, such as those presented in Figure 1, have already been created[1]. Interpretation of Figure 1 is as follows: $f_X(X)$ and $f_Y(Y)$ are the simulated probability density functions for parameters X and Y; the shaded area represents an approximate (1-γ) confidence interval; and $L_{1-\gamma}(.)$ and $U_{1-\gamma}(.)$ depict the lower and upper bounds of this confidence interval. Although the two distributions lie on the same number line, they are separated in the figure in order to isolate the degree of overlap between the two confidence intervals.

The dichotomous choice format asks individuals if they would be willing to pay a specified amount, or bid value, for a public good. Bid values (A) are randomly assigned across survey participants, and the yes/no (1/0) responses across participants and bid values can be modeled using a random utility framework (Hanemann, 1984).

The following linear logit distribution is frequently used to model the cumulative distribution function (G(A;θ)) of willingness to pay

$$G(A;\theta) = \Pi^N(A) = [1+e^{\theta}]^{-1} \tag{1a}$$

where,

$$\theta = \alpha - \beta A + \xi X \tag{1b}$$

$\Pi^N(A)$ is the probability of a 'no' response to bid value A, X is a vector of other explanatory variables, and α, β and ξ are coefficients to be estimated (Hanemann (1984); Bowker and Stoll; Boyle and Bishop). Approximate empirical distributions of the mean that correspond to those presented in Figure 1 may then be calculated using resampling techniques (e.g. bootstrapping, Krinsky and Robb, monte carlo) for the estimated coefficients and the following closed-form solution presented in Hanemann (1989)

$$E(WTP) = \int_0^{\infty}[1-G(A;\theta)]dA = \frac{1}{\beta}\ln(1+e^{\alpha+\xi\bar{X}}) \tag{2}$$

In the above equation, E is the mathematical expectation operator and $\bar{X}$ is the mean value vector corresponding to X.

Two techniques for evaluating the significance of the difference between these distributions have been proposed in the literature. The first, as implicitly suggested by Krinsky and Robb and applied by Desvousges et al. (1992b), is that if the simulated distributions are approximately normal then classical statistical procedures for estimating differences can be applied. For instance, assuming equal and known standard errors (STDERR) for two normal distributions, the null hypothesis that the 'true' mean of the first distribution is equal to the 'true' mean of the second distribution is tested using the following difference formula,

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{2} * STDERR} \qquad \sim N(0,1) \tag{3}$$

where Z is the test statistic and $\bar{X}_i$ are the sample means (Snedecor and Cochrane, p. 101). As noted, the Z value has a standard normal distribution. In the bootstrapping framework, the standard error of the mean is given by the estimated standard deviation of the empirical distribution of the mean estimate.

Objections to this normality assumption occur at both a theoretical and empirical level. First, much effort has been given to developing these empirical approaches in order to capture non-linearities, and subsequent non-normalities, of the functions of parameters used to calculate the desired distribution. It would seem counterproductive to impose unneeded parametric assumptions at this stage. Second, our experience suggests that empirical distributions estimated following the method associated with Equations (1) and (2) are skewed, and the assumption of 'approximately normal' is often inappropriate. More generally, there is little reason to assume that non-linear functions of normal parameters will approximate a normal distribution, a fact that is equally relevant to elasticities, flexibilities, and welfare measures.

Park, Loomis and Creel (hereafter PLC) avoid the assumption of normality by employing a non-overlapping confidence interval criterion to evaluate differences in point estimates. That is, PLC judge the differences in mean willingness to pay across estimates to be statistically significant at the 10 percent level if their empirical 90 percent confidence intervals do not overlap. This approach is also used in Desvousges et al. (1992a) who state that "overlapping confidence intervals imply that significant differences do not exist...between WTP estimates" (p. 22). With respect to Figure 1, the distributions are judged, using this criterion, to be significantly different at the 5 percent level if $U_{0.95}(Y)$ lies to the left of $L_{0.95}(X)$ on a number line. If $L_{0.95}(X)$ lies to the left of $U_{0.95}(Y)$ then the two central confidence intervals overlap and the distributions are not judged to be significantly different at the 5 percent level using the non-overlapping confidence interval criterion.

In general, the actual significance of this non-overlapping confidence interval approach will not correspond to the stated level of the test. This point is demonstrated most simply for normal distributions using the analytical solution presented in Equation (3). Recall that for a single normal distribution the 95 percent confidence interval for the mean of an estimate is defined as $\bar{X}_i \pm 1.9600$*(STDERR). Again assuming that the standard errors for both distributions are known and equal, this implies that the critical difference in means, $(\bar{X}_1 - \bar{X}_2)$, associated with the non-overlapping 95 confidence intervals would have to be at least 3.9200 standard errors apart before they would be judged to be significantly different. Making this substitution, Equation (3) becomes

$$Z=\frac{(3.9200*STDERR)}{\sqrt{2}*STDERR}=\frac{3.9200}{\sqrt{2}}=2.772 \tag{4}$$

The estimated z value of 2.772 corresponds to a significance level (which shall be referred to as γ') of 0.0048 rather than the stated value of γ=0.05.

Conversely, Equation (4) can be rearranged and solved for the difference between two means that corresponds to a non-overlapping confidence interval for γ=0.05. Simple algebra and a critical value of 1.9600 indicate that the point where the two means is significantly different occurs when the means are approximately 2.772 standard errors apart. At this distance, the non-overlapping two-sided confidence intervals only encompass about 87 percent of their respective distributions.

Clearly the non-overlapping confidence interval criterion given by $(1-\gamma)$ confidence intervals does not correspond to the γ level of significance for the normals case. In general, a lack of correspondence between γ and γ' is expected. For the normal distribution above, the significance level is understated (i.e. $\gamma > \gamma'$) and the test is more conservative than indicated. The degree of this difference between γ and γ' will depend upon the shape of the empirical distributions that are being compared.

In sum, the two methods currently being applied in DC-CVM either involve inappropriate assumptions or are statistically biased.

The Method of Convolutions

Another alternative - one that accommodates any distributional form - is based on the method of convolutions. This technique is used in statistics and mathematics to evaluate the sum of distributions of random variables and series (Feller; Mood, Graybill and Boes).

Let X and Y be independent random variables[2], with respective probability density functions $f_X(x)$ and $f_Y(y)$. Then, for all values of X and Y

$$f(x,y) = f_X(x)f_Y(y) \tag{5}$$

Define the difference V = X-Y to be a new random variable. The probability of the event V=v is defined as the union of all the possible combinations of x and y which result in a difference of v. For continuous functions this relation is given explicitly as

$$f_V(v) = f_{X-Y}(v) = \int_{-\infty}^{\infty} f_Y(x-v)f_X(x)dx = \int_{-\infty}^{\infty} f_X(v+y)f_Y(y)dy \tag{6}$$

which is a variant of the convolution formula (Mood, Graybill and Boes). Using only the far right hand side of Equation (6), the cumulative distribution function $F_V(v^o)$ of the difference of X and Y is

$$F_V(v^\circ) = \int_{-\infty}^{v^\circ} f_V(v)dv = \int_{-\infty}^{v^\circ}\int_{-\infty}^{\infty} f_X(v+y)f_Y(y)dydv \tag{7}$$

For empirical applications with discrete observations, the dimensions of Equation (7) can be reduced substantially. If $f_Y(y)=0$ or $f_X(v+y)=0$ then $f_V(v)=0$ also. This implies that the range of the first integrand can be bounded by the minimum of the ordered y vector and the value of y for which (v+y) exceeds the range of the empirical distribution of $f_X(x)$. These values shall be denoted infy and supy, respectively. Similarly the second integral can be bounded from below by the minimum possible value for X-Y, denoted here as infv. In this manner Equation (7) can be restated for discrete probabilities obtained from simulation procedures as

$$\hat{F}_V(v^\circ) = \sum_{infv}^{v^\circ}\sum_{infy}^{supy} \hat{f}_X(v+y)\hat{f}_Y(y)\Delta y \Delta z \tag{8}$$

where $\hat{F}_V(v^\circ)$, $\hat{f}_X(x)$ and $\hat{f}_Y(y)$ are discrete approximations of $F_V(v^\circ)$, $f_X(x)$ and $f_Y(y)$. The incremental values for y and z are defined by the desired level of precision and computational power.

The above equations can be directly applied to the information provided from the simulated distributions. As in the simulation methods, the distribution of the differences will generally not be known, and an empirical approach to estimating confidence intervals is necessary. Adopting a 'percentile approach' (Effron) the lower bound and upper bound of the 1-γ confidence intervals are respectively defined as

$$\hat{L}_{1-\gamma}(Z)=\hat{F}_Z^{-1}(\gamma/2) \qquad \hat{U}_{1-\gamma}(Z)=\hat{F}_Z^{-1}(1-\gamma/2) \tag{9}$$

And,

$$[\hat{L}_{1-\gamma}(Z),\hat{U}_{1-\gamma}(Z)] \tag{10}$$

is the approximate (1-γ) central confidence interval for Z. This range will often be non-symmetric around the mean.

Combining the principle of the two sided difference in means test with a percentile approach, the null hypothesis that the difference between X and Y equals zero is accepted at the γ level of significance if the approximate (1-γ) confidence interval of the convolution includes zero and rejected otherwise. Alternatively, assuming that the distributions are ordered in a descending fashion, the approximate significance of the difference between distributions is determined by twice the value of the cumulative distribution function at the convoluted value of zero.

A Simple Demonstration of the Convolutions Technique

This section demonstrates the application of the discrete convolution formula presented in Equation (8) and the suggested statistical test for estimating the significance of the difference between two approximate empirical distributions. Suppose that we are interested in evaluating the difference between the two approximate empirical distributions presented in Table 1. The probability density function (pdf), cumulative distribution function (cdf), and the calculations required to generate a convolution of these distributions are demonstrated in Table 2, where $f_V(.)$ and $F_V(.)$ are the pdf and cdf respectively and only the values that lie within the bounds set by infy, supy and infv are reported. Evaluating $F_V(0)$ indicates that the two distributions are different at the 17 (=2*0.085) percent level.

Application of the Convolutions Approach

This section applies the convolutions technique to evaluating differences in compensating variation associated with two different water flow levels in the Grand Canyon. In addition, this section further demonstrates that the normality based approach and the non-overlapping confidence interval criterion are inappropriate and may lead to misguided conclusions about the significance of the difference between distributions in policy relevant applications. In order to focus on the convolutions technique, the model presented in this example is intentionally simplistic -- only the bid value and cost of the trip are included as explanatory variables in the statistical analysis. More sophisticated models and a greater description of the study are presented in Bishop et al. [1987], Bishop et al. [1989], and Boyle, Welsh and Bishop.

Flow level in the Grand Canyon is a decision variable for the Glen Canyon dam, which generates electricity and regulates flows below the dam. These flow levels, measured in cubic feet per second (cfs), are outside the control of boaters but do affect the quality of whitewater rafting trips in the Grand Canyon.

> "Time at attraction sights, such as Indian ruins and side canyons with pleasing scenery, and for layovers, depends on the speed of the current. The size and the number of rapids are affected by dam releases. Boaters, particularly those on commercial trips, enjoy fairly large rapids that depend on substantial flows. At relatively low flows and flood flows, passengers, particularly those on commercial oar powered trips, may have to walk around rapids. This is generally considered undesirable by passengers" (Bishop et al., 1987, p. 11-12)

Given these considerations, Hicksian surplus values for different flow levels are implicitly defined as

$$V(P,Y-H_j;f_j) = V(P^m,Y;f_j) \tag{11}$$

where V(.) is an indirect utility function, P is the price, Y is income, f_j is the jth flow, H_j is Hicksian compensating surplus (WTP) for the jth flow, and P^M is the choke price at or above which the trip would not be taken (Boyle, Welsh and Bishop). In this simplified analysis other trip attributes and personal characteristics are assumed to be constant, and are subsumed here for notational convenience.

Two different flow ranges are considered in this analysis: 0 to 25,000 cfs and 26,000 to 33,500 cfs. These flow ranges are termed low and high flows respectively. The value of 33,500 cfs corresponds to the maximum flows that can be used to generate electricity by the dam, and thus represents the maximum of the policy relevant range. The 25,000 cfs cut-off point approximates the mean of the flow levels experienced by participants in the survey sample, and was used as an ad hoc division between low and high flows.

The linear specification of the logit model detailed in Equation (1) above was used to evaluate the distribution of willingness to pay from bid values and responses, with

$$\theta_j = \alpha_j + \beta_j A + \xi_j P \tag{12}$$

In the above equation α, β and ξ again represent coefficients to be estimated, j indicates the range of flows experienced, P is the price of the trip taken, and A is the dichotomous choice bid value. Respondents were grouped into flow level categories based on their mean flow level experienced during their trip taken from hydrological data. As presented in Table 3, the estimations are fairly robust. Although some individual coefficients are not significant, each estimated equation has highly significant χ^2 values.

In addition, log-likelihood ratio tests indicate that the estimated distribution of WTP for the low and the high flow levels are significantly different at the 10 percent level (LR=8.27 > $\chi^2_{3,10}$=6.25). Thus, we can conclude that, over these flow ranges, flow levels do have a significant effect on the distribution of WTP.

Whether there are significant differences in Hicksian surplus values is a different question, one that can only be answered by comparing distributions of mean willingness to pay estimates. Formally the hypothesis being tested is that

$$H_L = H_H \tag{13}$$

where H_L and H_H are the Hicksian surpluses associated with low and high flow conditions.

Estimated means and their distributions for each scenario were created by applying the Krinsky and Robb technique to the closed-form solution presented in Equation (2) above[3]. In calculating the empirical distributions, intervals for Δy and Δv were set at 1. Critical points on these distributions are presented in Table 4 and the distributions themselves are presented in Figure 2.

Evaluation of Table 4 and Figure 2 indicate two points of interest. First, the distributions evaluated here are significantly skewed and apparently deviate from normality. As a result of this observation, classical difference tests based on normality assumptions are not relevant here[4]. The second interesting point is that in spite of the statistical significance between WTP distributions for low and high flows, their distributions overlap considerably. Most notably, their 90 percent confidence intervals do overlap and thus, the application of the non-overlapping confidence interval criterion would lead to the conclusion that the mean WTP distributions are not significantly different at the 10 percent level.

A different conclusion is reached with the convolutions method detailed in this paper, for which the distribution is plotted in Figure 2 and critical points are identified in Table 5. In contrast to the non-overlapping confidence interval criterion, the mean willingness to pay values for low and high flows are judged to be significantly different at the 4.2 percent level and the 90 (and 95) percent confidence intervals for the difference do not include zero. This level of significance is clearly less than 10 percent, indicating that the non-overlapping confidence interval criterion would lead to erroneous conclusions of significance in policy relevant situations. In this instance, the deviation between the significance judgement of the non-overlapping confidence interval and the actual level of significance is substantial.

Summary and Conclusions

Economists have increasingly turned to resampling or simulation techniques to explore the variability in a wide range of estimated economic parameters, including elasticities, flexibilities and various welfare measures. Undoubtedly resampling and simulation techniques are a valuable tool for exploring the inherent variability of estimated parameters for which it is difficult (if not impossible) to develop analytic variance estimates. However by themselves, these techniques do not provide a way to compare the distributions that arise from applying the techniques in various contexts. Often it is this comparison that is of most interest. For example, is the price elasticity significantly different between two demographic groups? Do any of the various policy options result in a higher value of the estimated welfare measure?

Appropriate answers to these types of questions require appropriate statistical tests. The two approaches that have been used and reported by researchers to compare approximate empirical distributions are inappropriate in most applications. The method of convolutions, as presented in this paper, does provide a proper statistical test for assessing the significance of the difference between two distributions and represents a logical extension of resampling techniques.

Application of the convolutions approach is not costless, however. While some computing packages offer routines that will perform a convolution of two distributions[5], the actual calculation of the convolution can be computationally intensive if the distributions have many points.

The decision of whether to adopt the convolutions approach will depend upon the objective of the research and the nature of the distributions. With respect to the normality assumption the decision is obvious. For those cases in which the hypothesis of normality is rejected, then using a normality based approach is wrong. Under those circumstances a convolutions approach would seem justified. The answer is less clear when considering the non-overlapping confidence interval criterion. The criterion is conservative in the sense that if two differences are found to be non-overlapping at the 5 percent level the difference between the two distributions is certainly significant at that level. However it is possible that the 95 percent confidence intervals will overlap, and yet, the distributions will actually be different at the 5 percent level. If the consequences of declaring a significant difference insignificant is of little importance then the non-overlapping confidence criteria might be deemed an acceptable test. Yet, if one is interested in reducing the chance that a significant difference is missed or if the researcher desires to report the actual level of significance, then the convolutions approach may prove advantageous.

References:

Adamowicz, W. L., J. L. Fletcher, and T. Graham-Tomasi. "Functional Form and Statistical Properties of Welfare Measures", *American Journal of Agricultural Economics*, 71 (May 1989):414-421.

Bishop, R. C., *et al.* "Glen Canyon Dam Release and Downstream Recreation: An Analysis of User Preferences and Economic Values", Final Report to the U.S. Bureau of Reclamation, Salt Lake City, 1987.

Bishop, R. C., *et al.* "Grand Canyon Recreation and Glen Canyon Dam Operations: An Economic Evaluation", Paper Presented at the joint meetings of the Western Committee on the Benefits and Costs of Natural Resource Planning (W133) and the Western Regional Science Association, San Diego, CA., 1989.

Bishop, R. C. and T. A. Heberlein, . "Measuring Values of Extramarket Goods: Are Indirect Measures Biased?", *American Journal of Agricultural Economics*, 61 (Dec 1979):926-930.

Bowker, J. M., and J. M. Stoll. "Use of Dichotomous Choice Nonmarket Methods to Value the Whooping Crane Resource", *American Journal of Agricultural Economics*, 70 (May 1988):372-382.

Boyle, K. J., and R. C. Bishop. "Welfare Measurements Using Contingent Valuation: A Comparison of Techniques", *American Journal of Agricultural Economics*, 70 (Feb 1988):20-28.

Boyle, K. J., M. P. Welsh and R. C. Bishop. "The Role of Question Order and Respondent Experience in Contingent-Valuation Studies", forthcoming *Journal of Environmental Economics and Management*, 1993.

Creel, M. D., and J. B. Loomis. "Confidence Intervals for Welfare Measures With Application to Truncated Counts", *The Review of Economics and Statistics*, 73 (May 1991):370-373.

Cummings, R. G., D. S. Brookshire, and W. D. Schulze, eds. *Valuing Environmental Goods: A State of the Art Assessment of the Contingent Valuation Method*, Rowman and Allanheld, Totowa, 1986.

Desvousges, W. H., F. R. Johnson, R. W. Dunford, K. J. Boyle, S. P. Hudson, and K. N. Wilson. "Measuring Natural Resource Damages with Contingent Valuation: Tests of Validity and Reliability", paper presented at the Cambridge Economics, Inc. symposium titled "Contingent Valuation: A Critical Assessment", Washington, D.C., April 2-3, 1992a.

Desvousges, W. H., F. R. Johnson, R. W. Dunford, K. J. Boyle, S. P. Hudson, and K. N. Wilson. *Measuring Nonuse Damages Using Contingent Valuation: An Experimental Evaluation of Accuracy*, Research Triangle Institute Monograph 92-1, 1992b.

Dorfman, J. H., C. L. Kling and R. J. Sexton. "Confidence Intervals for Elasticities and Flexibilities: Reevaluating the Ratios of Normals Case", *American Journal of Agricultural Economics*, 72 (Nov 1990):1006-1017.

Eakin, B. K., D. A. McMillen and M. J. Buono. "Constructing Confidence Intervals Using the Bootstrap Application to a Multiproduct Logit Function", *Review of Economics and Statistics*, 72 (May 1990):339-344.

Effron, B. *The Jackknife, the Bootstrap and Other Resampling Plans*, Monograph 38, Society Industrial and Applied Mathematics, Philadelphia, 1982.

Feller, W. *An Introduction to Probability Theory and Its Application, Volume I*, 2nd edition, John Wiley and Sons, Inc., New York, 1957.

Hanemann, W. M. "Welfare Evaluation in Contingent Valuation Experiments with Discrete Responses" *American Journal of Agricultural Economics*, 66 (Aug 1984):332-41.

Hanemann, W. M. "Welfare Evaluations in Contingent Valuation Experiments with Discrete Responses: Reply", *American Journal of Agricultural Economics*, 71 (Nov 1989):1057-1061.

Kling, C. L. "Estimating the Precision of Welfare Measures". *Journal of Environmental Economics and Management*, 21 (Nov 1991):244-259.

Kling, C. L., and R. J. Sexton. "Bootstrapping in Applied Welfare Analysis", *American Journal of Agricultural Economics*, 72 (May 1990):406-418.

Krinsky, I., and A. L. Robb. "On Approximating the Statistical Properties of Elasticities", *Review of Economics and Statistics*, 68(Nov 1986):715-719.

Loomis, J. B. "Test-Retest Reliability of the Contingent Valuation Method: A Comparison of General Population and Visitor Responses", *American Journal of Agricultural Economics*, 71 (Feb 1989):76-84.

Loomis, J., M. Creel, and T. Park. "Comparing Benefit Estimates from Travel Cost and Contingent Valuation Using Confidence Intervals for Hicksian Welfare Measures," *Applied Economics*, 23 (Nov 1991):1725-1731.

Marquez, J. "Bilateral Trade Elasticities", *The Review of Economics and Statistics*, 72 (Feb 1990):70-77.

Mitchell, R. C., and R. T. Carson. *Using Surveys to Value Public Goods: The Contingent Valuation Method*, Resources for the Future, Washington D.C., 1989.

Mood, A., F. Graybill, and D. Boes. *Introduction to the Theory of Statistics*, McGraw Hill, San Francisco, 1974.

Park, T., J. B. Loomis and M. Creel. "Confidence Intervals for Evaluating Benefits Estimates from Dichotomous Choice Contingent Valuation Studies", *Land Economics*, 67 (Feb 1991):64-73.

Poe, G. L. "Information, Risk Perceptions and Contingent Values: The Case of Nitrates in Groundwater," Ph.D. dissertation, University of Wisconsin-Madison, 1993.

Schroeder, T. C. "Economies of Scale and Scope for Agricultural Supply and Marketing Cooperatives", *Review of Agricultural Economics*, 14 (Jan 1992):93-103.

Snedecor, G. W. and W. G. Cochrane. *Statistical Methods*, 7th ed. Iowa State University Press, Ames, 1980.

Endnotes:

1. A critical review and comparison of the three techniques currently being used in DC-CVM is provided in Poe. The techniques themselves are developed separately in Park, Loomis and Creel, Duffield and Patterson, and Desvousges et al. [1992b].

2. Independence is not necessary for the convolution formula itself, but this assumption facilitates the empirical application of this method. This is not meant to imply that the assumption of independence is inconsequential. Indeed, for contingent valuation it implies that these estimates should be derived from separate samples or by some other means that assures independence. The need for independent samples is shared by other statistical approaches. For example, in applying the classical techniques based on normality assumptions Desvousges et al. (1992b, p. 30) note that "using independent samples....is essential for the hypothesis testing".

3. Estimates of the significance using methods suggested in Duffield and Patterson and Desvousges et al. [1992b] provide similar results and are available from the authors.

4. Application of the normality based approach depicted in Equation 3 (after accounting for inequality in the standard errors) yields $z \approx 1.24 = (782.79-548.23)/(171.49^2 + 79.02^2)^{0.5}$. The corresponding two-sided significance level is approximately equal to 21.5 percent. When comparing this significance level to

those provided later in the paper it should not inferred from this one sample that the normality based approaches that use standard errors will grossly understate the true difference and the difference obtained from using the non-overlapping confidence interval criterion. The direction and the degree of the relation between these values will be particular to the distributions being compared.

5. The convolutions program used in this paper was performed in GAUSS, making use of the CONV routine. It is our understanding that the option of programming a convolution exists in other matrix based languages (e.g. SAS-IML).

Figure 1: Simulated Distributions and the Non-Overlapping Confidence Interval Criterion

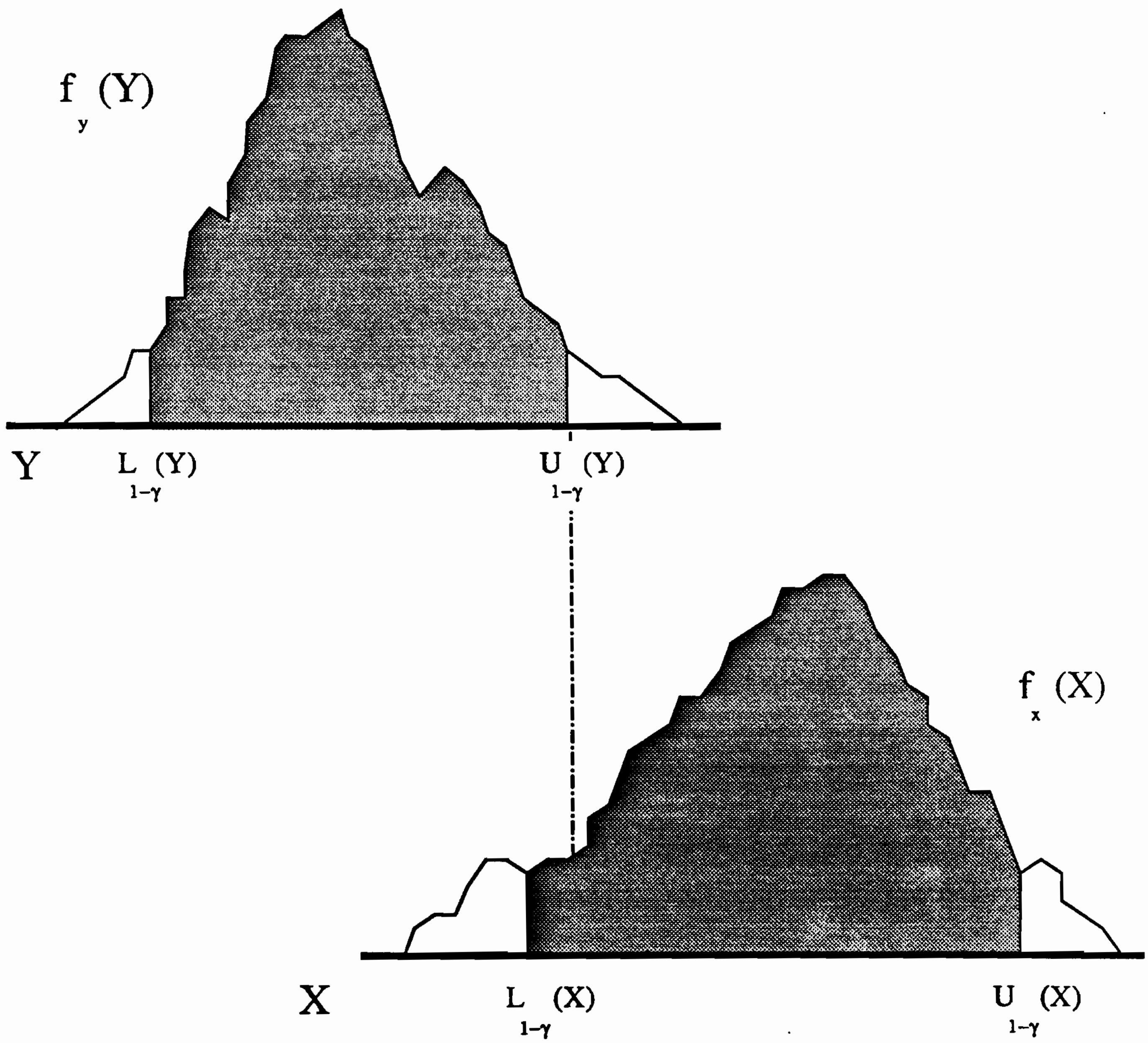

Figure 2: PDF and Convolution of Mean WTP for Low and High Flows

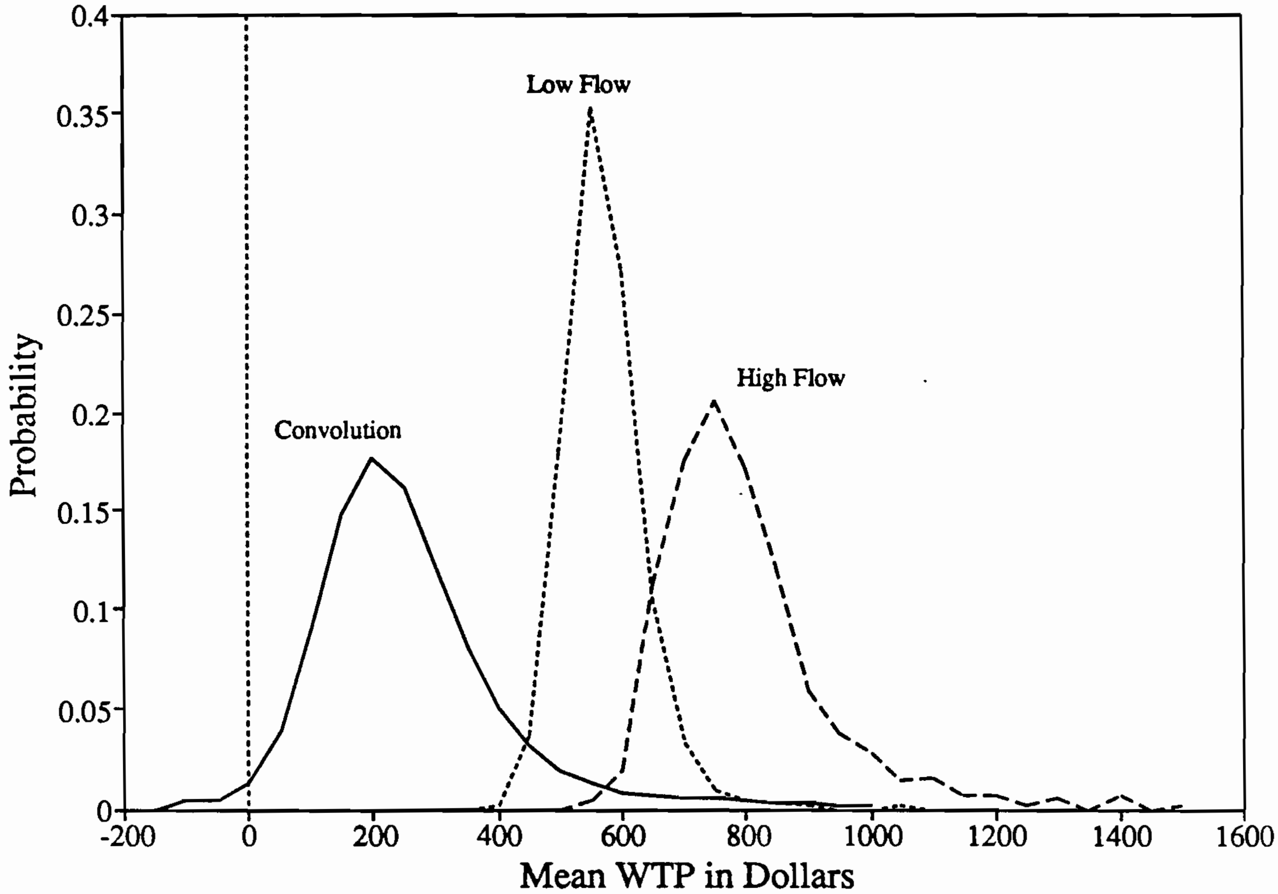

Table 1
Hypothetical Distributions

Range	$f_X(.)$	$f_Y(.)$
0	0.00	0.05
1	0.00	0.30
2	0.10	0.60
3	0.40	0.05
4	0.40	0.00
5	0.10	0.00

Table 2
Demonstration of Convolution for Simple Distributions

			$F_V(-2)$	= 0.000
$f_V(-1)$ =	$f_x(2)f_y(3)$	= 0.005	$F_V(-1)$	= 0.005
	(.1)(.05)			
$f_V(0)$ =	$f_x(2)f_y(2) + f_x(3)f_y(3)$	= 0.080	$F_V(0)$	= 0.085
	(.1)(.6) + (.4)(.05)			
$f_V(1)$ =	$f_x(2)f_y(1) + f_x(3)f_y(2) + f_x(4)f_y(3)$	= 0.290	$F_V(1)$	= 0.375
	(.1)(.3) + (.4)(.6) + (.4)(.05)			
$f_V(2)$ =	$f_x(2)f_y(0) + f_x(3)f_y(1) + f_x(4)f_y(2) + f_x(5)f_y(3)$	= 0.370	$F_V(2)$	= 0.745
	(.1)(.05) + (.4)(.3) + (.4)(.6) + (.1)(.05)			
$f_V(3)$ =	$f_x(3)f_y(0) + f_x(4)f_y(1) + f_x(5)f_y(2)$	= 0.200	$F_V(3)$	= 0.945
	(.4)(.05) + (.4)(.3) + (.1)(.6)			
$f_V(4)$ =	$f_x(4)f_y(0) + f_x(5)f_y(1)$	= 0.050	$F_V(4)$	= 0.995
	(.4)(.05) + (.1)(.3)			
$f_V(5)$ =	$f_x(5)f_y(0)$	= 0.005	$F_V(5)$	= 1.000
	(.1)(.05)			

Table 3
Estimated Logit Equations for Different Flow Levels for Commercial White Water Boaters

Flow Conditions[a,b]	Low	High
Constant	-1.368* (0.766)	0.418 (0.730)
Cost	-0.000373 (0.000436)	-0.00133*** (0.000423)
Bid	0.00380*** (0.000944)	0.00226*** (0.000732)
Model χ^2	21.78***	22.16***
n	98	128

[a] Asymptotic standard errors in ().
[b] Significance levels * (10%), ** (5%), *** (1%)

Table 4
Empirical Mean Willingness to Pay for Different Flow Ranges for Commercial White Water Boaters Using Krinsky and Robb Simulation Technique

Flow Level	Calculated from Parameter Means	Based on 1000 Draws							
		Lower Tail		Median	Mean	Upper Tail		Std.	Skew[a]
		5%	10%			10%	5%	Err.	
Low Flow	536.06	443.13	457.60	540.73	548.23	643.76	679.89	79.02	6.07
High Flow	735.14	609.10	623.22	745.75	782.77	1049.57	1221.05	171.49	4.18

[a] Skewness test: $g_{0.01,1000} = 0.180$ [Table 34b; Tables for Statisticians and Biometricians]

Table 5
Approximate Significance Levels and Confidence Intervals of Difference Between Mean Willingness to Pay Estimates for Different Flow Ranges for Commercial White Water Boaters Using Convolutions Technique

Estimated Significance Level of Difference	Confidence Interval Bounds			
	Lower Tail		Upper Tail	
	5 %	10%	10%	5%
0.043	7	41	526	696

OTHER AGRICULTURAL ECONOMICS WORKING PAPERS

No. 92-05	Oil Shocks and the Demand for Electricity	Edward C. Kokkelenberg Timothy D. Mount
No. 92-06	The Effects of Carbon Taxes on Economic Growth and Structure Will Developing Countries Join a Global Agreement?	Steven C. Kyle
No. 92-07	Climate Change, Sustainable Economic Systems and Welfare	Timothy D. Mount
No. 92-08	Welfare Effects of Improving End-Use Efficiency: Theory and Application to Residential Electricity	Jesus C. Dumagan Timothy D. Mount
No. 92-09	The Value of Segmenting the Milk Market into bST-Produced and Non-bST-Produced Milk	Loren W. Tauer
No. 92-10	World Oil: Hotelling Depletion, or Accelerating Use?	Duane Chapman
No. 92-11	Incorporating Demographic Variables into Demand Systems for Food: Theory and Practice	Deborah C. Peterson Timothy D. Mount
No. 92-12	U.S. Dairy Policy Alternatives Under Bovine Somatotropin	Fude Wang Richard N. Boisvert Harry M. Kaiser
No. 93-01	Empirical Analysis of Agricultural Commodity Prices: A Viewpoint	William G. Tomek Robert J. Myers
No. 93-02	Imperfect Competition Model and Deregulation: U.S. Dairy Policy	Nobuhiro Suzuki Harry M. Kaiser John E. Lenz Olan D. Forker