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An Exhaustible Resource Extraction Licensing Scheme Yielding Increasing Government Revenue

Robert W. Jefferson

Department of Agricultural Economics
New York State College of Agriculture and Life Sciences
A Statutory College of the State University
Cornell University, Ithaca, New York, 14853-7801

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ABSTRACT

A discrete-time model is used to simulate optimal extraction paths of a firm whose costs increase as reserves decline. When increasing extraction-based tax payments are required as a condition of license, a revenue tax is often preferred to a profit tax by both the firm and the government.

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When a poor country or region possesses a deposit of a commercially exploitable mineral, it may try to use the mineral to generate economic growth. The development of such a deposit is frequently left to private firms. Where the resource is government-owned, one major economic benefit often is public revenue (from taxation or through a government's share of the venture's profit) to reduce existing tax burden or to finance new economic initiatives.

When mineral exploitation is characterized by increasing production costs, as when a decline in remaining reserves causes unit costs to increase, maximization of the firm's discounted total profit over a fixed time horizon dictates high production in early stages of the economic life of the deposit, and declining production as time passes. Public revenues will decline also if the government receives a fixed proportion of profit each period, or levies a fixed-rate tax on either gross sales revenue or quantity extracted.

Elected government officials often have short planning horizons, and may be tempted to use all of the new revenue immediately for new spending or tax reductions. If so, the predictable decline in mineral revenues will force politically undesirable spending cuts or tax increases in the future. To extend the time over which a tax reduction or spending increase can take place, the government will need some guarantee that its new source of income will provide nondeclining or increasing revenues through time.

To induce compliance with its requirement of increasing revenue, the government may threaten to revoke the firm's production license. Long(1975) has shown that a firm facing a threat of expropriation, upon which its own activities exert no influence, will shift production forward even further. If, however, the threat of license revocation is made conditional on the firm's failure to meet the revenue-growth goal, the firm, by complying with the government's wishes, can eliminate it.

A discrete-time model is used in this paper to simulate the optimal production programmes of a firm facing a renewable extraction license, under both a tax on profit after extraction cost and a tax on gross sales revenue. Since the output price is held constant, the latter may also be interpreted as a severance tax. The results are compared with those obtained from models without such growth objectives.

THE MODEL

The model used for the simulations is an extension of the mine manager's problem in Conrad and Clark.(1987), in which production cost rises as the stock is depleted. It may also be viewed as a version of Pindyck's (1978) model, in which exploration is prohibitively expensive. In this paper, the model is extended to include multiplicative terms which represent, for each time period, the firm's subjective probability of retaining its license until that period. Thus, the firm's production profile influences not only the absolute after-tax profit in each period, but also the probability of receiving it.

The Conrad-Clark model, incorporating both revenue and profit taxes, is:

$$\underset{\{Y_t\}}{\text{MAX}} \sum_{t=0}^{T-1} \rho^t \left[Y_t \left\{ P(1-\chi) - Y_t/X_t \right\} (1-\tau) \right]$$

subject to:

$$X_{t+1} = X_t - Y_t \quad ,$$

$$X_o \text{ given} .$$

Where: X_t = stock at the start of period "t"; X_o = original deposit; Y_t = production during period "t"; P = constant output price; χ = tax on gross sales revenue; τ = tax on profit; δ = discount rate; ρ = discount factor = $[1/(1+\delta)]$; and T = date at which the deposit is abandoned.

When the government requires a certain minimum growth rate in the remittances it receives, the problem faced by the firm is altered. The firm may no longer plan to produce over the entire T-period horizon, since it may earn greater profit by producing large amounts in the early periods and foregoing entirely its profit in some later periods. The firm now chooses a production programme to maximize its expected profit, where the probability of having its license renewed in the future depends on its actions in the current and past periods. The firm's new problem is shown in Figure 1. The model now includes multiplicative terms describing the probability of survival, or license extension, conditional on its having been extended to the

Figure 1

Firm's Optimization Problems With Tax Payment Growth Objectives

$$\underset{\{Y_t\}}{\text{MAX}} \quad Y_o\left\{P(1-\chi)-Y_o/X_o\right\}(1-\tau)$$

$$+\sum_{t=1}^{T-1}\rho^t\left[\prod_{i=1}^{t}S_i(X_{i-1},X_i,Y_{i-1},Y_i)\right]\left[Y_t\left\{P(1-\chi)-Y_t/X_t\right\}(1-\tau)\right]$$

Subject to:

$$X_{t+1} = X_t - Y_t,$$

$$X_o \text{ given}, \quad S_o = 1.$$

With a tax on profit only, for t=1...T−1,

$$S_t(X_{t-1},X_t,Y_{t-1},Y_t) = 1-\left(\frac{bY_{t-1}(P-Y_{t-1}/X_{t-1})-Y_t(P-Y_t/X_t)}{(b-a)[Y_{t-1}(P-Y_{t-1}/X_{t-1})]}\right)^{\beta}$$

With a tax on gross revenue only, for t=1...T−1

$$S_t(X_{t-1},X_t,Y_{t-1},Y_t) = 1-\left(\frac{bY_{t-1}-Y_t}{(b-a)Y_{t-1}}\right)^{\beta}$$

Where:

X_t = Reserves at the start of period "t"
Y_t = Production during period "t"
P = Constant price of output
χ = Tax Rate on Gross Revenue
τ = Tax Rate on Profit
ρ = discount factor = $[1/1+\delta]$, where
δ = discount rate
T = date at which the deposit is abandoned
a = 1+(minimum acceptable tax growth rate)
b = 1+(upper bound on acceptable tax growth)
β = concavity coefficient on conditional probability distribution

current period: P(license extended|Tax Revenue Growth) = S_t = $F(Tax_t - Tax_{t-1})$, where $F'(\cdot) > 0$.

The form of $F(\cdot)$ chosen for the simulations is the cumulative distribution function of the Beta(α,β) distribution, with the parameter α set equal to one. The general form of the Beta(α,β) density function is:

$$f_X(x) = \Gamma(\alpha+\beta)\cdot[\Gamma(\alpha)\Gamma(\beta)]^{-1}\cdot\{x^{\alpha-1}(1-x)^{\beta-1}\} \text{ , } x\in[0,1].$$

Setting α equal to one, and using $\Gamma(t+1)=t\cdot\Gamma(t)$, $\Gamma(1)=1$, we have,

$$f_X(x) = \beta(1-x)^{\beta-1} \text{ , } x\in[0,1].$$

This function may be transformed for any $y=a+(b-a)x$:

$$f_Y(y) = \beta\{(b-y)/(b-a)\}^{\beta-1}\cdot(b-a)^{-1} \text{ , } y\in[a,b] \text{ ,}$$

which integrates to the cumulative distribution function:

$$F_Y(y) = 1-\{(b-y)/(b-a)\}^{\beta} \text{ , } y\in[a,b].$$

The variable y is used to represent two different ratios. If the firm faces a tax on profit, $y=Profit_t/Profit_{t-1}$. Under a revenue tax, $y=Production_t/Production_t$. The final expressions for S_t are shown in Figure 1.

The upper limit of the interval, b, is the government's growth objective, such that tax-payment growth of that magnitude is sufficient to ensure the extension of the firm's license (the minimum rate of tax-payment growth at which the subjective probability of license extension is one). Similarly, a is the rate at or below which the firm is certain of license revocation.

The government's willingness to revoke a firm's license for failing to satisfy the growth objective will depend on the number of other firms available to extract from the deposit. If that number is large, the government may easily dismiss any firm not meeting the requirements exactly. If the number is small, an incumbent firm may retain its license to extract by fulfilling the requirement only approximately. The firm's subjective beliefs concerning the government's willingness to forgive small deviations from the goal are captured in the parameter β. When $\beta=1$, S_t is linear in tax-receipts growth, the firm's "survival" probability falls in proportion to the magnitude of the deviation from the government's objective. Where $\beta>1$, S_t is strictly concave in growth, suggesting that small deviations from the goal are expected to be penalized much less harshly than are large deviations.

Conrad and Clark's original discrete-time problem can be solved by constructing and optimizing of a Hamiltonian. The problems described here cannot be, because production levels in previous periods influence the conditional probabilities associated with those periods, which are in turn multiplied to obtain the current period's total probability. Further, previous period production reduces the size of the reserve available in the current period, which influences the likelihood that a given rate of profit or revenue growth may be maintained.

Since the problems use discrete time, in principle, they can be solved using Lagrangian expressions. These are extremely lengthy, however, and for practical purposes, solutions were obtained using a numerical optimization package, the micro-computer version of GAMS-

MINOS (Brooke et al., 1988).

Three sets of simulations were computed, in which the desired rate of growth of tax payments, or the upper bound on the conditional probability distribution, was 10%, and the lower bound -5%. For comparability with Conrad and Clark's results, a time horizon of 10 periods was assumed, as was a 10% discount rate, an original deposit size of 1000 units, and a constant unit price of output.

SIMULATION RESULTS

As seen from Table 1, where there is no tax-receipts-growth goal, the imposition of a tax on profit after extraction cost causes no change in the optimal no-tax production plan, since such a tax cannot be avoided. In contrast, a tax on gross revenue does distort the optimal plan in comparison with the no-tax programme. The effect of the tax is to reduce the net price received by the firm, reducing the maximum royalty (marginal revenue less marginal cost) in the final period. The profit tax is preferable to the revenue tax on the basis of economic efficiency.

A growth objective introduces distortions into the firm's behavior, so the optimality of the profit tax from the standpoint of economic efficiency can no longer be assumed automatically. From Table 2, the simulation results show that, under a profit tax with a growth objective and a linear conditional probability function (β=1), the firm optimally produces in all ten periods, but under a tax on gross revenue with the same growth objective and value of β, its optimal programme precludes production in the final time period. For

TABLE 1

	PRODUCTION			TAX REVENUE			AFTER TAX PROFIT		
		Profit	Revenue		Profit	Revenue		Profit	Revenue
time	No Tax	Tax=30%	Tax=25%	No Tax	Tax=30%	Tax=25%	No Tax	Tax=30%	Tax=25%
0	283.23	283.23	199.58	$ 0.0	$ 54.44	$ 49.90	$181.48	$127.03	$109.85
1	183.68	183.68	162.13	0.0	41.82	40.53	139.39	97.57	88.76
2	141.82	141.82	131.92	0.0	32.11	32.98	107.03	74.92	71.67
3	109.71	109.71	107.55	0.0	24.64	26.89	82.12	57.49	57.82
4	85.09	85.09	87.91	0.0	18.88	21.98	62.92	44.04	46.55
5	66.22	66.22	72.08	0.0	14.42	18.02	48.06	33.64	37.35
6	51.74	51.74	59.32	0.0	10.98	14.83	36.46	25.53	29.76
7	40.61	40.61	49.00	0.0	8.18	12.25	27.26	19.08	23.38
8	32.03	32.03	40.61	0.0	5.90	10.15	19.65	13.76	17.82
9	24.43	24.43	33.73	0.0	3.81	8.43	12.72	8.90	12.65
								Discounted Sums	
							$580.30	$406.26	$390.67

TABLE 2

$\beta = 1$

	PRODUCTION			EXPECTED TAX REVENUE			EXPECTED AFTER TAX PROFIT		
	Profit Tax=30%	Revenue Tax = 25%		Profit Tax=30%	Revenue Tax = 25%		Profit Tax=30%	Revenue Tax = 25%	
time		Constrained[1]	Free[2]		Constrained	Free		Constrained	Free
0	42.05	57.62	64.04	$ 12.08	$ 14.41	$ 16.01	$ 28.20	$ 39.90	$ 43.93
1	46.57	63.38	70.44	13.29	15.85	17.61	31 01	43.27	47.53
2	51.67	69.72	77.48	14.62	17.43	19.37	34.12	46.76	51.18
3	57.45	76.69	85.23	16.08	19.17	21.31	37.53	50.25	54.71
4	64.07	84.36	93.76	17.69	21.09	23.44	41.28	53.56	57.81
5	71.87	92.80	103.13	19.46	23.20	25.78	45.41	56.31	59.89
6	81.27	102.08	113.45	21.41	25.52	28.36	49 95	57.80	59.65
7	93.40	112.29	124.79	23.55	28.07	31.20	54.94	56.40	53.91
8	111.74	123.52	137.27	25.90	30.88	34.32	60.44	47.90	53.91
9	189.95	135.87	0.0	28.49	33.98	0.0	66.48	17.04	0.0
							Discounted Sums		
							$281.95	$318.33	$323.72

Notes

1 Results of Conrad–Clark model subject to $Y_t / Y_{t-1} = 1.1$.

2 Results of model with probabilities of license renewal.

ease of comparison, a "constrained" model, consisting of the Conrad-Clark model with production constrained to rise at a rate of 10%, is constructed and solved as well.

If the government's utility function gives absolute tax receipts in all periods equal weight, it will still prefer the profit tax, because the final period tax receipt under the revenue tax is often zero. If the utility function places the smallest weight on tax receipts in the final periods, or if it includes minimization of the rate of growth of actual production (perhaps to minimize the adjustment costs of altering production rates), the revenue tax will be preferred. Somewhat surprisingly, the firm is able to earn greater total discounted profit under a tax on revenue than under a tax on profit, while at the same time making larger tax payments to the government.

This result obtains because, when faced with a growth-constrained profit tax, the firm must schedule production so that profit increases at the specified rate, implying that costs rise less rapidly than revenue. With stock-dependent costs, this necessitates reductions in early-period outputs, to allow higher production in later periods, and to ensure that enough of the mineral is left unextracted to keep later period costs down.

In contrast, a growth-constrained revenue tax places no restriction on the behavior of the firm's costs through time. The firm can maximize the discounted sum of after-tax profit, subject only to the constraint that gross revenue (a constant multiple of quantity

TABLE 3

$\beta = 3$

	PRODUCTION			EXPECTED TAX REVENUE			EXPECTED AFTER TAX PROFIT		
time	Profit Tax=30%	Revenue Tax=25%	Revenue Tax=30%	Profit Tax=30%	Revenue Tax=25%	Revenue Tax=30%	Profit Tax=30%	Revenue Tax=25%	Revenue Tax=30%
0	47.54	64.68	70.37	$ 13.58	$ 16.17	21.11	$ 31.70	$ 44.32	$ 44.31
1	52.31	70.60	76.79	14.83	17.65	23.04	34.60	47.62	47.41
2	57.44	76.78	83.49	16.12	19.18	25.02	37.61	50.72	50.22
3	63.03	83.23	90.46	17.46	20.77	27.08	40.74	53.53	52.58
4	69.18	89.93	97.67	18.84	22.39	29.19	43.95	55.75	54.10
5	76.07	96.84	105.05	20.24	24.02	31.26	47.22	56.92	54.11
6	84.06	103.84	112.45	21.62	25.60	33.20	50.44	56.27	51.33
7	94.00	110.74	119.52	22.90	27.00	34.76	53.42	52.12	43.03
8	108.65	117.02	125.23	23.93	27.85	34.89	55.83	40.57	21.77
9	173.86	120.38	0.0	24.20	25.55[1]	0.0	56.46	31.07	0.0
							Discounted Sums		
							$290.06	$333.77	$302.66

Notes

1 Tax revenue decreases only in expectation.

sold), increases each period at the specified rate of tax-receipts-growth. The firm will produce as long as it is able to meet the government's objective while earning positive after-tax profit, even though profit may be declining in later periods.

The effect on production, tax, and profit for values of $\beta>1$ can be seen in Table 3. At $\beta=3$, indicating great willingness by the government to forgive deviations from its goal, the firm facing a profit tax produces for all ten periods, but uses the government's leniency to shift production forward, increasing its total profit. The firm facing a revenue tax produces for nine or ten periods, depending on the tax rate. At tax rates at or below 25%, the firm's optimal plan is to produce for all ten periods. At tax rates above 30%, the firm again produces for only nine periods. In both cases, production is shifted forward compared with the results for $\beta=1$.

IV. CONCLUSION:

The government of a small, underdeveloped region can use a newly-discovered mineral deposit as a source of increasing revenues. It can do so by demanding, as a condition of a firm's license to extract, that activity-based tax payments to the government increase at a specified rate. Of the two types of tax considered here, the tax on gross revenue, or production, is preferred to the tax on profit. Under such a tax, the firm can remit larger amounts each period to the government, while earning greater discounted profit, than would be possible under a profit tax.

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