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Input Aggregation and Firm Efficiency

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Abstract

Technical efficiencies calculated using aggregated inputs are biased. It is shown empirically and analytically that input aggregation decreases calculated technical efficiencies and changes relative technical efficiencies among firms. Moreover, aggregation fails to separate technical efficiency effects from allocative efficiency effects. Thus, the calculated efficiencies are more properly economic efficiencies.

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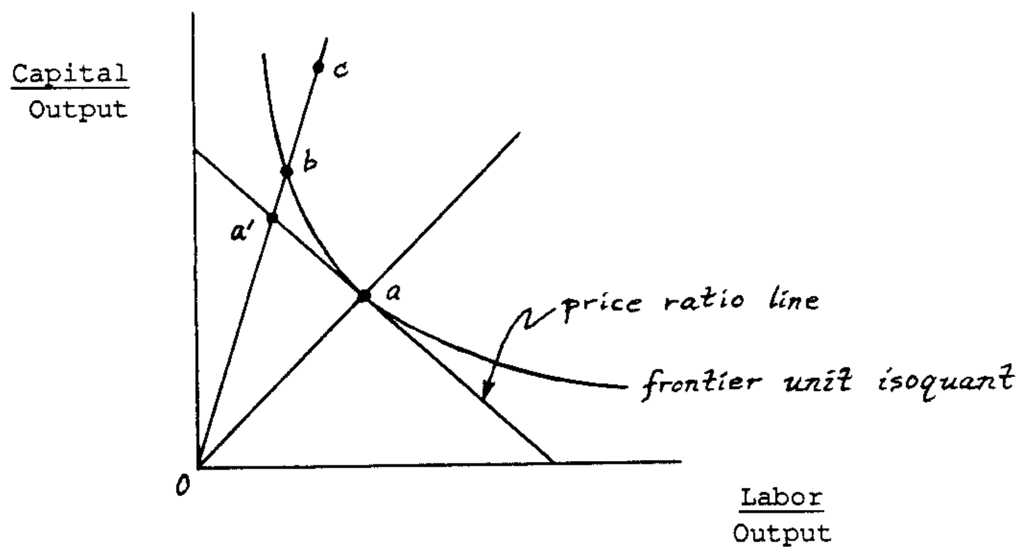
INPUT AGGREGATION AND FIRM EFFICIENCY

In recent years attention has been directed towards measuring technical and allocative efficiencies. Two general approaches commonly used are the econometric estimation of a frontier production function and the linear programming estimation of the efficient or frontier unit isoquant (Herdt and Mandac; Timmer). In selecting a method to empirically measure efficiencies, economists have been concerned with the assumptions required by a particular methodology as well as the data necessary for determining efficiency estimates. One major appeal of measuring either a frontier production function or a frontier unit isoquant is that technical efficiencies may be readily calculated using farm financial data aggregated into expense categories (Grisley and Mascarenhas). Assuming constant input prices across all firms, expenditure values can be used as input quantity values in empirical applications. However, even if the assumption is valid, substantial biases can occur when using value-aggregated data. Hence, this paper discusses the biases that occur when using aggregated inputs in estimating a frontier for calculating technical efficiencies. The purposes are threefold: 1) to empirically show the biases that can occur when aggregated inputs are used in measuring technical efficiencies with frontier approaches, 2) to explain the causes of these aggregational biases, and 3) to reveal that technical inefficiencies measured when aggregated inputs are used are truly inefficiencies due to both allocative and technical effects.

METHODOLOGY

The concept of measuring technical and allocative efficiency was first introduced by Farrell. According to Farrell, a unit isoquant could be defined for a firm by determining the amounts of inputs used by a firm to produce a given output level. Firms within an industry using the least amounts of inputs to produce the given output level identify the efficient or frontier unit isoquant. Figure 1 illustrates an efficient unit isoquant for the two input case of capital and labor. A firm using inputs

Figure 1.



in a combination that lies on the efficient unit isoquant is said to be technically efficient. Hence, firms at pt. a and pt. b are technically efficient in their use of capital and labor. A firm at pt. c is only (ob/oc)*100 percent technically efficient. Firms equating their marginal rates of technical substitution with their input price ratios are said to be allocatively efficient. A firm at pt. a is allocatively efficient.

However, a firm at pt. c is (oa'/ob)*100 percent allocatively efficient. Together, technical and allocative efficiency comprise what is commonly referred to as economic efficiency. A firm at pt. a is economically efficient in its use of capital and labor. If a firm is at pt. c, it is merely (oa'/oc)*100 percent economically efficient.

The theory set forth by Farrell was demonstrated using the two input case of capital and labor. Boles expanded Farrell's frontier unit isoquant framework to consider more than two inputs using a stepwise linear programming procedure. In his theory, Boles stated that if the standardized inputs utilized by one firm could be represented as a linear combination of the inputs of other firms, the original firm is technically inefficient. In other words, if more output can be produced by other firms using less than or equal amounts of the inputs used by the firm whose technical efficiency is being measured, the firm is said to be technically inefficient in its overall use of the inputs.

The actual technical efficiency measurement for the j^{th} firm is determined by solving the following linear program.

$$\text{Max } Z = CX$$

$$\text{subject to } AX <= B_j$$

$$X >= 0$$

In the linear program, the A matrix consists of column vectors of the inputs used by the firms in the analysis. Thus, for A_{m*n}, m inputs are specified for n firms. X is a column vector representing the activities, i.e. the firms. B_j is a column vector of the j^{th} firm's inputs, identical to the j^{th} firm's column vector in the A matrix. C is a row vector of ones. The technical efficiency of the j^{th} firm is found by the reciprocal of the objective function value. If 100 firms are present in the analysis, a series of 100 linear programs ($j = 1,\ldots,100$) must be solved in order to determine the technical efficiencies of all firms.

A limitation of this procedure is that it assumes that no scale effects are present among the firms in the analysis. If this assumption cannot be met, the problem may be surpassed by simply segregating the firms by size. Perceivably then, the efficient unit isoquants for the various firm size categories could be tested for significant differences.

TECHNICAL EFFICIENCIES OF NEW YORK DAIRY FARMS

For the purpose of illustrating the impacts of aggregating inputs on the technical efficiency measures determined by frontier approaches, the technical efficiencies of 125 New York dairy farms were calculated using Boles' linear programming procedure. The farms used in this sample represent farms that participated in the 1987 New York Dairy Farm Business Summary Program. Each of the farms had participated in the program since 1980.

Initially, the technical efficiencies were calculated using 28 inputs in total. These inputs are shown in Table 1. Of the 28 inputs, most were cash expense items reported on the income statements used in the summary forms. The cost of equity and labor values other than hired labor cash expense were also added. The cost of equity was determined by multiplying total equity by 9.44 percent, the average interest rate on the debt of all farms. Labor values were operator labor values reported by the operators (Oper Labrval) and unpaid family labor valued at $500 per month (Famupd Labrval).

The results of this initial technical efficiency calculation found only one farm inefficient out of 125. That inefficient farm was 94.5 percent technically efficient (Table 2). Therefore, the overall mean technical efficiency was quite high at 99.96 percent. Thus, it would appear that these New York dairy farmers technologically manage their farms quite efficiently.

A second set of technical efficiencies was then calculated for the same farms. However, during this efficiency estimation, the original 28 input categories were aggregated by value into 14 inputs (Table 1). Most of the aggregation involved combining smaller, related expense items. The resulting efficiencies had an overall mean of 98.21 percent. 27 farms were found inefficient with ranges from 99.9 percent efficient to 75.6 percent efficient (Table 2).

A third technical efficiency calculation was then made. This time inputs were further aggregated into eight input categories (Table 1). Results of this calculation found 77 of the 125 farms to be technically inefficient, ranging from 99.6 percent efficient to 65.3 percent efficient. The overall mean technical efficiency for all farms was 91.51 percent. Thus, the efficiencies determined in this calculation are far different from the efficiencies calculated using the original 28 inputs.

Table 1. Inputs Used in Measuring Technical Efficiencies of NY Dairies

28 Inputs	15 Inputs	7 Inputs	1 Input
Labor Exp	Labor Exp	Labor Exp	Total Cost
Oper Labrval	Family Labr Exp	Labor Exp	Labor Exp
Famupd Labrval	Oper Labrval	Oper Labrval	Oper Labrval
Dairy Feed Exp	Famupd Labrval	Famupd Labrval	Famupd Labrval
Othr Feed Exp	Feed Exp	Lvstk Exp	Dairy Feed Exp
Lvstk Purchase	Dairy Feed Exp	Dairy Feed Exp	Othr Feed Exp
Breed Exp	Othr Feed Exp	Othr Feed Exp	Lvstk Purchase
Vet Exp	Lvstk Purchase	Lvstk Purchase	Breed Exp
Othr Lvstk Exp	Other Lvstk	Breed Exp	Vet Exp
Milk Mkt Exp	Breed Exp	Vet Exp	Othr Lvstk Exp
Crop Fert Exp	Vet Exp	Othr Lvstk Exp	Milk Mkt Exp
Crop Seed Exp	Othr Lvstk Exp	Milk Mkt Exp	Crop Fert Exp
Crop Pest Exp	Milk Mkt Exp	Crop Exp	Crop Seed Exp
Mach Rent Exp	Crop Exp	Crop Fert Exp	Crop Pest Exp
Mach Parts Exp	Crop Fert Exp	Crop Seed Exp	Mach Rent Exp
Auto Exp	Crop Seed Exp	Crop Pest Exp	Mach Parts Exp
Mach Oil Exp	Crop Pest Exp	Machinery Exp	Auto Exp
Bldg Exp	Machinery Exp	Mach Rent Exp	Mach Oil Exp
Tax Exp	Mach Rent Exp	Mach Parts Exp	Bldg Exp
Ins Exp	Mach Parts Exp	Auto Exp	Tax Exp
Rental Exp	Auto Exp	Mach Oil Exp	Ins Exp
Telephone Exp	Mach Oil Exp	Mach Depr	Rental Exp
Elec Exp	Real Estate Exp	Real Estate Exp	Telephone Exp
Othr Exp	Bldg Exp	Bldg Exp	Elec Exp
Int Exp	Tax Exp	Tax Exp	Othr Exp
Mach Depr	Ins Exp	Ins Exp	Int Exp
Bldg Depr	Rental Exp	Rental Exp	Mach Depr
Equity Cost	Other Exp	Bldg Depr	Bldg Depr
	Telephone Exp	Other Exp	Equity Cost
	Elec Exp	Telephone Exp	
	Othr Exp	Elec Exp	
	Int Exp	Othr Exp	
	Mach Depr	Int Exp	
	Bldg Depr	Equity Cost	
	Equity Cost		

Finally, a fourth efficiency estimation was done. In this estimation the inputs were aggregated into one category--essentially, total cost. The number of technically efficient farms should be intuitively obvious. Only one farm was technically efficient, that being the farm with the least total cost. Thus, 124 farms were found inefficient, with their efficiencies ranging from 91.7 percent efficient to 33.4 percent efficient. The overall mean technical efficiency fell to 67.22 percent--significantly different from the efficiencies calculated using the original 28 inputs.

Table 2. Efficiency Results of New York Dairy Farms

Number of Inputs	Mean Technical Efficiency	Number of Inefficient Farms	Range of Inefficient Farms	Spearman Rank Correlation Coefficient
28	99.96%	1	94.5%	------
14	98.21%	27	75.6-99.9%	-0.047
8	91.51%	77	65.3-99.6%	0.597
1	67.22%	124	33.4-91.7%	0.526

The Spearman rank correlation coefficients were then calculated between farms' rankings in technical efficiencies for consecutive input aggregations (Table 2). The ranking of farms by technical efficiencies measured using 28 inputs had a correlation of -0.047 with the ranking of farms by technical efficiencies measured using 14 inputs. Likewise, the rankings with 15 inputs and eight inputs had a correlation of 0.597, and the rankings with eight inputs and one input had a correlation of 0.526.

AGGREGATIONAL EFFECTS ON EFFICIENCY MEASURES

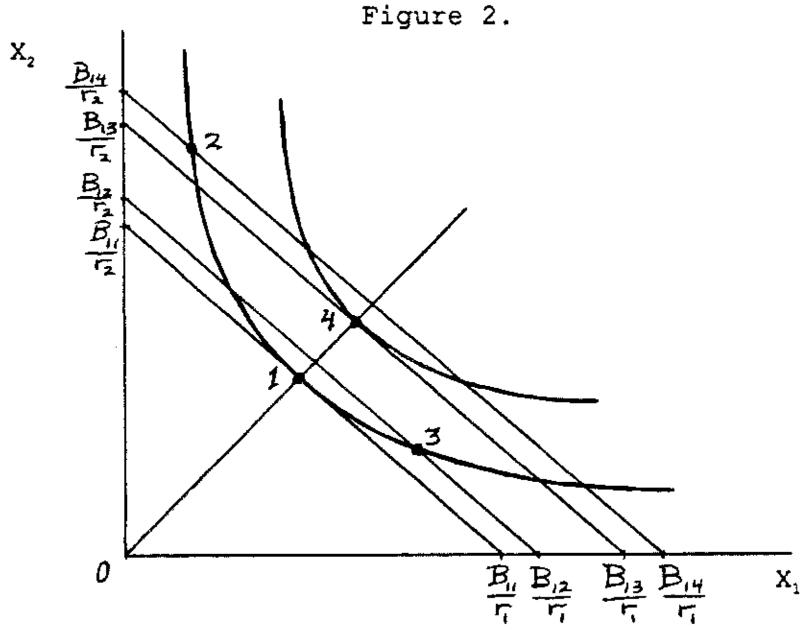
The illustration of the technical efficiencies of New York dairy farms clearly demonstrates two impacts of input aggregation on the estimation of technical efficiencies using Boles' approach. The first effect is that as inputs are aggregated, the technical efficiencies of many firms decrease as shown by a fall in the mean efficiency. The second effect is that as the inputs are aggregated, the relative efficiencies of firms change as shown by the Spearman rank correlation coefficients. In the example, the sole farm determined inefficient when 28 inputs were defined was found to be 100 percent efficient when 14 inputs and eight inputs were defined and 81.3 percent efficient when one input was defined. One would expect that a firm being least efficient when disaggregated inputs are used would also be the least efficient firm when aggregated inputs are used. Contrary to this expectation, this was not the case.

EXPLANATION OF AGGREGATION BIAS

The underlying question becomes why the bias? The key to the explanation lies in what apparently economists have assumed to be technical efficiency estimates. Are efficiencies calculated using frontier approaches actually technical efficiencies if the inputs used in one's analysis are aggregated? An examination of input aggregation through a simple graphical analysis reveals that they are not.

For demonstrative purposes, assume that a particular industry contains many firms. Each firm uses only four inputs, x_1, x_2, x_3, x_4, and the input prices are constant throughout the entire industry. Four specific firms within the industry will be used to explain the aggregational bias. Of the four firms, only firm 1 and firm 2 are truly technically efficient in their use of all four inputs.

Figure 2.



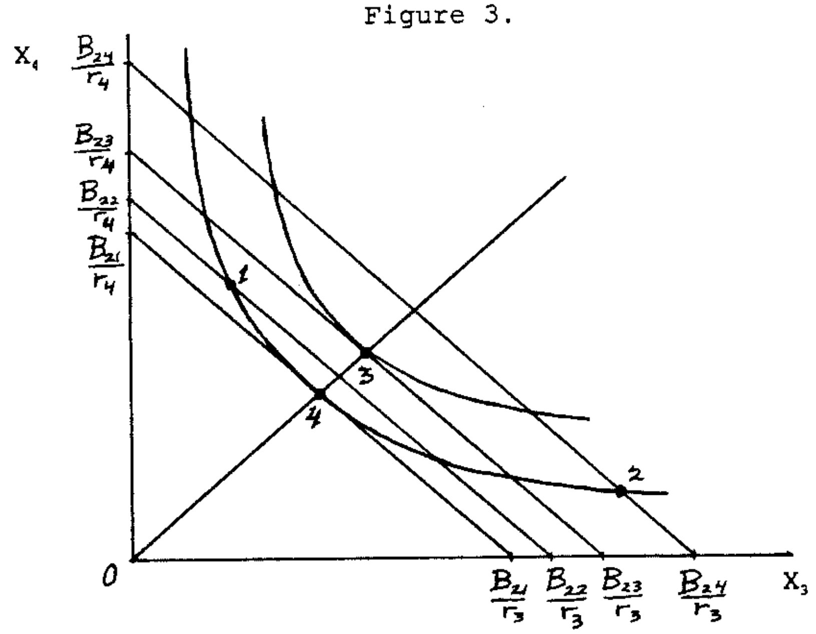
Following Farrell's line of thought, firm 1, firm 2, and firm 3 are all technically efficient in their use of x_1 and x_2. This is shown in the frontier unit isoquant in Figure 2. Firm 4, however, is technically inefficient in its use of the two inputs. Of the four firms, only firm 1 and firm 4 are allocatively efficient. If x_1 and x_2 are aggregated by value into a new input ($B_1 = r_1x_1 + r_2x_2$), then firm 1 is seen as using the least amount of the aggregated input (B_{11}), followed by firm 3 (B_{12}), firm 4 (B_{13}), and finally firm 2 (B_{14}).[1] Hence, although firm 2 is technically efficient in its use of the two inputs, it is measured as using the

[1]An additive aggregator function is used. Diewert has derived aggregation indices for commonly used production functions. Yet, unless the researcher collects data on all disaggregated inputs and aggregates, using expenditures implicitly assumes additive aggregation.

greatest amount of the aggregated input. Thus, input aggregation in this manner fails to separate technical effects from allocative effects. In a sense, the technical efficiency measure has become an economic efficiency measure in that it is comprised of components of both technical efficiency and allocative efficiency.

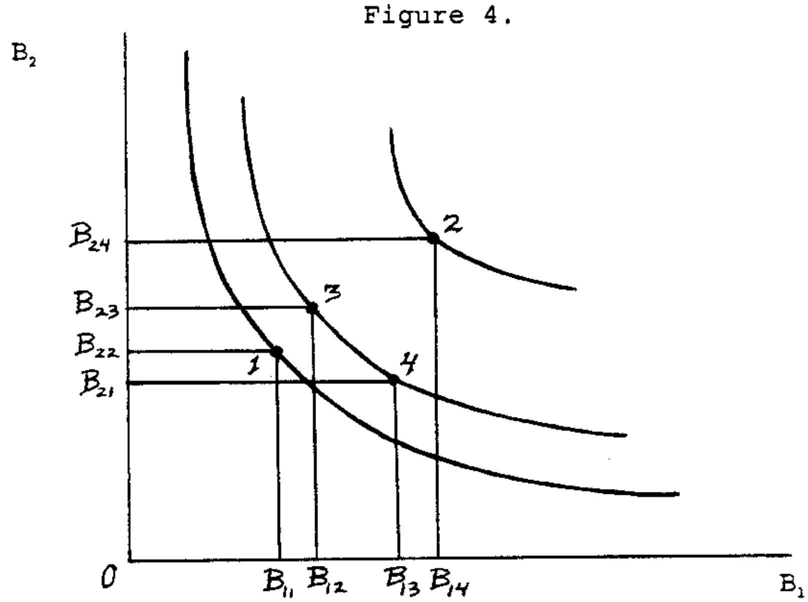
Figure 3 shows the firms' usage levels of x_3 and x_4, as well as the industry's frontier unit isoquant. In Figure 3, it is seen that firm 1, firm 2, and firm 4 are technically efficient in their use of x_3 and x_4. If the two inputs are aggregated into a new input ($B_2 = r_3x_3 + r_4x_4$), it can be seen that firm 4 uses the least amount of the aggregated input (B_{21}), followed by firm 1 (B_{22}), firm 3 (B_{23}), and firm 2 (B_{24}).

Figure 3.



The result of the input aggregation is illustrated in Figure 4. In Figure 4, only firm 1 lies on the industry's efficient unit isoquant and is, therefore, technically efficient. Firm 2, however, is now seen as technically inefficient when in fact it is actually technically efficient in the use of the four inputs. Thus, the aggregational bias of decreasing technical efficiencies as inputs are aggregated is explained. Also in Figure 4, it can be seen that firm 2 has become the least technically efficient firm of the four. Hence, its relative efficiency has changed dramatically as the result of input aggregation. It should be noted that the use of inputs aggregated by value in the estimation of a frontier production function will result in the same biases. These biases could be easily shown through this same graphical analysis with the addition of a frontier production function.

Figure 4.



SUMMARY AND CONCLUSION

Biases were shown to exist in technical efficiencies using New York dairy farms as data. First, as inputs were aggregated, the technical efficiencies of firms decreased. Second, the relative technical efficiencies among firms changed dramatically. This questions frontier measures as true technical efficiencies if aggregated data is used. It was demonstrated graphically that aggregating inputs by input prices fails to distinguish between the allocative aspects and technical aspects of efficiency. Thus, frontier approaches do not elicit a true measure of technical efficiency if value-aggregated data is used. Rather, these efficiencies must be interpreted as economic efficiency measures, the composite of technical and allocative efficiency. These findings are of significant importance since numerous empirical studies have been conducted using frontier approaches in conjunction with aggregated data.

One may then pose the question of whether frontier approaches are valid measures of technical efficiencies even if raw, disaggregated data is used. The answer to this particular question is embedded in the way in which the word disaggregated is defined. How disaggregated is disaggregated? With a large number of inputs it is conceivably possible to find that each firm uses an input that no other firms use. Thus, it would be impossible to represent one firm's inputs as a linear combination of the other firms' inputs, and all firms would be technically efficient. Therefore, it seems necessary to aggregate at least some inputs. But then biases occur, and a true measure of technical efficiency is not determined. Hence, it would appear that technical efficiencies are least valid for small sample sizes or for industries in which a wide range of inputs are used. The issue then becomes whether some firms are operating off the economic efficient point, or are they operating under a different production function?

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