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ABSTRACT

This paper examines some of the economic issues underlying the control of pollution and the management of pollution-sensitive resources. The economics of a stock pollutant are briefly reviewed for a simple model with a quadratic damage function and linear dynamics (Section II). A two-state model, where pollution adversely affects the growth of a renewable resource is presented in Section III. Section IV presents differential and difference equation models for two types of pollutants in a stylized model of krill, whales and seals in the Southern Ocean. The paper closes with a section on the need for integrating physical models of pollution with ecosystem models and the likely role of physical scientists, biologists and economists in both modelling and formulating policy for an equitable evolutionary strategy.

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A Bioeconomic Model of Pollution and Resource Management

I. Introduction

The summer of 1988 may mark the emergence of marine pollution as a major issue in U. S. environmental policy. On Sunday, July 3, the *New York Times* ran a front page article on a fish kill in Leonardo, New Jersey apparently the result of oxygen depletion in the stratified waters of Sandy Hook Bay (Figure 1). The article went on to discuss the recurring problem of "brown tide", an algal bloom which wiped out the 1987 bay scallop fishery in the Peconic Bay system located on eastern Long Island.

On August 1st *Time* had a cover story entitled "The Dirty Seas" which detailed a broad spectrum of ocean pollutants that included the run-off of agricultural fertilizers and pesticides, municipal and industrial wastes, garbage from boats and ships, oil pollution, plastics and acid rain. In addition to the Hudson River-Raritan estuary, Boston Harbor, Chesapeake Bay, Galveston Bay, San Diego, Santa Monica, San Francisco Bay and Puget Sound were also identified as coastal areas suffering from high levels of toxic chemicals, low dissolved oxygen and contaminated shellfish.

Marine pollution is by no means new nor restricted to affluent industrialized countries. Untreated sewage has flowed into the Mediterranean, Adriatic and Aegean seas since 2000 BC. Coastal cities in Africa, Central and South America and the Soviet Union face many of the same esturine and marine pollution problems confronting the US and Europe. Even the remote Arctic and Southern oceans may not be immune. The depletion of stratospheric ozone by chloroflurocarbons, while not polluting these oceans per se, leads to increased ultraviolet radiation, which may, in turn, adversely affect the production of krill (Euphausia superba), a basic food source in both Arctic and Antarctic ecosystems (Figure 2).

The costs of marine pollution are both direct and indirect. Contamination by medical wastes has forced the closure of bathing beaches on Long Island. While not presenting a health hazard, plastic and other debris greatly reduce the value (utility) from a visit to a beach or coastal area. Toxic chemicals or municipal waste may render shellfish unfit for human consumption. The productivity of nearshore coastal areas has declined as a result of reduced habitat and lower rates of net recruitment. A less obvious effect, but one which seems to emerge from computer simulations, is that pollution may reduce the resiliency and stability of a marine ecosystem.

The remainder of this paper is organized as follows. In the next section we examine some equations that might be used to describe the dynamics of stock pollutants. A simple optimization problem is posed and solved.

The third section examines a two-state model where a pollution stock adversely affects the growth of a renewable resource. Equations defining a steady state optimum are identified and interpreted. The optimal approach to steady state is more problematical. Differential equations which must hold for a "separable" problem are presented.

The fourth section presents simulations for two types of residuals in a multispecies system. The first is a biodegradable residual, like sludge, which can accumulate and decompose in the marine environment. The second residual is one that depletes a physical attribute (say, ozone) that in turn supports, or in some sense protects, a marine ecosystem.

The fifth and final section poses a series of questions that might aid in identifying the type of research needed to forecast and manage large scale ecosystems and the likely roles that biologists, physical scientists and economists will play in performing that research and shaping public policy.

II. Stock Pollutants

Economists frequently dichotomize variables as stocks or flows. Certain inputs and outputs are regarded as flow variables if they are "used up" or "consumed" within a single period. Stock variables are longer-lived and provide a flow of utility or productive services over more than one period. Factories, equipment, inventories and certain "consumer durables" would be regarded as stocks. The distinction is somewhat arbitrary (How should I classify my thirteen year old tie?) but useful when modelling economic processes.

In the models considered in sections three and four we will presume a production process which employs a fixed (given) resource to produce flows of output and waste. Denoting the flow of output by Q and the flow of a residual waste by S, we presume that the transformation function may be written implicitly as $\phi(Q,S) = 0$ The transformation function confronts society with tradeoffs between Q and S as shown in Figure 3. With care and caution society can obtain $Q = Q_0$ and $S = 0$ (no residual discharge). To increase Q, resources must be diverted from residual reduction to output production. By devoting all of the fixed input to production of Q it is possible to obtain Q_{MAX} but with the trade-off that $S = S_{MAX}$. By convention we assume the partial derivatives $\phi_Q > 0$ and $\phi_S < 0$.

Suppose Z is the stock of a degradable pollutant. If the rate of degradation is a constant proportion of the existing pollution stock and if S is measured in the same units as X then the equation of motion for the pollution stock may be written

$$\dot{Z} = -\gamma Z + S \tag{1}$$

This equation is analogous to the capital stock equation in the neoclassical growth model where the degradation rate γ is comparable to a depreciation rate and the residual discharge level is comparable to the level of gross investment.

Let W(Q,Z) be a welfare or net benefit function associated with the flow of output of Q and the stock of pollution Z. We will assume W(·) to be concave in Q with partials derivatives $W_Q > 0$ and $W_Z < 0$. The problem of maximizing the present value of welfare subject to the transformation function and the dynamics of the pollution stock may be stated as

$$\text{Maximize} \int_0^\infty W(Q,Z)\, e^{-\delta t}\, dt$$

$$\text{Subject to } \dot{Z} = -\gamma Z + S$$
$$\phi(Q,S) = 0$$

where δ is the instantaneous rate of discount. The current value

Hamiltonian may be written as

$$\tilde{H} = W(Q,Z) + \mu[-\gamma Z + S) - \omega\phi(Q,S) \tag{2}$$

where $\mu < 0$ is the costate variable associated with the pollution stock and $\omega > 0$ is the multiplier associated with the transformation function. The first order necessary conditions are derived in Appendix A. When evaluated in steady state they collapse to a three equation system defining the optimal values for Z, S and Q. These three equations are

$$W_Q = \frac{W_Z[\phi_Q/\phi_S]}{(\delta + \gamma)}$$

$$S = \gamma Z$$

$$\phi(Q,S) = 0$$

The first equation in this set requires a balancing between the marginal benefits from an extra unit of Q and the present value of marginal cost, in perpetuity, from the increase in Z. Recall, to increase Q society would divert some of the fixed resource from emissions control, increasing S which in turn leads to a steady state increase in Z.

Suppose that we are dealing with a small country or region which can export the commodity Q at the constant world price p, but which incurs local environmental costs proportional to the square of pollution stock (ie, a quadratic damage function). The welfare or net benefit function may be written as $W(Q,Z) = pQ - \alpha Z^2$ where $p > 0$ and $\alpha > 0$. If our transformation function is linear so that $Q = c + nS$ (thus $c = Q_0$ and $\phi_Q/\phi_S = 1/n$) then the first equation may be solved for the optimal pollution stock yielding

$$Z^* = \frac{np(\delta + \gamma)}{2\alpha} \tag{3}$$

The comparative statics are immediately deduced; namely, the optimal pollution stock increases with an increase in n, p, δ or γ and decreases with an increase in α. In this special case the current value Hamiltonian will be linear in Q (or S via substitution of $Q = c + nS$ into the objective function) and the "most rapid approach path" (MRAP) is optimal. The solution to this case is shown in Figure 4. If $Z_0 > Z^*$, then $S = 0$ and $Q = Q_0$, while if $Z_0 < Z^*$, then $S = S_{MAX}$ and $Q = Q_{MAX}$. When $Z = Z^*$, then $S = S^* = \gamma Z^*$ and $Q = Q^* = c + nS^*$.

Equation (1) presumes that γ is independent of the size of the pollution stock. A nonlinear specification would result if the rate of degradation depended on the pollution stock, in which case $\gamma = \gamma(X)$. It is also possible for the degradation rate (γ) to exhibit discontinuities. Conrad (1988 a) has considered the extreme case of "irreversible accumulation" where if the pollution stock exceeds some unknown critical value the degradation rate drops to zero.

When spatial considerations are important, say when dealing with pollution from a point source, a model of diffusion may be appropriate. Let $Z = Z(D,t)$ now denote the concentration of the pollutant at distance D from a point source. Diffusion might be characterized by the partial differential equation

$$\frac{\partial Z}{\partial t} = \sigma^2 Z_{DD} - \gamma(Z,D) \tag{4}$$

where σ^2 is the diffusion coefficient and Z_{DD} is the second partial of $Z(\cdot)$ with respect to distance. The function $\gamma(Z,D)$ allows degradation to vary according to concentration and distance from the point source. When $\gamma(Z,D) = \gamma$ and there is a constant rate of residual discharge leading to $Z(0,t) = Z_0$, the equilibrium solution is simply exponential decay as the distance from the point source increases with

$$Z(D) = Z_0\, e^{-\sqrt{\gamma}D/\sigma} \tag{5}$$

Finally, residual discharges may affect a physical or chemical attribute of an ecosystem. Chloroflurocarbons (CFCs) deplete stratospheric ozone which will increase the amount of ultraviolet radiation into the biosphere. The increase in ultraviolet radiation may adversely affect the growth of lower trophic level organisms which support higher level species. In the case of ozone depletion there is a lag between discharge in the lower atmosphere and depletion in the stratosphere. Let Z now denote the ozone concentration in the stratosphere, S the discharge of CFCs and τ the lag period. Then the change in ozone might be modeled by the difference equation

$$Z_{t+1} = Z_t + \Psi(Z_t , S_{t-\tau}) \qquad (6)$$

In Section IV systems of differential and difference equations will be used in two stylized models to describe the qualitative effect of pollution on population dynamics in the Southern Ocean.

III. Pollution and a Single Species

Suppose that the stock pollutant adversely affects the growth rate of a renewable resource whose biomass is denoted by the the variable X. We will assume that the pollutant does not pose an immediate risk to human health and that the resource is valued solely on the basis of its yield (harvest). The equation describing the dynamics of the resource is written as

$$\dot{X} = F(X,Z) - Y \qquad (7)$$

where Y is yield and the partial of the growth function with respect to pollution is negative ($F_Z < 0$). For example, the logistic function takes the form $F(X) = rX(1 - X/K)$, where r is the intrinsic growth rate and K is the environmental carrying capacity. The pollution stock could adversely affect either parameter [$r = r(Z)$ or $K = K(Z)$ with $r'(Z) < 0$ or

$K'(Z) < 0]$.

Suppose now that net benefits at instant t are given by $W(Q,Y)$ where Q is again the positively-valued commodity and $Q > Q_0$ results in residual discharge $S > 0$. With pollution dynamics given by equation (1) and resource dynamics given by equation (7) the problem of maximizing the present value of net benefits may be stated mathematically as

$$\text{Maximize} \int_0^\infty W(Q,Y)\, e^{-\delta t}\, dt$$

$$\text{Subject to } \dot{X} = F(X,Z) - Y$$

$$\dot{Z} = -\gamma Z + S$$

$$\phi(Q,S) = 0$$

The current value Hamiltonian is written as

$$\tilde{H} = W(Q,Y) + \lambda[F(X,Z) - Y] + \mu[-\gamma Z + S] - \omega\phi(Q,S) \qquad (8)$$

The first order necessary conditions are derived in Appendix B. In steady state these conditions collapse to five equations in five unknowns (X, Z, Y, S and Q) taking the form

$$F_X = \delta$$

$$W_Q = \frac{W_Y F_Z [\phi_Q/\phi_S]}{(\delta + \gamma)}$$

$$Y = F(X,Z)$$

$$S = \gamma Z$$

$$\phi(Q,S) = 0$$

The first of these equations is a familiar expression in resource economics. In the nonlinear bioeconomic model (Clark 1976, p.95) it requires that the optimal stock equate the biological growth rate to the rate of discount. In the present model the partial derivative F_X may involve both X and Z and by itself would not determine the optimal population level for the renewable resource.

The second equation in the steady state set may be interpreted as equating the marginal social benefit of Q to the discounted marginal social cost of an increase in S. Note that $[\phi_Q/\phi_S]$ is the reciprocal of the marginal rate of transformation of the residual S into commodity Q. An incremental increase in S, allowing Q to increase, will also result in an increase in the steady state pollution stock, Z. This will reduce net growth and sustainable harvest, Y, with a marginal loss of W_Y in perpetuity.

While it is possible to identify and interpret the equations defining the steady state optimum it is much more difficult to determine the properties of an optimal approach from (X_0,Z_0). If $W(Q,Y)$ and $\phi(Q,S)$ are additively separable functions then it can be shown that the first order necessary conditions of Appendix B will imply that the optimal approach to a steady state optimum must satisfy

$$\dot{Y} = \frac{W_Y}{W_{YY}}[\delta - F_X]$$

$$\dot{S} = \frac{W_Q\,\phi_S\,\phi_Q^2\,(\delta + \gamma) - \phi_Q^3\,W_Y\,F_Z}{W_Q\,\phi_{SS}\,\phi_Q^2 - \phi_S^2\,W_{QQ}\,\phi_Q + W_Q\,\phi_S^2\,\phi_{QQ}}$$

$$\dot{X} = F(X,Z) - Y$$

$$\dot{Z} = -\gamma Z + S$$

Even with the assumption of separability it was not possible to deduce the movement of a point in (X,Z)-space, from (X_0,Z_0) to (X^*,Z^*). With additional structure [linearity and separability in W(·) and φ(·)], Conrad (1988 b) gives a numerical example showing how harvest (Y) and residual discharge (S) can be used to drive (X_0,Z_0) to (X^*,Z^*) in a "rapid" approach. How closely this algorithm approximates the optimal approach is not known and additional numerical analysis is being undertaken.

IV. Pollution and Multispecies Systems

The reduction in one species, through harvest or pollution, is likely to alter the population dynamics of other species within an ecosystem. For example, the reduction in baleen whale populations following World War I and II is thought to have increased the amount of krill available to seal, seabird and penguin populations in Antarctica

(Figure 2). We will consider two models of pollution based on a multispecies system developed by May et. al. (1979), initially constructed to examine the effect of commercial krill harvests on the dynamics of baleen whales and seals in the Southern Ocean.

The results presented here are based on computer simulations of a four dimensional dynamical system. As was seen in the preceding section, it becomes increasingly difficult to derive analytical results when optimizing a dynamical system of two or more state variables. By restricting ourselves to simulation the most we can expect is to get a "feel" for the effect of pollution within such systems.

We begin by reconstructing the two-predator one-prey model of May et. al. fashioned on the likely qualitative relationships between krill, seals and whales in the Southern Ocean. We proceed directly to the "dimensionless form" where the population of each species is expressed as a fraction of its carrying capacity. In terms of notation, let X_1 and E_1 denote the biomass and (suitably scaled) level of effort expended in the harvest of krill, while X_2 and E_2 will denote biomass and effort to harvest baleen whales and X_3 and E_3 the biomass and effort to harvest seals. May et. al. were interested in the the implications of krill harvest both with and without a moratorium on whaling. The dynamical system used to explore various "constant harvest" regimes took the form

$$\dot{X}_1 = r_1X_1\,(1 - E_1 - X_1 - \upsilon X_2 - \eta X_3) \qquad \text{(Krill)}$$

$$\dot{X}_2 = r_2X_2\,(1 - E_2 - X_2/X_1) \qquad \text{(Whales)}$$

$$\dot{X}_3 = r_3X_3\,(1 - E_3 - X_3/X_1) \qquad \text{(Seals)}$$

Figures 5A-5C reproduce the results of May et. al. for the parameters $r_1 = 1$, $r_2 = 0.1$, $r_3 = 0.3$, $\upsilon = 1$, $\eta = 1$ and $E_3 = 0$ (no harvest of seals). Figure 5A shows the effect of a moratorium on whaling ($E_2 = 0$) with no harvest of krill ($E_1 = 0$) from initial conditions that correspond to the equilibrium after intensive whaling (ie, $X_{1,0} = 0.4545$, $X_{2,0} = 0.0909$ and $X_{3,0} = 0.4545$ when $E_2 = 0.8$). Figure 5B shows the results of initiating krill harvest ($E_1 = 0.5$) while maintaining intensive whaling effort ($E_2 = 0.8$). Figure 5C shows the effects of a moratorium on whaling ($E_2 = 0$) but initiation of commercial krill harvest at $E_1 = 0.5$. (The time paths shown in Figures 5A-5C were obtained via an Euler approximation using a mesh size of 0.1).

The simulations show that harvesting krill during a moratorium on whaling will slow the recovery of whale populations to an ultimately lower equilibrium level (5C). Initiating krill harvests with continued whaling would have caused a further decline in baleen whale populations.

To this stylized system for the Southern Ocean we now introduce a stock pollutant adversely affecting the growth of krill. The system is modified to

$$\dot{X}_1 = r_1X_1\,(1 - E_1 - (1 + \varepsilon Z)X_1 - \upsilon X_2 - \eta X_3) \qquad \text{(Krill)}$$

$$\dot{X}_2 = r_2X_2\,(1 - E_2 - X_2/X_1) \qquad \text{(Whales)}$$

$$\dot{X}_3 = r_3X_3\,(1 - E_3 - X_3/X_1) \qquad \text{(Seals)}$$

$$\dot{Z} = -\gamma Z + S \qquad \text{(Pollution)}$$

Inspection of the first equation in this system reveals that the pollution stock reduces the environmental carrying capacity for krill according to the the term $(1 + \varepsilon Z)$. Figure 6 shows the effects of a constant discharge rate of $S = 0.1$ with $\varepsilon = 1$ when all other parameters are the same as for the case shown in Figure 5A. The populations of all species are seen converging to 0.2857 which is less than the value 0.3333 for the comparable case of no pollution shown in Figure 5A.

As a final model we consider a system of difference equations. Let S_t now represent the discharge of chloroflurocarbons and $X_{4,t}$ the concentration of ozone in period t. As before $X_{1,t}$, $X_{2,t}$ and $X_{3,t}$ will represent the stocks of krill, whales and seals. The carrying capacity for krill decreases if the concentration of ozone declines below one, which is the equilibrium when there has been no prior discharge of CFCs. There is a lag of τ periods before ozone reaches the stratosphere. The system is written as

$$X_{1,t+1} = X_{1,t} + r_1 X_{1,t} (1 - E_1 - X_{1,t}/X_{4,t} - \upsilon X_{2,t} - \eta X_{3,t}) \quad \text{(Krill)}$$
$$X_{2,t+1} = X_{2,t} + r_2 X_{2,t} (1 - E_2 - X_{2,t}/X_{1,t}) \quad \text{(Whales)}$$
$$X_{3,t+1} = X_{3,t} + r_3 X_{3,t} (1 - E_3 - X_{3,t}/X_{1,t}) \quad \text{(Seals)}$$
$$X_{4,t+1} = X_{4,t} + r_4 X_{4,t} ((X_{4,t}/\alpha - 1)(1 - X_{4,t}) - S_{t-\tau}) \quad \text{(Ozone)}$$

With no discharge of chloroflurocarbons ($S_t = 0$), ozone at its maximum ($X_{4,0} = 1$) and the same parameters as in the original Krill-Whales-Seals system, the same equilibrium as in Figure 5A is achieved. The ozone level is constant at one. The time paths are shown in Figure 7 for a simulation of 50 periods.

With $S_t = 0.1$, $r_4 = 0.35$, $\alpha = 0.1$ and $\tau = 7$ the system would appear to become chaotic (see Figure 8 for a simulation of 50 periods). The aperiodic fluctuations in ozone induce fluctuations in krill and seals and to a lesser extent in whales.

Based on the last simulation one might be tempted to conclude that pollution can induce fluctuations and instability into a system which had been stable and predictable. In truth the last simulation was *rigged* in the sense that a bit of time was required to find a nonlinear difference equation for ozone which would lead to chaotic fluctuations. When an Euler approximation is used with mesh size of 0.1 the system will smoothly approach an equilibrium without overshoot.

Perhaps the only conclusion which can be drawn from this last simulation is that structure of the model, the form of individual equations, period length and lags are important. When stock pollutants affect ecosystem dynamics it is unlikely that biologists (and even less likely that economists) would know the structure of equations appropriate for describing the physical modifications to the environment. If we are entering an age of large scale, possibly global pollution problems interdisciplinary modeling will be imperative. What are the likely roles for physical scientists, biologists, and economists?

V. Global Pollution and Interdisciplinary Research

An agenda for interdisciplinary modeling might be based on the following questions.

1. What are the important physical parameters underlying major terrestrial and marine ecosystems?

2 How and on what time scale are stock pollutants thought to change these physical parameters within a particular ecosystem?

3. Within the major ecosystems, what species are most immediately affected by stock pollutants, and how is the "life history" of these species altered (ie, changes in the rate of growth, mortality, fecundity, etc.)?

4. How are pollution stocks, physical parameters and the ecosystem likely to evolve with different emission rates and pollution control strategies?

5. What are the costs of pollution in terms of human health and altered ecosystems?

6. What are the costs of alternative pollution control strategies, which are the most cost effective and what policies and institutions are needed to achieve the "desired" evolutionary strategy?

Developing better models of pollution dynamics, their transport and their physical and chemical change in water, soil and the atmosphere is critically important. Not any model will do, however. The level of detail and scale must be such that they can relate and describe changes in parameters that are in turn relevant and underlie the ecosystem being studied. Biologists must learn something about the physics and chemistry of pollution transport and the engineers, soil physicists, an oceanographers must learn something about population dynamics and ecosystem modeling.

Biologists must begin to consider how increased concentrations of certain pollutants or the induced change in other physical or chemical attributes affect various species within the ecosystem. Attributes that were never considered in ecosystem models or thought to be constant or changing on a geologic time scale, may need to be explicitly considered if such change is being accelerated by pollution. An example would be temperature or salinity in a marine ecosystem changing as a result of the greenhouse effect. This is again an area where both physical scientists and ecologists must work together.

Field observations and possibly laboratory studies must be designed to determine the physiological and behavioral changes in those species thought to be most immediately affected by pollution or an altered physical attribute. How do rates of mortality, morbidity, growth or fecundity change with increased concentrations of certain pollutants or what defense mechanisms, mobile or evolutionary, might species adopt in response to a pollution altered environment?

Models of the physical environment must be designed to interface with models of the ecosystem in order to simulate the effects of different emission rates. Even with the accelerated rate of change in physical systems there are likely to be widely varying time scales when coupling a physical model of pollution with a model of some ecosystem. We may be dealing with real time scales that require 50, 100 or 200 years for the full effects of pollution control policies to dynamically work themselves out. Lack of historical data on climatic change and the lengthy future horizon create obvious estimation validation problems. Two or three research teams, charged with the same broad modelling objectives but working independently of one another, may collectively provide bounds for the evolution of physical and biological systems. Given the uncertainty inherent in such forecasts these groups would then critique and provide feedback on each other's research.

The uncertainty of the physical and biological models makes the economist's job (questions 5 and 6) all the more difficult. How is society to value greater risks to public health and more rapid (and likely undesirable) changes in terrestrial, aquatic and marine ecosystems? Costs of prevention and remediation will be high and while a more sensitive public may be willing to pay more for pollution control, they will want to know what the benefits are, if not in monetary terms then in terms of dissolved oxygen, edible shellfish, and potable and swimable water.

The global and large scale systems designed to examine the physical, biological and economic dimensions of pollution are unlikely to lend themselves to optimization. Simple optimization models might still be useful in differentiating between alternative policies in terms of their likely effectiveness or cost of administration. Economists will need to extend their policy analysis to spatial models and to satisfy both transboundary and intergenerational constraints on the generation, transport and concentration of stock pollutants.

Enormous strides have been made in man's ability to mathematically model his physical and biological environment. In looking to the future two broad questions remain to be answered. Will our ability to model resource systems be adequate to task of accurately describing the future consequences of man-made pollutants? Will we have the wisdom and self-restraint that will allow "equitable evolution"; that is, an evolution where future generations have meaningful choice in managing the quality and diversity of their physical and biological environment?

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Figure 1. Location Map for Leonardo, New Jersey and the Great Peconic Bay on Eastern Long Island

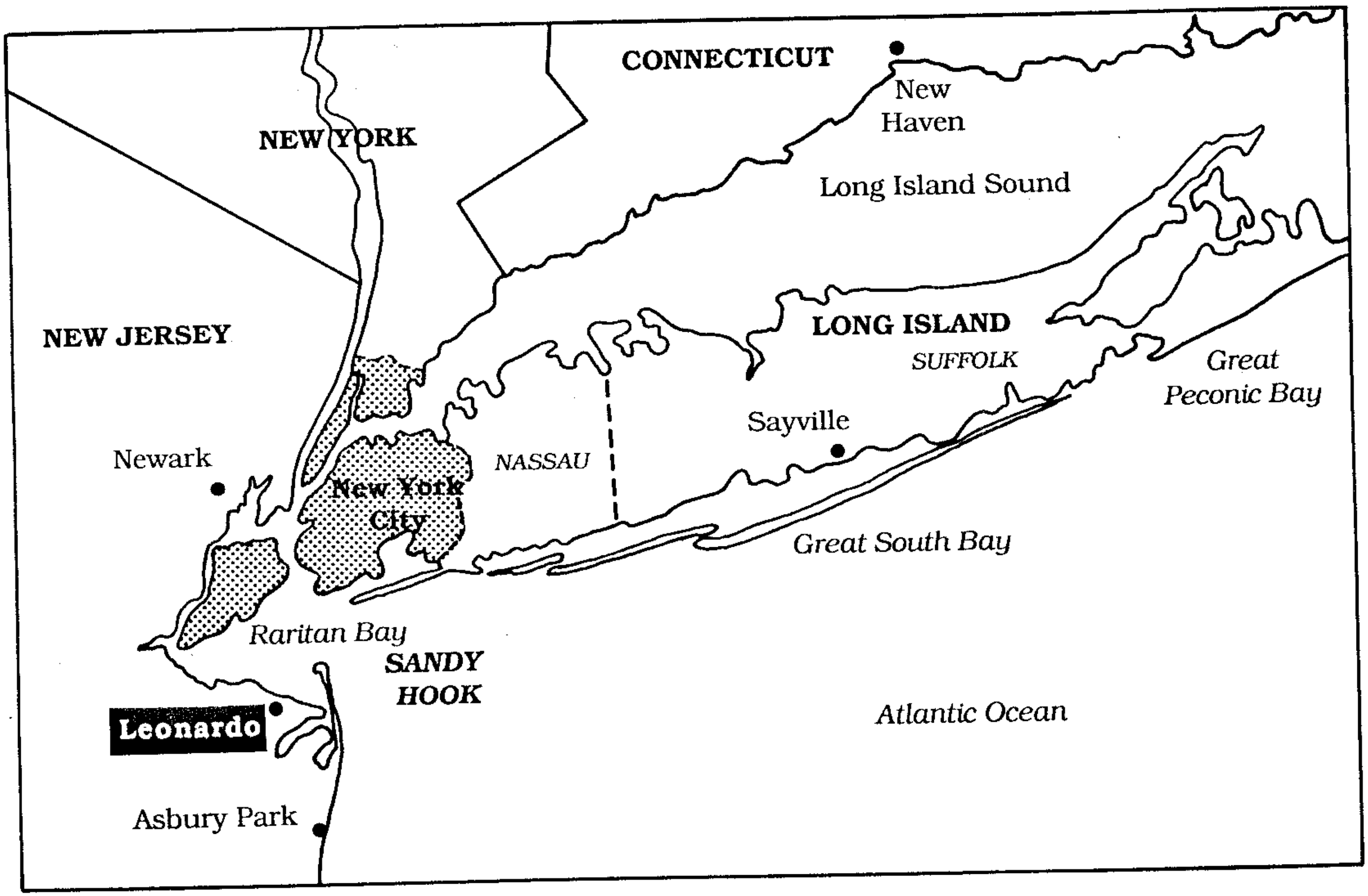


Figure 2. The Food Chain in the Southern Ocean

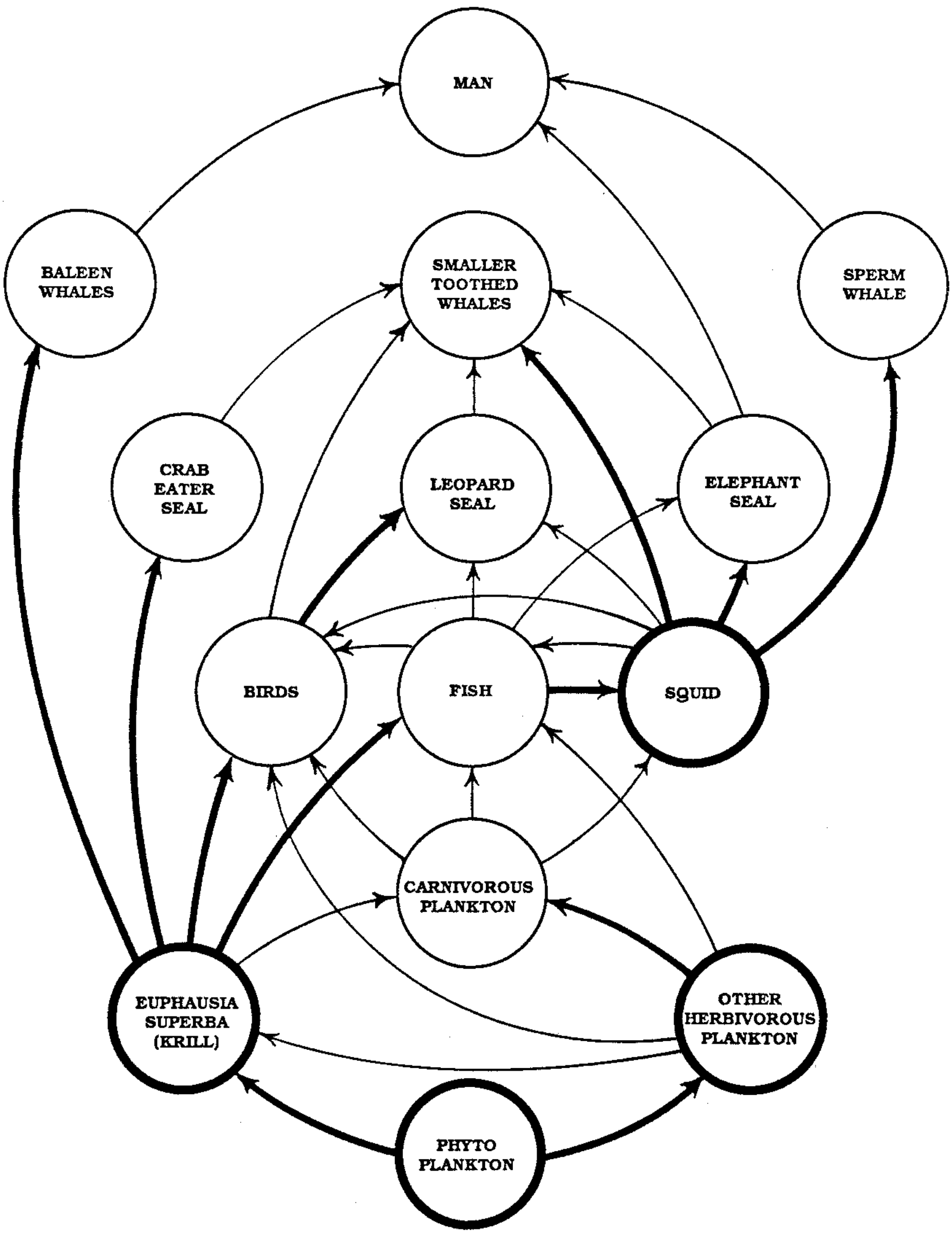


Figure 3. A Graph of the Commodity-Residual Transformation Curve Implied by $\phi(Q,S) = O$

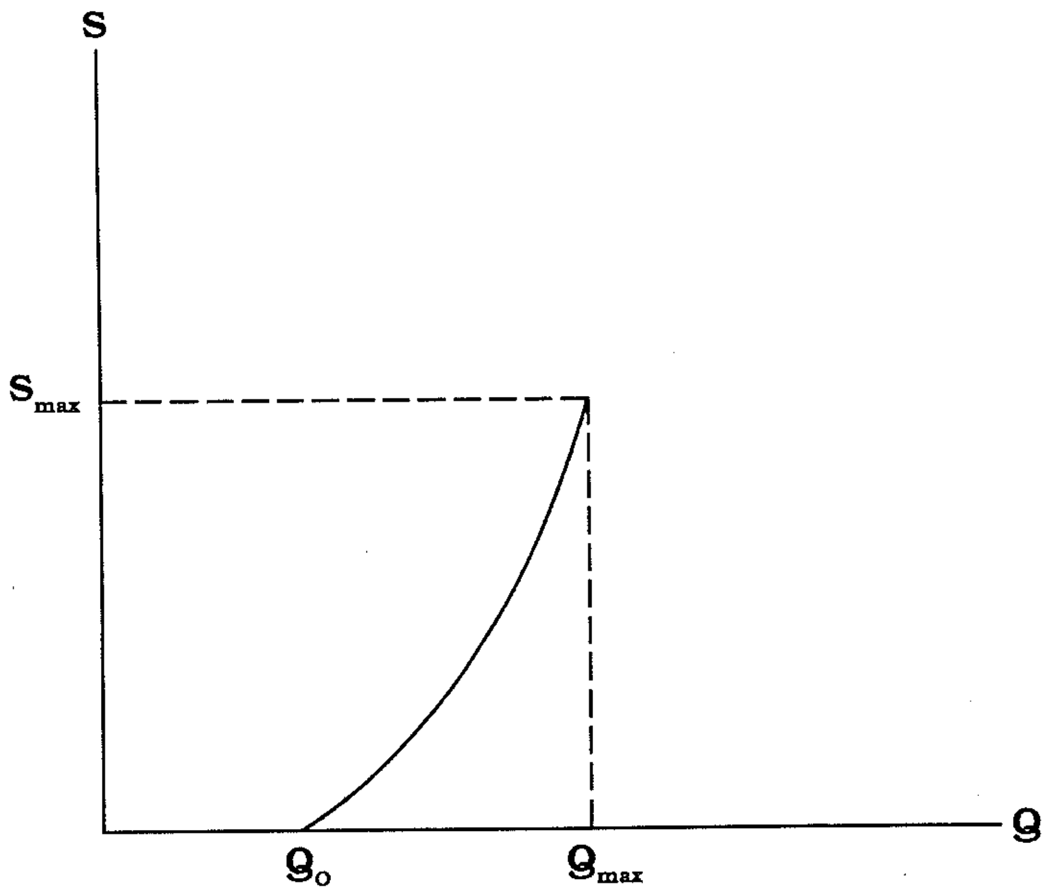


Figure 4. The Optimal Pollution Stock and the MRAPs from $Z_o > Z^*$ and $Z_o < Z^*$

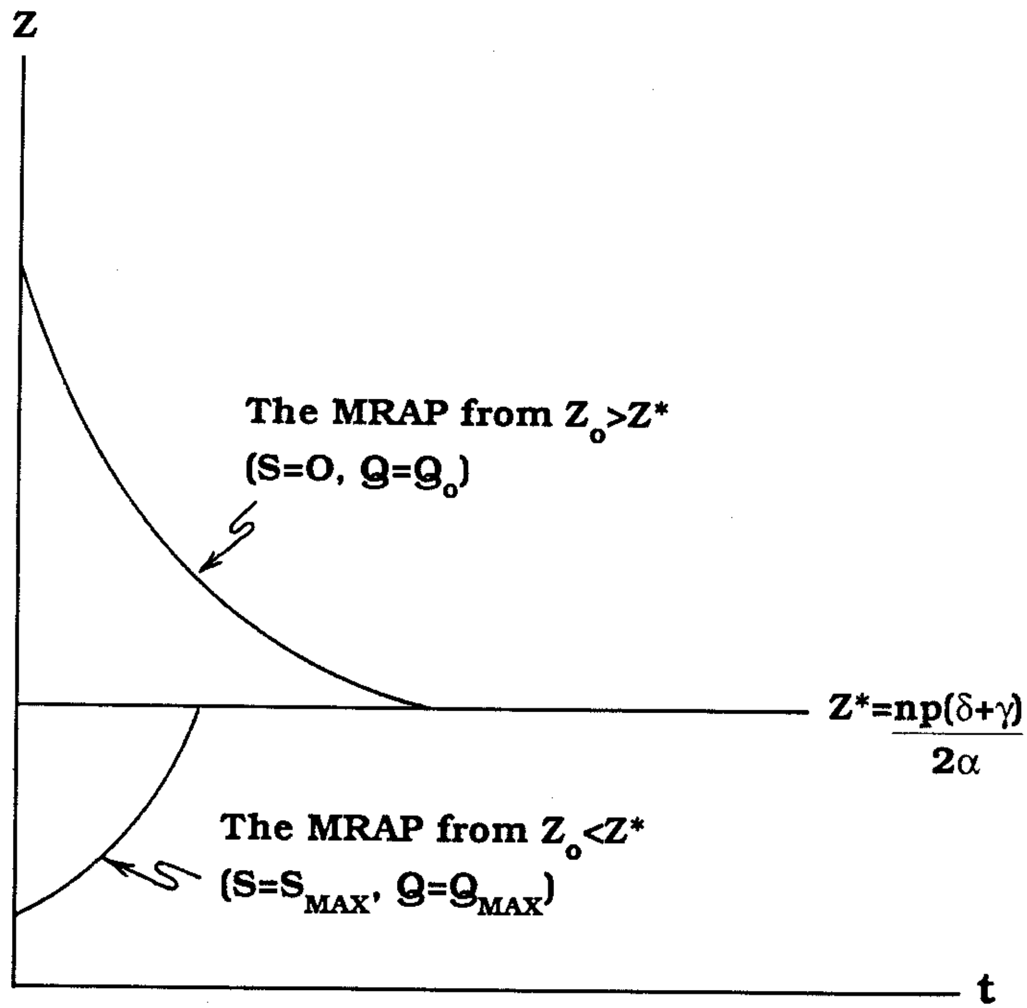


Figure 5. The krill (X_1, solid curve), whale (X_2 dashed) and seal (X_3 dot-dash) model for $r_1 = 1$, $r_2 = 0.1$, $r_3 = 0.3$, $\upsilon = 1$, $\eta = 1$, and $E_3 = 0$ when (A) there is no harvest of krill ($E_1 = 0$) and a moratorium on whaling ($E_2 = 0$) after intensive whaling had led to $X_{1,0} = 0.4545$, $X_{2,0} = 0.0909$ and $X_{3,0} = 0.4545$, (B) maintaining intensive whaling ($E_2 = 0.8$) and initiating the commercial harvest of krill ($E_1 = 0.5$) and (C) a moratorium on whaling ($E_2 = 0$) but initiation of krill harvesting ($E_1 = 0.5$).

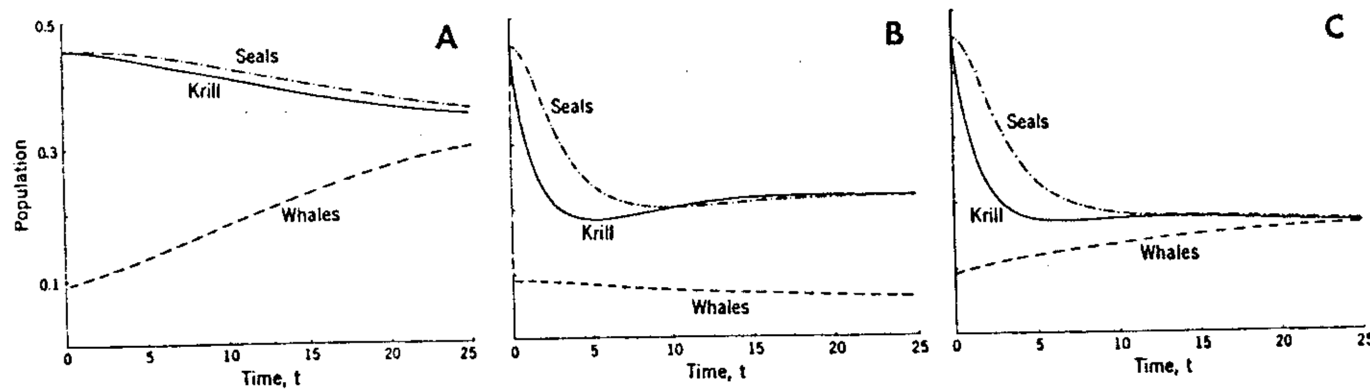


Figure 6. A simulation of the krill-whale-seal model with a degradable pollution stock adversely affecting the carrying capacity of krill when $S=0.1$, $\gamma=0.2$, $\varepsilon=1$ and all other parameters are the same as in Figure 5A

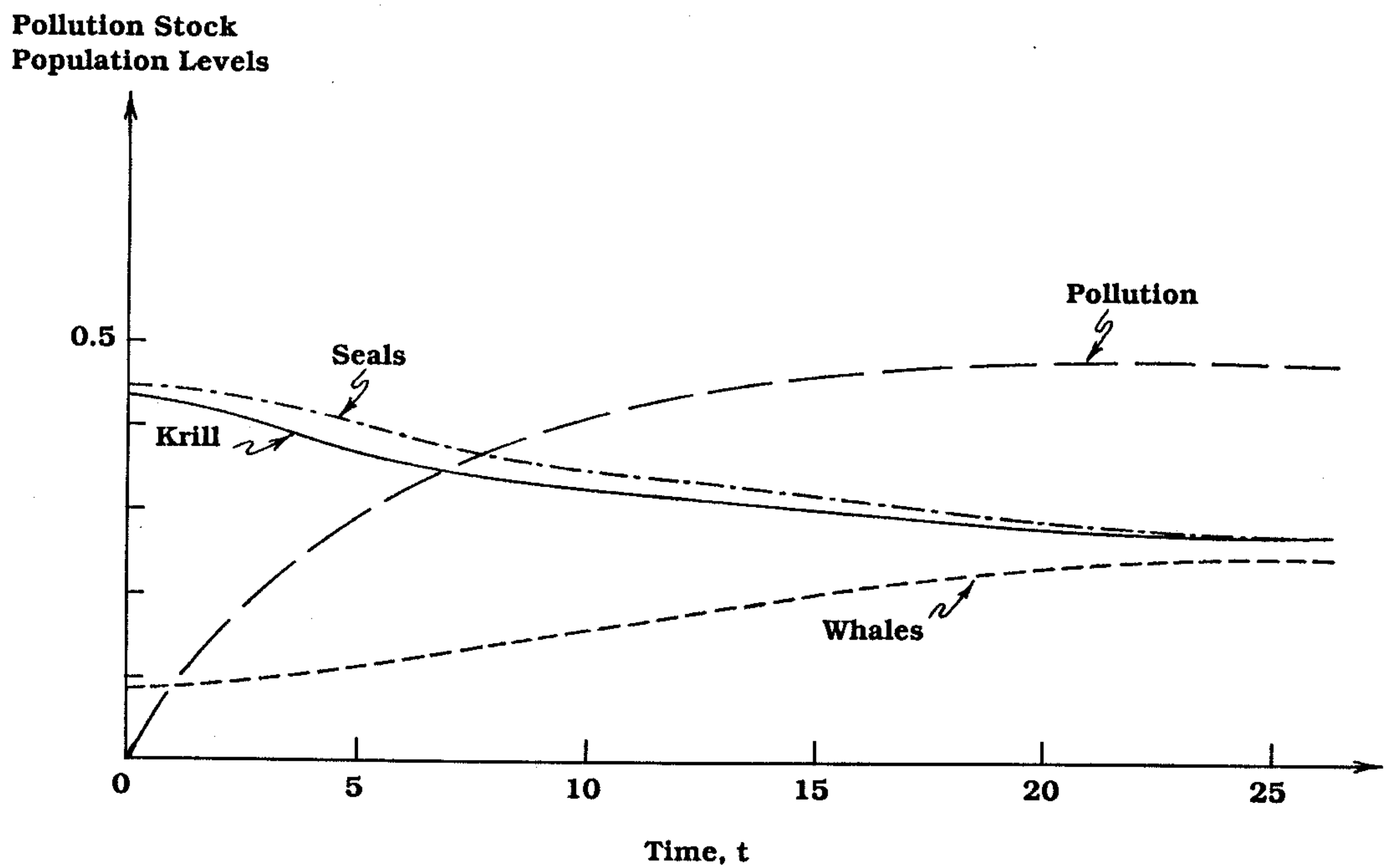


Figure 7. The difference equation model of krill, whales, seals and ozone ($X_{4,t}$) when there has been no prior emissions of CFCs (S_t=0). The equilibrium is the same as in Figure 5A for krill, whales and seals with ozone stable at $X_{4,t}$=1)

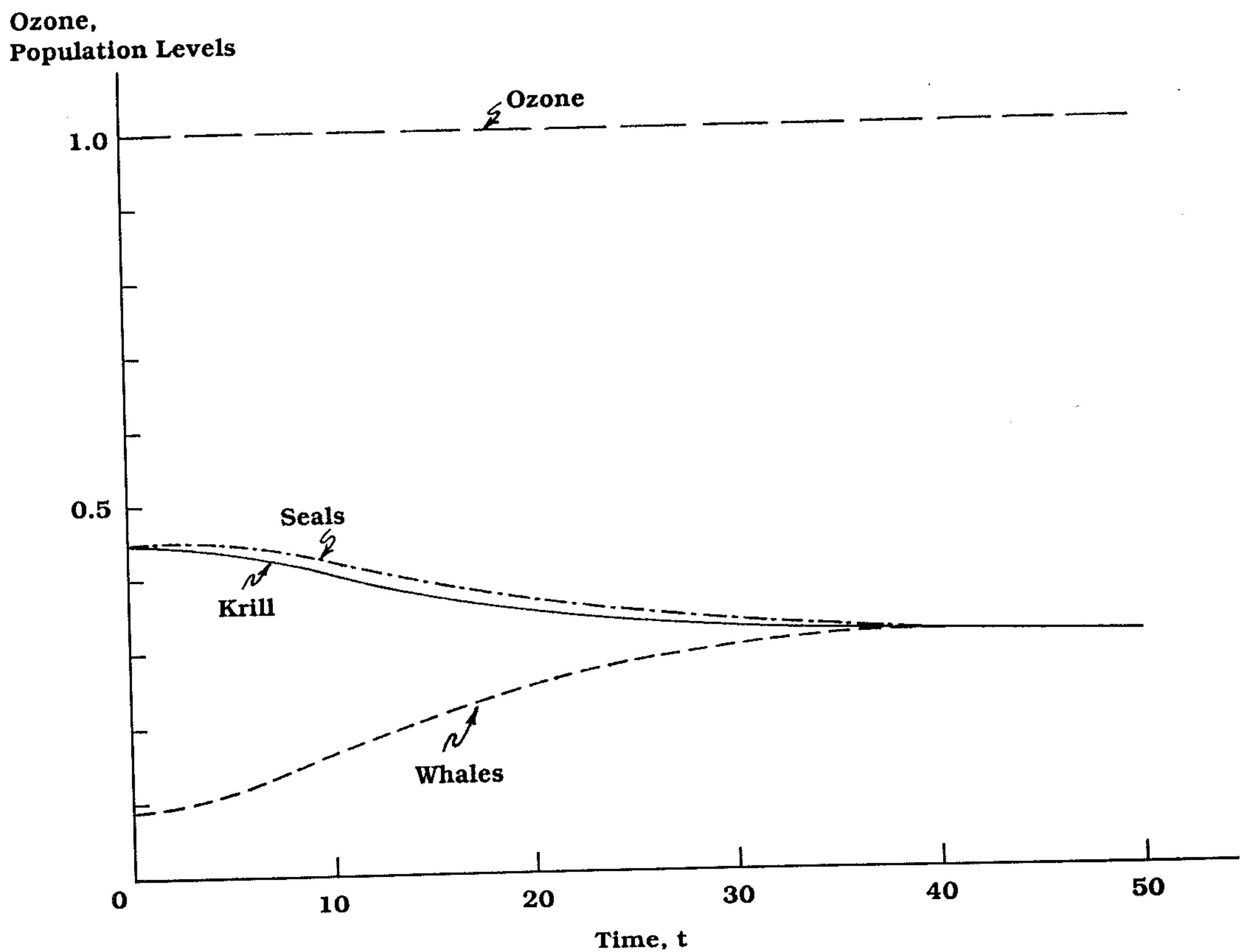
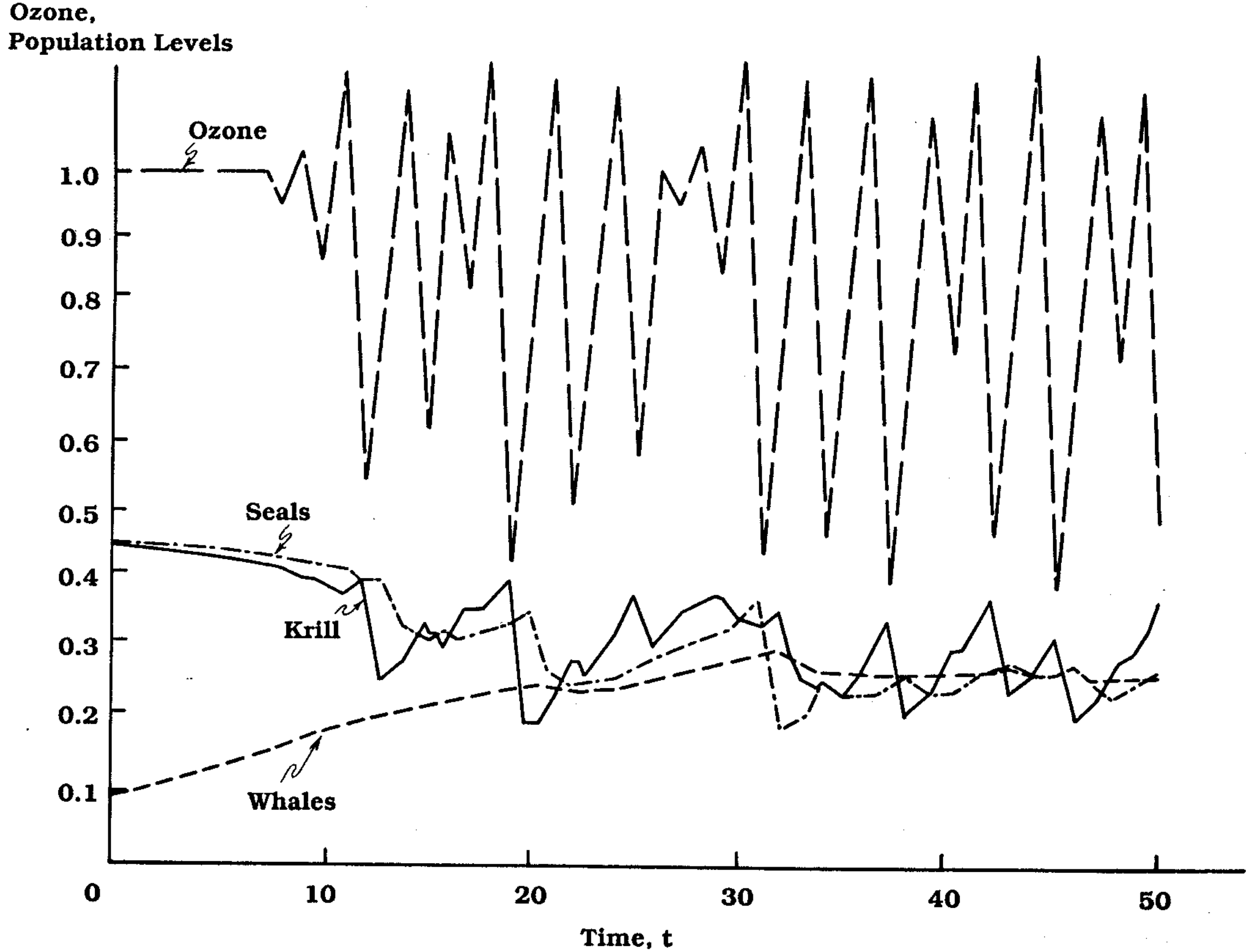


Figure 8. The difference equation model of krill, whales, seals and ozone with CFC emissions at S_t=0.1 when r_4=0.35, α=0.1, τ=7 and all other parameters the same as in Figure 5A



Appendix A

This appendix derives the first order necessary conditions for the single-state pollution problem in Section II of this paper. The problem was to

$$\text{Maximize} \quad \int_0^\infty W(Q,Z)e^{-\delta t}dt$$

$$\text{Subject to} \quad \dot{Z} = -\gamma Z + S$$

$$\phi(Q,S) = 0$$

The current value Hamiltonian is written as

$$\tilde{H} = W(Q,Z) + \mu[-\gamma Z + S] - \omega\phi(Q,S)$$

with first order conditions that include

$$\frac{\partial \tilde{H}}{\partial Q} = W_Q - \omega\phi_Q = 0$$

$$\frac{\partial \tilde{H}}{\partial S} = \mu - \omega\phi_S = 0$$

$$\dot{\mu} - \delta\mu = -\frac{\partial \tilde{H}}{\partial Z} = -[W_Z - \mu\gamma]$$

In steady state the first of these conditions implies $\omega = W_Q/\phi_Q$ while the second implies $\mu = \omega\phi_S$ and thus $\mu = [W_Q/\phi_Q]\phi_S$. The third equation leads to $W_Z = (\delta + \gamma)\mu$. Substituting the expression for μ into the right-hand-side and solving for W_Q yields

$$W_Q = \frac{W_Z\,[\phi_Q/\phi_S]}{(\delta + \gamma)}$$

When $W(Q,Z) = pQ - \alpha Z^2$, so that pollution damage is quadratic in the stock, and $Q = c + nS$, the above equation may be solved for Z yielding

$$Z^* = \frac{np(\delta + \gamma)}{2\alpha}$$

and the comparative statics are immediately apparent; namely that the optimal pollution stock increases with an increase in n, p, δ, or γ and decreases with an increase in α. The MRAP is optimal.

Appendix B

In this appendix we derive the first order conditions for the two-state model in Section III of this paper. The problem was to

$$\text{Maximize} \int_0^\infty W(Q,Y)\, e^{-\delta t}\, dt$$

$$\text{Subject to } \dot{X} = F(X,Z) - Y$$

$$\dot{Z} = -\gamma Z + S$$

$$\phi(Q,S) \equiv 0$$

The current value Hamiltonian is written as

$$\tilde{H} = W(Q,Y) + \lambda[F(X,Z) - Y] + \mu[-\gamma Z + S] - \omega\phi(Q,S)$$

and the first order necessary conditions include

$$\frac{\partial \tilde{H}}{\partial Y} = W_Y - \lambda = 0$$

$$\frac{\partial \tilde{H}}{\partial Q} = W_Q - \omega\phi_Q = 0$$

$$\frac{\partial \tilde{H}}{\partial S} = \mu - \omega\phi_S = 0$$

$$\dot{\lambda} - \delta\lambda = -\frac{\partial \tilde{H}}{\partial X} = -\lambda F_X$$

$$\dot{\mu} - \delta\mu = -\frac{\partial \tilde{H}}{\partial Z} = -\lambda F_Z + \mu\gamma$$

where $\lambda > 0$ and $\mu < 0$ are the costate variables on the resource and pollution stock, respectively. In steady state the fourth condition immediately implies $F_X = \delta$, while the fifth implies $\lambda F_Z = \mu(\delta + \gamma)$. Substituting $\lambda = W_Y$ and $\mu = [W_Q/\phi_Q]\phi_S$ into this last expression and solving for W_Q yields

$$W_Q = \frac{W_Y F_Z [\phi_Q/\phi_S]}{(\delta + \gamma)}$$

The remaining steady state conditions on page 10 are implied by $\dot{X} = 0$, $\dot{Z} = 0$ and the transformation function.