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THE ROLE OF ALTERNATIVE RISK PROGRAMMING MODELS IN EMPIRICAL RESEARCH

by

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For at least 30 years, agricultural economists have increasingly incorporated risk considerations into empirical economic analysis. These efforts have recognized the risk inherent in biological production processes, the potential for discrepancies between planned production and realized production, uncertain demand, and the need to allocate scarce resources before prices are known. The research agenda has been equally diverse, ranging from the study of decision processes and the identification of preference functions, to the interaction between public policy and the risk environment faced by the agricultural sector. The micro, or farm level, studies have dominated the agricultural economics literature and have been largely prescriptive, focusing on strategies for risk efficiency. Much less attention has been given to aggregate supply or demand studies which are descriptive. Pope (1982) attributes much of this neglect to the poorly developed literature on economic welfare under risk.

It would be impossible within the space available to describe, let alone evaluate, the numerous empirical approaches involving agricultural risk. To reduce the task to manageable proportions, this paper focuses primarily on the prescriptive models at the firm level. Some attention is given to their potential for resolving problems at a more aggregate level. By necessity, the discussion is limited almost exclusively to

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models underpinned by the expected utility paradigm. Although there is a legitimate debate about the appropriateness of the paradigm, alternatives to it have received little attention in the empirical literature.

To limit further the scope of the paper, little attention is given to empirical procedures for estimating utility functions or risk aversion parameters. These latter issues are important but have recently been evaluated at the 1981 meetings of the AAEA-AEA (e.g. Robison; Pope; Hazell; Binswanger; Roe). Other articles on the subject have appeared in the AJAE as well, the most recent by Knowles, who claims to resolve some longstanding econometric problems in the measurement of utility.

Since numerous other authors, including some on this technical committee (e.g. Barry, 1984), have evaluated alternative risk models, my intent is not merely to reiterate what most of us probably know already. Nor is it to be unnecessarily critical of past research. Rather, I shall attempt to assess some of the recent developments in risk analysis and discuss how they are likely to shape future research. Much of the discussion centers around the historical role of mathematical programming and how its role is likely to change in the face of new developments in risk analysis.

The paper begins with a brief statement of the micro decision framework and assessment of the traditional role of programming models in risk analysis. This is followed by a general discussion of stochastic dominance and how it has changed the need for probability information in risk analysis. The remainder of the paper attempts to assess the usefulness of both active and passive stochastic programming models within this context. Some recent developments in passive stochastic programming that have found application in business and central planning are discussed. Under specific conditions, they can generate important probability information about decision alternatives, and in recent years, they have become computationally more tractable. Their application to problems in agriculture, particularly at the micro level, is an open question.

The Micro-Decision Framework

Since the first applications of decision theory based on the expected utility maxim to firm-level decisions in agriculture in the 1950's, agricultural economists have worked to refine the theory and apply it empirically to a wide range of risk situations. Developments in statistics, operations research and computer technology, have lead to increasingly complex empirical analysis.

Regardless of the level of complexity, the analytical framework for any risk analysis at the firm-level must contain several components. The framework must have some mechanism for identifying a set of alternative courses of action. The states of nature that circumscribe the decision environment must be articulated, along with their probabilities of occurrence. This information is then used in conjunction with some performance measure to estimate payoff probabilities. Finally, a decision criteria is applied to identify a preferred course of action or, in the absence of complete information about preferences, a subset of actions that are risk efficient.[1] Viewed

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[1]This refers to a partial ordering of alternatives into two subsets, those which are clearly inferior and can be eliminated from consideration and those for which no clear preference can be determined without more complete information on preferences.

from a slightly different perspective, the decision involves the selection of a probability distribution, although in most applications decision rules do not require complete information about the density or distribution functions.

The Traditional Role of Programming Models

Rarely in empirical work are these components to the decision framework accommodated in this lock-step fashion. Once the data are assembled, a model is constructed which simultaneously processes the relevant information on probabilities and the alternative courses of action and identifies an optimal strategy or a set of efficient ones. This strategy is most apparent in those risk applications where it is appropriate to characterize the alternative courses of action by a programming model. An attractive feature of a programming approach is an ability to consider many risk situations within a whole farm planning context. Optimal or efficient choices are then determined from the infinite set of feasible alternatives which differ both in terms of the individual production components of the "portfolio" and each component's relative importance. The complexities that can be incorporated into the model are governed, of course, by computational considerations and other limitations common to activity analysis based almost exclusively on Leontief-type production technology.[2] With one or two exceptions (Cocks,

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[2]With few notable exceptions (e.g. Johnson, 1979; Whitson et al. 1976), most quadratic programming applications have been limited to the consideration of a single production period. Additional complications arise if the model is designed to cover a multiple period planning horizon. These concerns relate to the discounting of costs and revenues whose components may be random and to the appropriateness of assuming the stochastic processes are stationary and independent over time.

1968, and Rae, 1971) most programming applications have focused on decisions which must be made prior to knowing the values of the problem's random components. Sequential programming models are concerned with problems where decisions at stage t make use of information known at all previous stages. Generally, these applications add enormous complexity to the analysis.

Computational limitations, particularly for quadratic programming applications, are less severe than they were 10 to 15 years ago. However, the recent experimentation with alternative decision criteria has important implications for the role of nonlinear programming in empirical research. In the case of the quadratic programming model, one's ability to apply the expected utility maxim is conditioned by the appropriateness of assuming normally distributed gross margins (or other suitable performance measure) and exponential utility, or quadratic utility. When gross margins are normally distributed, the information needed to generate the E-V efficiency locus can also be used to make direct application of the safety-first criteria as put forth by Roy (1952) and Kataoka (1963). By appealing to Chebyshev's inequality, the safety-first rules can be applied, but the precision associated with the probability statements falls dramatically.

The second type of programming model most often associated with micro-level risk analysis is the MOTAD model developed by Hazell (1971). The procedure is designed to generate risk efficient sets by identifying alternatives that minimize the total absolute deviations about specified levels of expected gross margin or income. Although not necessary, the criteria is most often applied to problems formulated in a linear programming (LP) framework. In this situation, MOTAD becomes a special case of the goal programming model originally developed by Charnes and Cooper (1961). Each goal is the deviation about mean income for one year in the data series of gross margins. Equal weighting of the deviations is logical so the numerous problems faced in other goal programming applications discussed by Willis and Perlack (1980) are avoided.

MOTAD is also often described as a linear alternative to quadratic programming. This is true operationally, but it is important to remember that it was not developed as a method for approximating the E-V efficiency locus. Rather, in the case of normally distributed returns, Hazell argues that a statistic $d\ (\Pi s/2(s-1))^{1/2}$ involving $\Pi = 22/7$, $s =$ the sample size and $d =$ mean absolute deviation, can be used to estimate the population standard deviation. Thus, discrepancies between MOTAD and E-V efficient solutions derive from differences between this and the more usual sample standard deviation estimate. The more usual estimate is more efficient, although both are unbiased estimators. On this basis, one might expect quadratic programming to discriminate more reliability between efficient and inefficient alternatives. Any added reliability comes at a substantial loss both in computational ease and in the complexity of the programming model.

Both of these programming strategies have stood the test of time. They have been used successfully in gaining a much better understanding of alternative actions that are "risk" efficient (e.g. Freund, 1956; Lin, Dean and Moore, 1974; Brink and McCarl, 1978; and Mapp et al. 1979). They have been used to estimate risk preferences indirectly (e.g. Wiens, 1976; Brink and McCarl) and they have been employed in more aggregate models as well (e.g. Scandizzo et al., 1984). Nevertheless, given the increasing reluctance to assume normality or quadratic utility and the advantages of defining risk efficiency in terms of second-degree stochastic dominance, the role of quadratic programming and MOTAD models in empirical risk analysis has diminished in recent years and will probably continue to decline in the future.

Operationally, the reason is quite simple. These models have been developed to generate only that information about the probability distributions on the performance variable needed to apply the decision rules that underpinned the models. To the extent the empirical programming models can be developed to generate more information about the nature of the probability distributions on the performance variables, these analytical frameworks will remain valid even in the face of theoretical developments in decision theory such as stochastic efficiency.

Stochastic Dominance In Perspective

Without doubt the most pervasive development in the empirical risk literature over the past 10 to 15 years has been the increasing reliance on stochastic dominance as an efficiency criterion. Developed independently by several individuals during the 1960's, both first and second degree stochastic dominance or efficiency (FSD and SSD) have been used in numerous empirical studies to examine alternative production activities at the farm level, to identify risk preferences that are consistent with participation in agricultural programs (Kramer and Pope, 1981) and to devise methods for constructing interval measurements of a decision maker's absolute risk aversion (King and Robison, 1981).[3] The latter application draws heavily on Meyer's (1977a, b) work on stochastic dominance with respect to a function.

The intuitive appeal of stochastic efficiency derives in large measure from one's ability to separate alternatives into efficient and inefficient sets based only on the assumption that marginal utility for the performance measure be positive (e.g. for first degree stochastic efficiency) and that additionally the marginal utility be decreasing for second degree stochastic efficiency. To put it another way, second degree stochastic efficiency can be applied to risk situations regardless of the shape of the underlying probability functions. All that is required is the assumption of a concave utility function; E-V efficiency simply becomes a special case of second degree stochastic efficiency when probability distributions are normal.

By placing few restrictions on the utility functions and probability distributions, these efficiency criteria are potentially applicable to a broad range of problems. However, both first- and second degree stochastic efficiency in general must be stated in terms of the cumulative probability functions.[4] That is, as Anderson et al. (1977)

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[3]Absolute risk aversion is defined by

$$a(y) = -u''(y)/u'(y)$$

where u(y) is a utility function in some performance measure y, and u'(y) and u"(y) are first and second derivations.

[4]This is not quite true for some well known parametric distributions such as the normal, log normal and gamma, in which SSD can be evaluated by "plugging" into expressions that depend only on the parameters of the probability functions (Pope and Ziemer, 1984).

suggest a risky prospect F^1 is said to dominate F^2 by FSD if $F_1{}^1(y) \leq F_1{}^2(y)$ for all possible y in the range [a,b] with the strong inequality holding for at least one y and

$$(1) \quad F_1{}^i(y) = \int_a^y f^i(x)\, dx \qquad (i=1,2)$$

For F^1 to dominate F^2 by SSD, one requires that $F_2{}^1(y) \leq F_2{}^2(y)$ for all y and the strong inequality hold for at least one y and

$$(2) \quad F_2{}^i(y) = \int_a^y F_1{}^i(x)dx \qquad (i=1,2).$$

When compared to E-V or MOTAD efficiency, the application of stochastic efficiency without knowing the utility functions is extremely demanding in terms of the probability specification. In the general case, successive integrations are required if one assumes the probability functions are continuous. If one has some basis for assuming one of several well known parametric distributions (e.g., normal, log normal, gamma), the SSD ordering rules can be evaluated by a "plug in" method once the distribution's parameters have been estimated (see footnote 4). The remaining alternative that is followed in most cases is to estimate a discrete empirical distribution function and evaluate the analog to equation (2) for discrete probability distributions. This SSD decision rule is developed by defining $\Delta x_k = x_k - x_{k-1}$, where the x's are ranked in ascending order and x_n is the largest x. Then the analog of $F_2{}^i$ is given by

$$(3) \quad F_2{}^i(x_r) = \sum_{k=2}^{r} F_1{}^i(x_{k-1})\Delta x_k, \; r = 2,\ldots,n;$$

where,

$$F_2^{\,i}(x_1) = 0;$$

$$F_1^{\,i}(y) = P(x_k \leq y) = \sum_{x_k \leq y} f_*^{\,i}(x_k); \text{ and}$$

$f_*^{\,i}(x_i)$ = a probability mass function (Anderson et al., 1977).

This need for additional probability information, combined with the fact that SSD essentially requires pairwise comparisons without benefit of information on other alternatives, is undoubtedly among the most serious limitations of the procedure. Anderson et al. (1977) were among the first to allude to empirical problems that can result from the pairwise comparisons inherent in the SSD.

> If all prospects are 'pure prospects' the process [SSD] says little about the efficiency of possible mixtures of pure prospects. The only way of establishing the efficiency or otherwise of such mixtures is to specify the distributions pertaining to the mixtures and to test these as for other prospects (p. 294).

As a result of these needs for additional probability information and the reliance on pairwise comparisons of alternatives, many recent applications of SSD have been confined to specific production or investment decisions, abstracting somewhat from the overall farm-firm environment. This is a natural research strategy during the years following a major development in empirical analysis. Carried to its logical extreme, the search for better probability information could lead to an examination of a narrower and narrower set of production and investment alternatives. To fall into this trap would be a serious mistake at any time, but is especially so at a time with the agricultural sector under tremendous financial stress. Our ability to "fine-tune" the ordering of specific risky production and investment decisions will be of less value to our clientele if the empirical models fail to consider the alternatives within the context of the whole farm firm.

The practical difficulties encountered in applying second degree stochastic efficiency to complex decision problems suggests three avenues of potential research. The first relates to resolving the empirical questions surrounding the consensus requirement implied by the pairwise comparison strategy in SSD. A second issue relates to the identification of the relationship between the accuracy of the probability information available and one's ability to order decision alternatives correctly. The third relates to identifying ways of generating the probability information needed to evaluate complex decision alternatives.

At last year's technical committee meeting, Cochran et al. (1984) described a computer program for convex set stochastic dominance (CSD) to eliminate some problems of these pairwise comparisons that are made without benefit of information from other comparisons. The CSD eliminates the consensus requirement by comparing a given action with convex combinations of other actions. The authors claim that this procedure reduces type II errors without a corresponding increase in type I errors.[5] They are also quick to point out that CSD cannot address the diversification issue by creating "portfolios" or mixtures of pure prospects and thereby enlarging the set of alternatives. Although this procedure does have the potential for reducing the size of the SSD

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[5]A type I error is made when one ranks an alternative as inefficient when it is in fact efficient. A type II error is when an alternative is dominated by others but is included in the efficient set erroneously.

efficient set, the iterative search requirements could be cumbersome, particularly where numerous alternatives are involved.

Regardless of how the SSD comparisons are made, one must also be concerned with the ability of the probability information generated for this purpose to order alternatives correctly. Pope (1982) has been one of the few economists to be concerned with sampling errors in estimating means and variances in returns from historical data and the implications of these errors on the validity of the E-V efficiency frontier.

The same issue arises in SSD analysis, but is potentially more complex in that one must estimate the entire probability density, or the parameters of selected parametric distributions so that SSD can be performed using the "plug in" method described above. Meyer and Pope (1980) argue that unbiased estimates of the probability function and expected utility are desirable, but in some cases, biased estimates may lead to an appropriate ordering of alternatives, at least on average (Pope, 1982).

The recent paper by Pope and Ziemer (1984) is one of the few to address empirically, and in a comprehensive fashion, the issues surrounding sampling errors in efficiency analysis. These authors conduct a Monte Carlo experiment for three distributions, the normal, log normal and gamma, and a number of parameter values and sample sizes. They use this experiment to order distributions by the SSD and E-V criteria. The SSD results are generated in two different ways: a) by "plugging in" the maximum likelihood estimates of the parameters of these functions into an appropriate formula and b) by estimating the empirical distribution functions. They concluded that

> the empirical distribution function (a) performs about as well or better than ML methods, which presume knowledge of the underlying distributions, and (b) is substantially more robust than the usual EV rule, given nonnormal distributions. In addition, the empirical distribution function generally yields more correct rankings than the ML methods when sample size is small (p. 39).

On the basis of this one study alone, it would be premature to conclude that researchers should abandon efforts to identify and test for an underlying parametric structure for the probability distribution in risk analysis. There are other reasons for knowing as much as possible about these distributions apart from their use in stochastic efficiency orderings. The fact that empirical distributions performed well relative to other procedures is encouraging and allays the concerns expressed by Anderson et al. (1977) about reliance on discrete probability distributions. However, the analyst must still make some judgment about the additional accuracy gained through increasing the sample size with the additional costs of expanding the research effort.

Despite the encouraging results for the empirical distribution, the most disturbing conclusion is that the probability of correctly ranking distributions was relatively low regardless of sample size. Rarely did this probability exceed 0.7 and it was much lower even when the underlying distribution parameters differed by as much as 25%. As the underlying distributions become more similar, the probability of incorrect orderings is obviously exacerbated, but in these cases, the "cost" of a wrong decision may be reduced as well.

This observation seems to suggest an additional item for the research agenda. In most studies involving stochastic dominance, the empirical analysis is so oriented around the ordering of alternatives, little attention is given to the implications of a wrong decision, either in terms of some simple summary statistics or in terms of other characteristics of the alternatives that may have implications for long-term production and investment decisions of the decision maker. At some point, we need to devise methods for better understanding the costs of wrong decisions and be able to articulate in a more heuristic way to our clientele how to choose among alternative stochastically efficient choices.

Regardless of how one displays and interprets the results from stochastic efficiency analysis, the feasibility of the research itself depends critically on one's ability to estimate the necessary probability information. Anderson (1975) recognizes these difficulties, particularly as they relate to whole farm planning and the general inability of well structured programming models to generate the necessary probability information for SSD. At that time, he experimented with Monte Carlo programming for generating feasible farm plans.

> Briefly the method consists of selecting activities by pseudo-random (Monte Carlo) sampling and expanding the levels to the limits of the available resources. The feasible plans may then be subjected to some quality tests and those that pass stored" (p. 95).

These plans can then be ranked by SSD by constructing empirical cumulative distribution functions. The major disadvantage to this Monte Carlo method is that one cannot assume that all risk efficient plans will be identified, particularly those for which it is desirable to leave some resources unused.

To some extent, the growth in the use of simulation methodologies for SSD analysis is explained by this shortcoming in well-structured programming models. The flexibility inherent in simulation models also accommodates a more realistic representation of biological growth processes, stochastic variables and the dynamic aspects of investment and firm growth. However, due to the lack of any preexisting structure "... few simulation models in agricultural economics have been generalized and documented for modification and reuse" (Mapp and Helmers, 1984, p. 123). Thus, one can argue effectively for additional research into mathematical programming structures that can either accommodate a broader range of decision criteria or more general probability information about the performance measure.

One important attempt to accommodate a broader range of decision criteria into a programming framework is Tauer's (1983) Target MOTAD model. This LP model, which is designed to maximize expected return subject to a constraint on the probability weighted negative deviations from a specified target return, has been shown to generate solutions that are efficient according to second-degree stochastic dominance (SSD). The ranking is accomplished essentially by evaluating discrete cumulative probability functions.

As in the case of either the E-V or the original MOTAD model, Target MOTAD is a two-attribute risk model. However, in the former two cases, one attribute is in the objective function, while the other is in the right hand side of one constraint. For Target MOTAD, both attributes are in the constraints. Thus, as McCamley and Kliebenstein (1984) suggest, the procedure for determining all Target MOTAD solutions is more complex than generating the efficient set in the other two cases. The complete set of Target MOTAD solutions can still be described by a finite number of extreme points, but must be generated by a selective parameterization of target return and acceptable levels of probability weighted absolute deviations about the target.

The jury is still out on how valuable the Target MOTAD model will become in empirical risk analysis, but I suspect the number of individuals beginning to experiment with the model is increasing steadily. From a conceptual point of view, the most serious question that remains unanswered, has to do with the extent to which there are SSD efficient portfolios associated with the programming model that cannot be identified through Target MOTAD. In light of the current enthusiasm for SSD efficiency criteria, efforts to contribute to a better understanding of the formal properties of the Target MOTAD and test its usefulness in empirical analysis should be high on the research agenda.

This discussion of Target MOTAD was motivated in large part by a recognition that traditional risk programming models generate too little information about the probability distributions on performance measures to facilitate the application of SSD efficiency criteria. Target MOTAD results have a direct SSD interpretation, whereas, an alternative is to develop programming procedures for estimating the density and or distribution functions on the performance measure so that SSD or other decision criteria can be applied expost to the programming results. The next section of this paper summarizes the work by one of my students and me to contribute to the literature in this way. In so doing, the model also contributes to our understanding of a complex general class of stochastic programming problems which seems to have received little attention in the recent past.

New Opportunities for Stochastic Programming?

In general, stochastic linear programming concerns the behavior of linear programming optimization models when one or more of the coefficients is random. This obviously complicates the decision problem and also requires a careful distinction between the timing of the outcome and the realization of the random variables in the model.

Active and Passive Approaches

In this sense, one can think of active (here and now) models where decisions must be made before the uncertainty in the model is resolved. In contrast, the passive (wait and see) model assumes that the decision maker knows the values of the random components prior to the time the decision is made.

Most applications of stochastic programming in agricultural economics (including QP, MOTAD and Target MOTAD, as well as chance-constrained models) are active formulations. Their solutions provide initial recommendations for courses of action but offer little advice on how plans should be modified as the uncertainty is resolved over the course of the production period. This issue is particularly important where the resource vector and/or the technology matrix are assumed to have important random components. In these cases, modifications may be required as the production process evolves because initial plans might not be feasible. It is also clear that the decision criteria in the chance constrained or recourse programming models are somewhat ad hoc and may be unsupported by axioms of rational behavior.

At the other extreme, the main inquiry of passive models is into the behavior of the optimal solutions as the random variables range over all possible outcomes. For the problem,

$$(4) \quad z(u) = \max_{x} \; (c(u)x: \;\; A(u)x = b(u), \; x \geq 0)$$

where c(u), A(u) and b(u) are vectors of stochastic coefficients, a family of deterministic programs is generated as u ranges over its set of possible outcomes. In turn, the optimal value of the program z(u) becomes a random variable whose probability distribution function F_z can in theory be estimated. In practice, this is extremely difficult.

Viewed in this way, passive decision problems are not concerned with risk management, because at the time of optimization, there is no remaining uncertainty. The importance in solving the problem derives from the present value of obtaining information on the future distribution of costs and/or revenues. This could be particularly important at the firm level for budgetary purposes, for evaluating large scale investments or for planning or policy purposes at a more aggregate level.

Discrete Sequential Programming

There have been few, if any, applications of passive programming in the agricultural economics literature, although Tintner (1955) uses an agricultural example in his original paper on stochastic programming. Cocks (1968) and Rae (1971), on the other hand, combine elements of both the active and passive approaches in their discrete stochastic sequential programming model. In this model, the decision maker's knowledge of the outcomes from random events changes over time. Initial optimal production plans are provided, as are recommendations for modifying those plans as the uncertainty is unraveled through time.

For these reasons, this discrete sequential programming strategy has tremendous appeal intuitively. Additionally, one can use it to generate the empirical distribution function of the performance measure. However, the underlying programming model has a block diagonal structure with a subproblem embedded in the model for each node on the decision tree representing individually each value taken on by all random variables. The potential size of the problem is enormous; to date there have been only a handful of applications to problems in agriculture.

Although this sequential programming formulation suffers from a "curse of dimensionality", the notion of combining active and passive components of stochastic programming is appealing, particularly in evaluating major investments where production policy over the life of the investment can be changed in response to economic conditions. In an attempt to contribute to this literature, a student of mine, Richard Luckyn-Malone (1984), and I have attempted to solve a capital investment decision problem within the stochastic programming framework.[6]

A Capacity Expansion Problem

This problem is designed to facilitate the comparison of a set V of alternative investment projects by means of the present value of the firm's profits resulting from each alternative v. The stochastic version of the problem is given by

$$(5)\quad v^* = \arg \sup_{v \in V} \{g(F_p^{\,v}, F_y^{\,v})\}$$

where

$$(6)\quad F_p^{\,v}(\alpha) = P\{p(v) \leq \alpha\}$$

$$(7)\quad F_y^{\,v}(B) = P\{Y_t^{\,v} \leq B\}$$

$$(8)\quad p(v) = -c(v) + \sum_{t=1}^{T} a^T Y_t^{\,v} \;;\; a = 1/1{+}r$$

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[6]I am indebted to Richard for letting me summarize some of the major results of his thesis in this paper. Any errors in interpretation etc. are mine alone.

(9) $Y_t^v(u_t) = \max_x (cx: \quad Ax \leq b(u_t) + v, \; x \geq 0)$

(10) $u_t \sim F_{u,t}$

$u_t \varepsilon R^k.$

This formulation assumes that the yearly production alternatives once the investment decision is made are given by the linear program in (9). Each investment, v, affects the production capacity through the right hand sides of the constraints, which contain the only random components. If the objective function or technical coefficient matrix contained random elements (9) would look like (4).

The decision problem is to choose one project out of the set V based on the probability distribution functions of net present value, F_p^v, and yearly profit, F_y^v. The investment decision is an active problem and the solution to the nested passive problem gives the decision maker maximum information at the planning stage.

As Bereanu (1980) points out, the passive stochastic inner program (equation (9)) has a two-fold structure: one parametric and one probabilistic. The parametric structure partitions the problem into decision regions which correspond to the set of values on u for which a given basis is primal and dual feasible (e.g. optimal). By defining the ith basis of A as B_i, the decision regions U_i on the domain of u are defined in terms of the simplex criterion as

(11) $$U_i^v = \{u: \quad u\varepsilon R^k, \; B_i^{-1}(b(u_t)+v) \geq 0; \quad c_d - c_b B_i^{-1} D \leq 0\}$$

where D = columns of A not in B_i; c_b, c_d = components of c corresponding to vectors in B_i and D, respectively. We know that the set of U_i is finite because there exists at most $\binom{m+n}{n}$ bases. The actual number under consideration will be reduced significantly because for many bases, the decision regions are empty or values of u will be such that (9) is infeasible.

Although the notion of a decision region is an integral part of solving the stochastic program, these regions are of intrinsic interest for other purposes as well. They may indicate which production activities will never be optimal over a large range in u or those which will dominate.

Turning now to the probabilistic structure of the problem, the strategy is to find the distribution function on Y_t^v. The general and exact results are due to Bereanu (1967) and begin with two regularity conditions.[7] For decision regions i=1,...,q, we require

(12) $P(U_i \cap U_j) = 0; \quad i \neq j$

(13) $P(\bigcup_{i=1}^{q} U_i) = 1$

and define

(14) $Y_i^v(u) = \begin{cases} c_b B_i^{-1}(b(u) + v) & \text{if } u \in U_i^v \\ 0 & \text{if } u \notin U_i^v. \end{cases}$[8]

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[7]The necessary and sufficient conditions for this regularity are given by Gleit (1977). From a practical standpoint, equation (12), is the most difficult. Decision regions that intersect give rise to multiple optima and potentially lead to double counting the mass and moments of the distribution. The problem can be resolved by constructing an arbitrary sequence of disjoint sets out of the decision regions.

[8]By ignoring the t, we are essentially assuming that u is a stationary stochastic process.

From this, it can be shown that

(15) $P_i = P(U_i) = \int_{U_i^v} d\, F_u(u) \quad i = 1,\ldots,q$

(16) $P_{i\alpha} = P(U_{i\alpha}) = \int_{U_{i\alpha}^v} d\, F_u(u) \quad i = 1,\ldots,q$

and the distribution function (by assumptions (12) and (13))

(17) $F_y^{\ v} = P\{Y^v(u) \leq \alpha\} = \sum P(U_{i\alpha}) = \sum_{i=1}^{q} \{ \int_{U_{i\alpha}} d\, F_u(u)\}$

where

$U_{i\alpha} = \{u: \ u \varepsilon u_i, \ Y_i^{\ v}(u) \leq \alpha, \ \alpha \ \varepsilon \ R\}.$

The expected value and variance of Y^v become

(18) $E(Y^v(u)) = \sum_{i=1}^{q} \{ \int_{U_i} Y_i(u)\, d\, F_u(u)\};$

(19) $Var\ (Y^v(u)) = \sum_{i=1}^{q} \{ \int_{U_i} Y_i^{\ v2}(u)\, d\, F_u(u) - (E[Y^v(u)])^2\}.$

These equations constitute the most general form for the exact solution to the passive stochastic program in equation (9). The computational considerations remain unresolved for this general case, but numerous individuals have worked to resolve them for special cases.

Luckyn Malone (1984), for example, has solved this problem when u_t has an independent, stationary multivariate normal distribution. By pruning the t and v from the notation, we can simplify the algebra, keeping in mind that the solution procedure would be applied to all projects. The production model in (9) can be written more simply as

$$
\begin{aligned}
(20)\quad Y(u) &= \max\,(c\cdot x:\ \ Ax = b(u) + v,\ x \geq 0) \\
b(u) &= b^{o} + b_{1}u^{(1)} + \ldots + b^{k}\ u^{(k)} = b^{o} + \beta\ u \\
h(u) &= \beta u + (b^{o} + v) \\
u &\sim N_{k}(\mu_{u}, M_{u})
\end{aligned}
$$

where u has a non-singular distribution.[9]

The exact solution to the distribution problem in (20) is found by finding the a) distribution law of the resource vector; b) distribution law of basic solutions; c) probability on decision regions; and d) distribution law of the optimal values. These exact calculations are summarized in Appendix A. An important result is that

> ... the multivariate normal law of the random vector u carries through to the resource vector, the basic solutions, and the value of the objective function at each basic solution" (Luckyn Malone, 1984, p. 97).

Unfortunately, unless the number of random variables is less than three, it is not possible to evaluate these multivariate normal integrals for empirical applications that could surely lead to singular distributions and further complications in the calculations. The best that can be hoped for is an approximate solution that can be obtained algorithmically without difficulty.

Computational Considerations

Of the several approximation methods, perhaps the most direct computationally is a Monte Carlo method for evaluating F_y by estimating the multinomial probabilities in (18A) and (19A) for a progression of values of α. Another approach is to generate a large sample from the multinomial distribution and solve the associated linear problems.

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[9]Luckyn Malone also considers the implications of u being singular, but this is of little consequence here.

Luckyn Malone (1984) on the other hand, proposes an alternative method for approximating F_y which accounts for the double counting of the probability mass where the regularity conditions (12) and (13) are violated. The procedure involves two levels of approximation. In the first, a new discrete variable Y* is used to approximate Y by taking on values equal to the conditional expectation of Y on each decision region, $E[Y|U_i]$, with probability p_i*.

By defining I(0) as the set of all <u>decision regions</u> with positive probability p_i* and Ω be the union of U_i for $i \varepsilon I(0)$ then

$$(21)\quad E[Y|U_i] = \frac{1}{p_i*}\int_{U_i} Y(u)dF_u \quad \text{for } u_i \ \varepsilon\ \Omega.$$

From this, one can define a piecewise constant random variable which for each outcome of the random process u gives the conditional expectation of profit on U_i.

$$(22)\quad Y*(u) = \sum_{i\varepsilon I(0)} E[Y|U_i]\cdot 1_{\sim u_i}(u)$$

where $1_{\sim u_i}$ is the indicator function

$$(23)\quad 1_{\sim u_i}(u) = \begin{cases} 1 & \text{if } u\ \varepsilon\ U_i \\ 0 & \text{otherwise.} \end{cases}$$

Luckyn Malone goes on to show that this new random variable provides unbiased estimates of E[Y] and var [Y]. The cumulative distribution function is then given by

$$(24)\quad F_y*(a) = P\{Y* \leq a\} = \begin{cases} 0 \quad \text{for} \quad a < E[Y|U_1] \\ p_i* \text{ for } E[Y|U_1] \leq a < E[Y|U_2] \\ p_1*+p_2* \text{ for } E[Y|U_2] \leq a < E[Y|U_3] \\ - - - - - - - - - - - - - - - - - - \\ \sum_{i=1}^{q-1} p_i* \text{ for } E[Y|U_{q-1}] \leq a < E[Y|U_q] \\ 1 \text{ for } E[Y|U_q] \leq a. \end{cases}$$

The second level approximation is in estimating Y*, which in the process requires the construction of I(0)[10] and estimates of p_i* and $E[Y|U_i]$. Since the direct estimation of p_i* is not possible for the case of more than three random variables, upper and lower bound probabilities are determined using Bonferroni-type inequalities (Tong, 1980). These inequalities rely on pairwise interactions of the joint probabilities that individual components of the bases corresponding to the i and j decision regions are less than zero (Luckyn Malone, p. 110-18). For each decision region, these estimates reduce to the computation of m probabilities from marginal univariate distributions and m(m-1)/2 probabilities from bivariate distributions (m being the number of constraints). The conditional expectation of Y over U_i can be determined directly by the expected value of x_i under the truncated multinormal distribution. To avoid evaluating multinormal distributions directly, this conditional expectation is facilitated by approximating the jointly normal x_i by m independent normal variates.

Others have also developed strategies for resolving the distribution problem in stochastic linear programming. Ewbank, et al. (1974) developed a closed form expression for the CDF (equation (17)) in the case where "c" or "b" is random by using a Jacobian transformation to simplify the regions of integration. This recognizes that the limits of the integrals are derived from a set of simultaneous linear equations representing the space contained by m or n hyperplanes (p. 227).

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[10]For unimodel distributions, it may be expected that the set of decision regions may cluster around the one associated with the optimal basis where u takes on its mean or modal value. It has been suggested by Dempster and Papagaki-Papoulias (1980) that one should start with the mean value problem, chaining in a rational way through adjacent bases.

Foote (1980) has adapted this idea into a simplex based algorithm for generating the bases which determine the decision regions for when the c or b vectors are random. Following a strategy similar to that by Dempster and Papagaki-Papoulias (1980), his procedure moves to an adjacent decision region, by selecting the incoming activity leading to the extreme point with the highest probability mass associated with it.

The most extensive computational methods have been developed by Bereanu (1980). His methods are based on numerical methods and computer routines and derive inputs from any LP code which includes parameterization procedures. It appears that he has used PARAOBJ and PARARHS from IBM's MPS or MPSX codes. Much of his computational experience has been with what he calls simple randomization in which there are numerous random coefficients in the model, all functions of a single random variable. Under these conditions, the program can handle random variables with an arbitrary normal, exponential or uniform distribution, along with a distribution given by a histogram with up to 100 values.

In extending his computational methods to include additional random variables, Bereanu argues that

> ... it seems unrealistic to assume the knowledge of the joint d.f. of the thousands of coefficients of a linear program corresponding to a real life problem, and even less to use such a d.f. in computations, it seems appropriate to devise approximate methods ... depending on a 'small' number of random variables (critical random vectors, but otherwise having the dimensions met in practice) (p. 184).

According to this strategy, the components of his general stochastic linear programming problem are articulated by

$$A(e) = A_0 + e_1A_1 + e_2A_2 + \dots + e_rA_r$$

(25) $$b(e) = b_0 + e_1b_1 + c_2b_2 + \dots + e_rb_r$$

$$c(e) = c_0 + e_1c_1 + e_2c_2 + \dots + e_rc_r;$$

where A_i, b_i, c_i are known matrices and $(e_1,\ldots,e_r)$ is a random vector with known distribution function. Thus, the major limitations imposed on the calculations depend on r and not on the number of random coefficients. Empirically, the challenge is in specifying a set of known matrices that will accommodate the major random components of the model using the fewest number of underlying random variables.

Bereanu's computational experience is on the IBM 370/168 computer and, to this author at least, was surprising. In the simple random experiments, problems of dimension up to 226 x 508 (including slacks) and with 185 random coefficients, ran in under one second. For the complex cases involving more than one random variable, the computational time increased substantially, but was still only 13 seconds for a problem with dimensions 102 x 181, with 48 random coefficients and two random variables. For a problem of the same size as discussed above for the simple random experiment but with 4 random variables, run time increased to 12 minutes. It appears that this could be reduced if the capacities of MPSX to revise coefficients, and store optimal bases for restart, the time could have been reduced substantially.

More About Applications

Although the discussion in the previous section demonstrates that substantial progress has been made recently for approximating the solution in the distribution problem in stochastic linear programming, the methods developed by Luckyn Malone have not been computerized, nor have we had any first hand experience with other algorithms. Only through some extensive experimentation can the accuracy and computational feasibility of these alternative approximation methods be assessed.

At this time, a more important question surrounds the applicability of these methods for empirical work. Most applications to date of the capacity expansion or similar problems have been related to business decisions, where, for example, investments had to be evaluated but the passive inner problem dealt with uncertain prices or demands which become known prior to annual production decisions. The other important application has been for central government planning. However, much of my interest in the topic stems from a logical extension of this kind of application for planning capacity expansion in a regulated industry such as the electric utilities. The model also seems quite attractive for evaluating large scale public investment projects. There are certainly public investments with significant agricultural components, such as irrigation or drainage, but at this time, they may be most relevant in a development context.

The applicability of the methods for micro decisions in agriculture is less clear than for business decisions and planning in a regulated industry. In agriculture, it is more likely that, in addition to price variability, resource supplies and/or some technical coefficients constitute the important random components of the problem and that the inner passive problem of the formulation above only approximately reflects the knowledge situation of the decision maker during any production period. In a sense it is somewhere on the continuum between active formulations on the one hand and discrete sequential formulations on the other.[11] In the case where a small number of elements in the "A"

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[11]If one assumes that the random variables are discrete, the capacity expansion problem above is equivalent to the discrete stochastic formulation by Cocks (1968) when a singular stage of production is assumed in estimating the value of information in investment planning.

matrix or "b" matrix are random, it may represent one possible way of generating more information about the consequences of investment decisions than is forthcoming through chance constrained programming and other active decision models. As mentioned earlier, the solution to the passive inner problem contains important information about production activities that are optimal with high probability as well as those that are not likely to obtain under any circumstances. This is a first step to understanding how production plans might be altered from year to year in response to changing values of the model's random components. Finally, since the solution to the inner problem assumes that the uncertainty is resolved prior to the production decisions, a comparison of its solution to the probability distribution implied by a solution to the active programming counterpart, may be important.

Conclusions

When I agreed to prepare this paper, I was given a working title of "Strengths and Weaknesses of Alternative Models in Empirical Research: Fitting the Tool to the Job." Within this context, my first strategy was to attempt a general evaluation of many different approaches to risk, including those employed at the micro and macro levels. In addition to being put off by the enormity of the task, I soon concluded that such an exercise would result in just one more interpretation of what is generally known by economists working in risk.

Therefore, in preparing the paper, I took a very different approach. In the spirit of the working title given to me earlier, I have tried to state in generic terms what I think is now the biggest "job" in risk analysis. Given our preoccupation with stochastic efficiency, we are continually faced with the problem of generating detailed probability information about risky alternatives. Research by others has given us an indication of the acceptability of certain types of probability information for ordering alternatives. I have tried to speak to the importance of generating this information for risk alternatives that constitute a significant part of the whole farm operation. Advances in our methods for evaluating risky alternatives will mean little if they are used only to evaluate a narrower set of production and investment alternatives.

The remainder of the paper has tried to assess the role of mathematical stochastic programming in generating the probability information needed for stochastic efficiency. Several important contributions to the set of "active" programming models are mentioned, as are some recent developments in solving investment problems that combine active and passive programming analysis. Although recent advances in solving these models have been made, our ability to "fit" these latter models to the "job" at hand is less clear.

Appendix A

Four Stages of the Distribution Problem for Equation (20)

Distribution Law of the Resource Vectors

(1A) $h(u) \sim N_m(\mu_h, M_h)$

(2A) $\mu_h = E[h(u)] = \beta\ \mu_u + (b^o + v)$

(3A) $M_h = Var[h(u)] = \beta\ M_u\ \beta'$

Distribution Law of Basic Solutions

For any given u*, a basic solution to (20) is defined for non-singular basis B_i of rank m (i=1,...,q) as

(4A) $x(u^*, i) = (x_i(u^*), 0);\ x_i(u^*) = B_i^{-1} h(u^*) \quad i=1,\ldots,q$

(5A) $x_i \sim N_m(B_i^{-1}(\beta\ \mu_u + b^o + v),\ B^{-1}\beta\ M_u\ \beta'\ B_i^{-1'}) \quad i=1,\ldots,q$

Probability on Decision Regions

Let p_i be the probability that B_i is primal feasible and p_i^* be the probability that B_i is both primal and dual feasible. Then for i=1,...,s

(6A) $$p_i = (2\Pi)^{-1/2)m} \left|M_i\right|^{-1/2} \int_0^\infty \cdots \int_0^\infty \exp\{-1/2(\xi-\mu_i)' \cdot M_i^{-1}(\xi-\mu_i)\} \cdot \prod_{j=1}^{m} d\xi_j$$

if the distribution law on x_i is non-singular. If it is singular, then p_i must be expressed in terms of the pdf on u by noting that

(7A) $x_i(u) = B_i^{-1}\ \beta u + B_i^{-1}(b^o + v)$

hence

(8A) $Pr\{x_i \geq 0\} = Pr\{u:\ B_i^{-1}(\beta u + b^o + v) \geq 0\}$

and

(9A) $p_i = (2\Pi)^{-(1/2)k} \left|M_u\right|^{-1/2} \int_{S_i} \cdots \int \exp\{-1/2(u-\mu_u)'$

$\cdot M_u^{-1}(u-\mu_u)\} \prod_{j=1}^{k} du_j$

where $S_i = \{u: \; B_i^{-1}(\beta u + b^o + v) \geq 0\}$.

Now turning to the decision region, $c_d - c_b B_i^{-1} D$ does not depend on u and B_i is either dual feasible <u>or</u> not dual feasible for all $u \varepsilon R^k$. Denoting

(10A) $\underset{\sim}{1}_i = \begin{cases} 1 \text{ if } c_d - c_b B_i^{-1} D \leq 0 \\ 0 \text{ otherwise} \end{cases}$

it follows that

(11A) $p_i^* = \underset{\sim}{1}_i \, p_i \qquad i=1,\ldots,s$

This holds for all bases in A and $p_i^* > 0$ only if B_i is optimal on some set of $u \varepsilon R^k$.

<u>Distribution Law of the Optimal Value</u>

For a particular outcome of the random process, u* say, the value of the objective function at each basic solution to program (20), denoted by $z_i(u^*)$, is given by:

(12A) $z_i(u^*) = c_b \cdot x_i(u^*) = c_b B_i^{-1} h(u^*) \qquad i=1,\ldots,q$

Since $c_b x_i$ defines a linear sum of normal variates,

(13A) $E[z_i] = c_b \mu_i \qquad i=1,\ldots,q$

(14A) $\text{var}[z_i] = c_b \, \text{var}[x_i] c_b' \qquad i = 1,\ldots,q$

$= c_b B_i^{-1} \beta \, M_u \quad \beta B_i^{-1}{}' c_b' \qquad \text{(using 5A)}$

Recalling the results of equations (14) - (17), the probability that Y = Z is less than or equal to α for a given decision region i is

(15A) $p_{i\alpha} = P(U_{i\alpha}) = \int_{U_{i\alpha}} dF_u(u) = \Pr\{x_i: \; x_i \geq 0, \; c_b \cdot x_i \leq \alpha\} \cdot \underset{\sim}{1}_i.$

Then the distribution function of profit is defined for all $\alpha \varepsilon R$ as

(16A) $F_y(\alpha) = P(\bigcup_{i=1}^{q} U_{i\alpha}) = \sum_{i=1}^{q} p_{i\alpha}$.

From (15A), $p_{i\alpha}$ is the probability of the orthant in x_i-space bounded by the linear hyperplane $c_b \cdot x_i = \alpha$.

We may write

(17A) $p_{i\alpha} = \underset{\sim}{1}_i \int_{V_{i\alpha}} \cdots \int \Phi_i(dx) \quad i=1,\ldots,q; \; -\infty < \alpha < \infty$

where $V_{i\alpha} = \{x_i : \; x_i \geq 0, \; c_b x_i \leq \alpha\}$, a convex polygon, and Φ_i is the distribution function of x_i from (5A).

Using results by Ewbank et al. (1974) to construct the limits of integration, and if indeed x_i has a non-singular distribution, then

(18A) $$F_y(\alpha) = \sum_{i=1}^{q} p_{i\alpha}$$

$$= (2\Pi)^{-(1/2)m} \sum_{i=1}^{q} \{|M_i|^{-1/2} \cdot \underset{\sim}{1}_i \cdot \int_0^{\infty} \cdots \int_0^{g_p^{(i)}} \cdots \int_0^{g_1^{(i)}}$$

$$\cdot \exp\{-1/2(\xi-\mu_i)'M_i^{-1} \cdot (\xi-\mu_i)\} \prod_{j=1}^{m} d\xi_j$$

where

$$g_j^{(i)} = \frac{1}{c_b^{(j)}} [\alpha - \sum_{k=j+1}^{m} c_b^{(k)} x_i^{(k)}] \quad j=1,\ldots,p$$

In the case of a singular distribution on x_i (e.g. when the number of constraints exceeds the number of random variables)

(19A) $$F_y(\alpha) = (2\Pi)^{-(1/2)m} \sum_{i=1}^{q} \{|M_i|^{-1/2} \cdot \underset{\sim}{1}_i$$

$$\int_{U_{i\alpha}} \cdots \int \exp\{-1/2(u-\mu_u)'M_u^{-1}(u-\mu_u)\} \prod_{j=1}^{k} du_j .$$

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