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GEN3--A SIMULATION MODEL FOR
RESOURCE POLICY EVALUATION

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GEN3--A SIMULATION MODEL FOR RESOURCE POLICY EVALUATION

GEN3 is a third generation computer simulation model designed for natural resource policy analysis. A brief history of the earlier versions may be useful for readers who have followed the model development. The original version was a simple discounted cash flow program (97 source statements) which was used for Outer Continental Shelf (OCS) and oil shale leasing policy analysis (Kalter, Stevens, and Bloom; Kalter, Tyner, and Stevens; Stevens and Kalter). The next model version included new policy options and added Monte Carlo simulation for price and cost variables. This version was first used for an analysis of policy options which could be used to stimulate oil shale production (Kalter and Tyner, Shale Oil, 1975) and for OCS leasing policy (Kalter and Tyner, Contingency Leasing Options, 1976). Price guarantee, purchase guarantee, and investment subsidy options were included. This program contained about 600 source statements.

Subsequently, the model was developed into the GEN1 version of the generalized resource policy evaluation model. This version performed Monte Carlo simulation with risked variables for the chance of no resource, reserves, prices, investment cost, and operating cost. Exploration and development were separated and development was not undertaken if the discovered reserves were uneconomic or no resources were found. Advance royalty options (Kalter and Tyner, Advanced Royalty, 1975) were included and a number of new policy options were added. Capital recovery profit share systems and variable rate structures for both royalty and profit share were added. The GEN1 version was about 1300 Fortran source statements. It was used for an analysis of OCS oil leasing policy (Kalter, *et al*., 1975) and for other analyses.

In 1976-77, the GEN2 version was developed. New policy options were added and the entire model was reprogrammed using structured programming to facilitate understanding of the model and to increase efficiency. All calculations in this version were performed on an annual basis, and annual streams for output variables could be obtained. The GEN2 version contained about 2300 source statements.

Between 1977 and 1981, GEN2 was used extensively for leasing policy analysis by both the government and private sectors. It has been used by at least three federal government offices in the Departments of Energy and the Interior, three states, Congressional offices, consulting firms, and several major oil companies. During these five years, the authors have received numerous questions and comments on the model. With the creation of GEN3, we hope to accomplish two major objectives:

(1) To incorporate several new features that were deemed important by users over the past few years. These include 1) the ability to run the model in real or nominal terms and to explicitly account for inflation, 2) the ability to handle different debt/equity ratios, 3) modifying the depreciation and depletion subroutines so that they work correctly when the model is run in real terms, 4) inclusion of new input and output options, and 5) allowing the investment and production periods to overlap.

(2) To expand and clarify program documentation so that the meaning of model inputs and outputs will be clearer to users, and the model will be more accessible to new users.

This paper is the documentation for GEN3. It is designed to provide readers with an in-depth understanding of how the model works. It is not written with reference to a specific resource; rather, a generalized model description is retained which can be applicable to any resource or similar investment situation. Regardless of the investment being considered, private sector response to public resource policies will normally follow a similar logic. Assuming competitive markets and a profit maximization objective function for the private sector, discounted cash flow techniques (appropriately constrained for public rules and market rigidities) can be used to simulate these responses. Both the theoretical and mechanical aspects are covered in great detail, in order that the reader will understand not only the rationale behind the relationships modeled but also will comprehend the means used to translate the theoretical structure into actual equations and solution procedures.[1]

Model Logic and Procedures

This model incorporates a number of factors important for public policy decisions into a framework of private market behavior. Economic, geological and engineering considerations relevant to private producer decision making are included so that the model may be useful for quantitatively testing the effects of numerous public policy alternatives. To this end, a wide range of leasing policy alternatives are incorporated to enable analysis of alternative leasing strategies for publically owned resources.

To determine the after tax net present value of the leasehold, the model utilizes exogenously supplied reserve distributions on an individual or group of leaseholds, along with

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[1]The computer code for GEN3 is available from the authors.

distributions for the associated production costs (investment and operating) and market prices.[2] In so doing, the model determines the productive capacity to be installed on the leasehold and the length of time that capacity is used. Uncertainty with respect to the key variables supplied exogenously (reserves, production costs and market prices) is incorporated via use of Monte Carlo simulation. Net present value calculations are carried out using discounted cash flow techniques with exogenously supplied rates of return on debt and equity capital as discount rates.

Installed Capacity Options

By using a control variable (NQØ), the model can be set to solve for optimal installed capacity (plant size) either endogenously (all possible values for installed capacity are evaluated) or exogenously (only prespecified capacities are permitted in the model solution). This distinction, in large part, leads to the different model algorithms. In the former situation, equations are specified which solve for and optimize installed capacity simultaneously with other model outputs. In the latter, the discrete installed capacities are entered into the model and the optimal capacity among these is determined. One advantage of the latter approach is that it permits economies of scale with respect to installed capacity to be considered in model solutions since unique cost relationships can be entered with each capacity examined.[3]

Figures 1 and 2 are flow diagrams for the two alternative solution algorithms contained in the GEN3 program. Both approaches have been programmed for model execution. The model description will follow these two flow diagrams and separately describe the solution algorithm with endogenous and exogenous installed capacity (q_o). It may be helpful for the reader to refer back and forth between these two flow diagrams and the text. To make the description easier to follow, a list of all model input variables used in this description is provided in Table 1. All symbols in the text and future references to variable names will refer to the variable definitions in Table 1.

Time Indices

The model relationships consist of both discrete and continuous time calculations. Both integral and summation forms are used in

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[2]If the model is being used for analysis of flow resources such as a biomass alcohol plant, reserve calculations may be by-passed by fixing reserves at some arbitrarily high level, and uncertainty in prices and costs handled in the usual way.

[3]Economies of scale with respect to reserve size can be considered under both approaches.

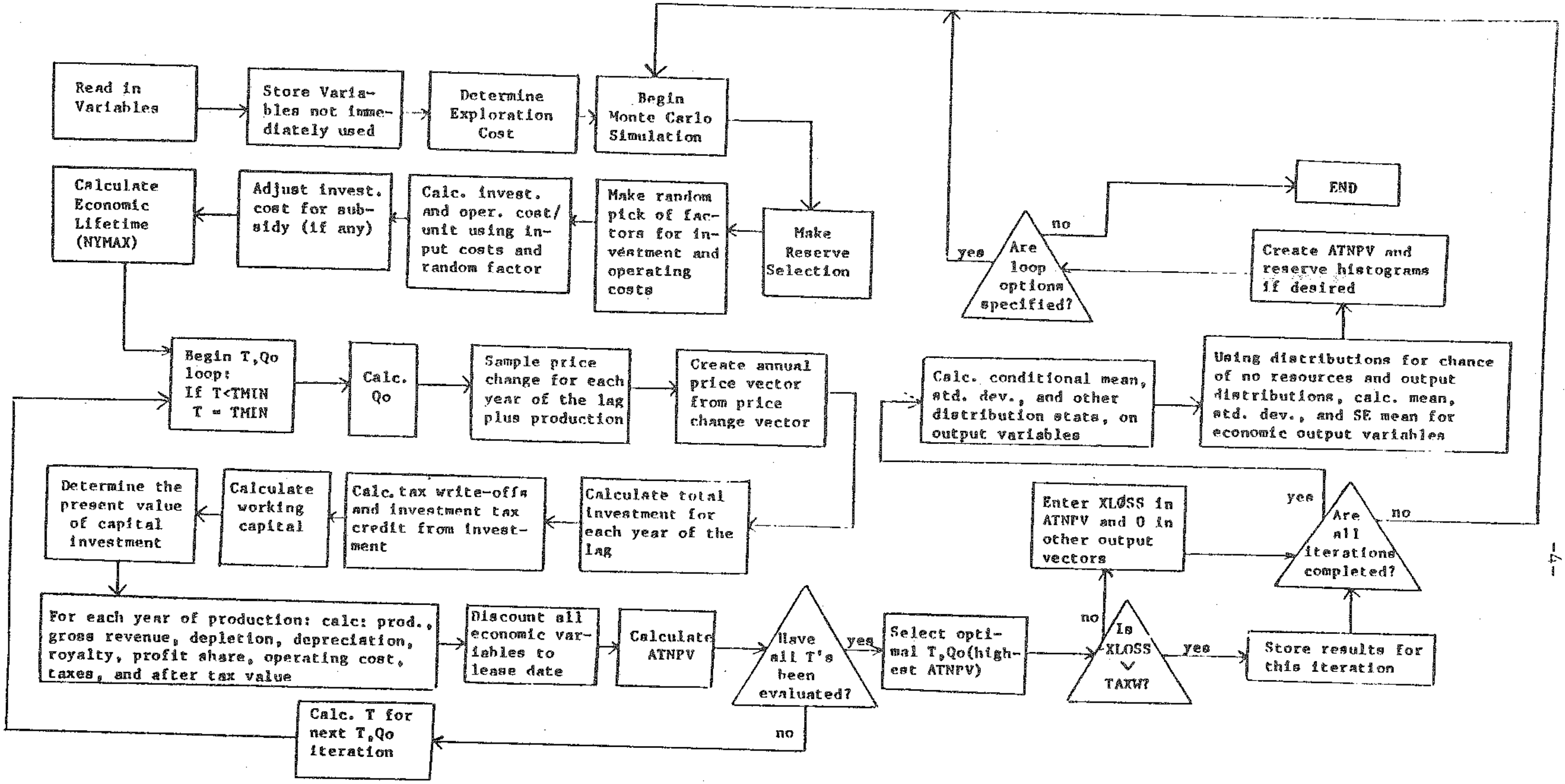

FIGURE 1.--FLOW DIAGRAM FOR SIMULATION MODEL WITH ENDOGENOUS Q_o

Qo = installed annual capacity
ATNPV = after-tax net present value
TAXW = tax write-off available if lease is not developed after exploration
XLØSS = loss incurred from exploration if lease is not developed (exclusive of bonus)
TMIN = minimum production time period

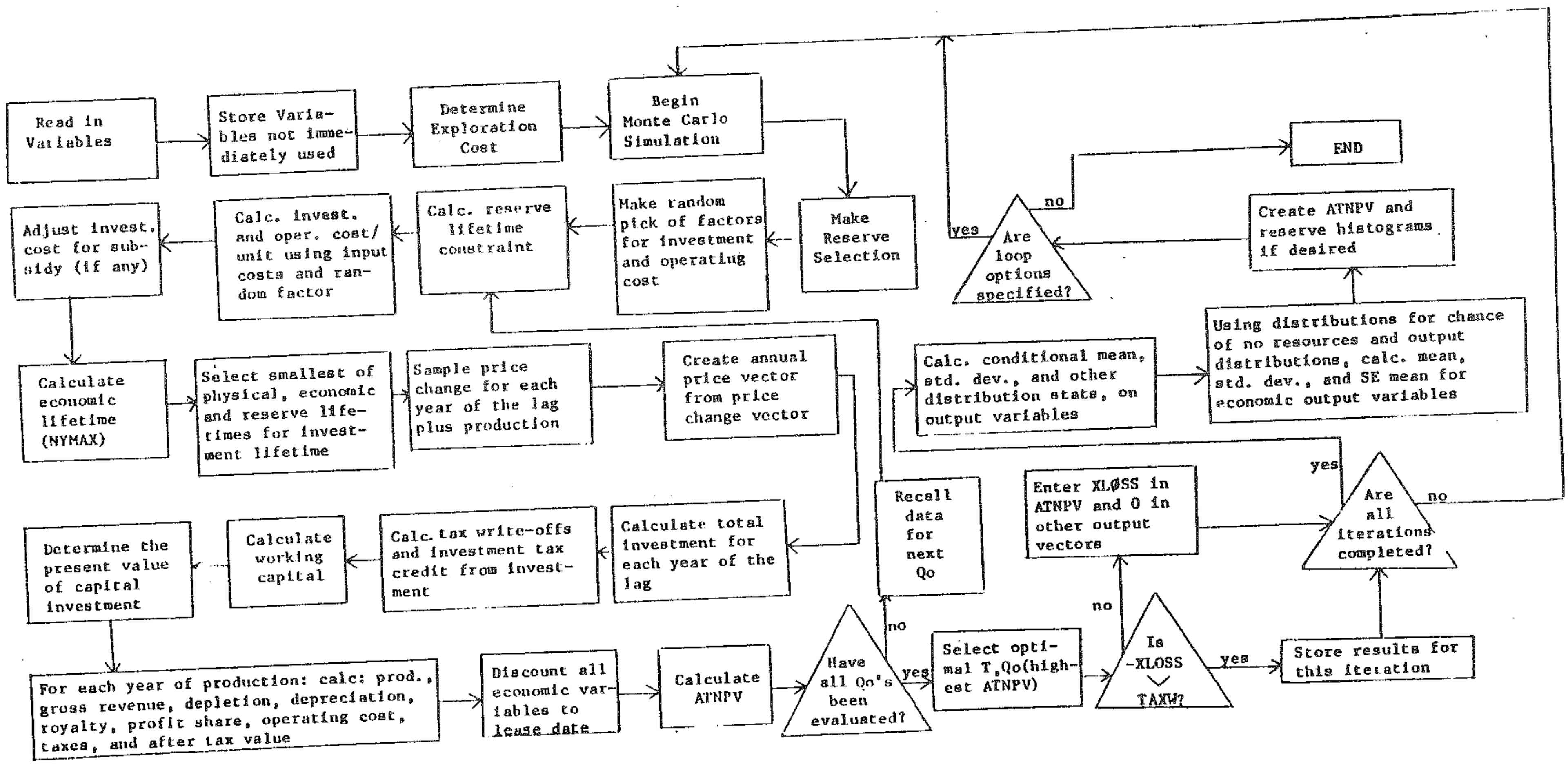

FIGURE 2.—FLOW DIAGRAM FOR SIMULATION MODEL WITH INPUT Qo

Qo = installed annual capacity
ATNPV = after-tax net present value
TAXW = tax write-off available if lease is not developed after exploration
XLØSS = loss incurred from exploration if lease is not developed (exclusive of bonus)

TABLE 1

IMPORTANT INPUT VARIABLES FOR GEN3

Symbol	Definition
Price related:	
P_o	Initial price for the primary resource
GP_o	Initial price for the secondary resource
P_1	Mean of the normal distribution for annual price change (primary resource)
GP_1	Mean of the normal distribution for annual price change (secondary resource)
Geologic:	
R	Mean of reserve distribution
β	Geologic parameter
AGFAC	Factor for calculating the amount of associated gas or other resource
F	Length of the initial flat production period (years)
a	Production decline rate (%)
Economic and tax related:	
λ	Royalty rate (%)
s	Severance tax rate (%)
α	Investment salvageable (%)
η	Investment tax credit rate (%)
z	Depletion rate (revenue) (%)
ϕ	Federal tax rate (%)

TABLE 1
(continued)

Symbol	Definition
N	Depreciation period (years)
r	Discount rate--weighted after tax cost of capital* (%)
B_o	Constant used to calculate BØNUS
B_1	Factor used to calculate BØNUS
T_p	Maximum physical lifetime (years)
Production and cost related:	
q_o	Installed capacity (annual)
C	Cost per unit of installed capacity
K_o	Operating cost per unit of output, installed capacity, or the average of the two
θ	Annual change in operating cost (%)
L	Length of the development period (years)
f	Proportion of investment expended in each year
y	Proportion of yearly investment which is tangible
RENT	Annual rent per acre
h_j	Factor used to determine production during build-up period (IBP in program, B in Figure 3)

*This variable is calculated in the program using the after tax return on equity, the fraction of equity capital, the debt interest rate, and federal and state tax rates.

the calculations as appropriate to represent these two forms. Hence, it is necessary to define time variables in both discrete and continuous terms. Also, for some purposes, the investment life is divided into a development and a production period with separate time indices. For discounting purposes, only one time index is used which originates with the beginning of the investment or lease.

The time variable definitions are provided in Table 2, and the time scales are illustrated in Figure 3. The variable tt is used as a discrete time index for variables such as annual production, qq(tt), and yearly initial prices, $P_0(tt)$. The variables t and ν are time rate variables used in calculating changes in production and prices and in computing the discount factor.

TABLE 2

TIME VARIABLE DEFINITIONS

Variable	Definition	Range
t	Continuous time index for the production period	0 to T
tt	Discrete time index for the production period	1 to TT
ν	Continuous time index for lease life	0 to τ
L	Length of development period	
T	End of production*	
TT	Last year of production*	
τ	End of the lease (T+L)**	

* Numerically, T always equals TT because production time in years is set equal to an integer.
** When development and production overlap, τ is not equal to T+L.

<u>Discounting</u>

GEN3 uses a weighted average after-tax cost of capital as the discount rate for model calculations. Table 3 provides the

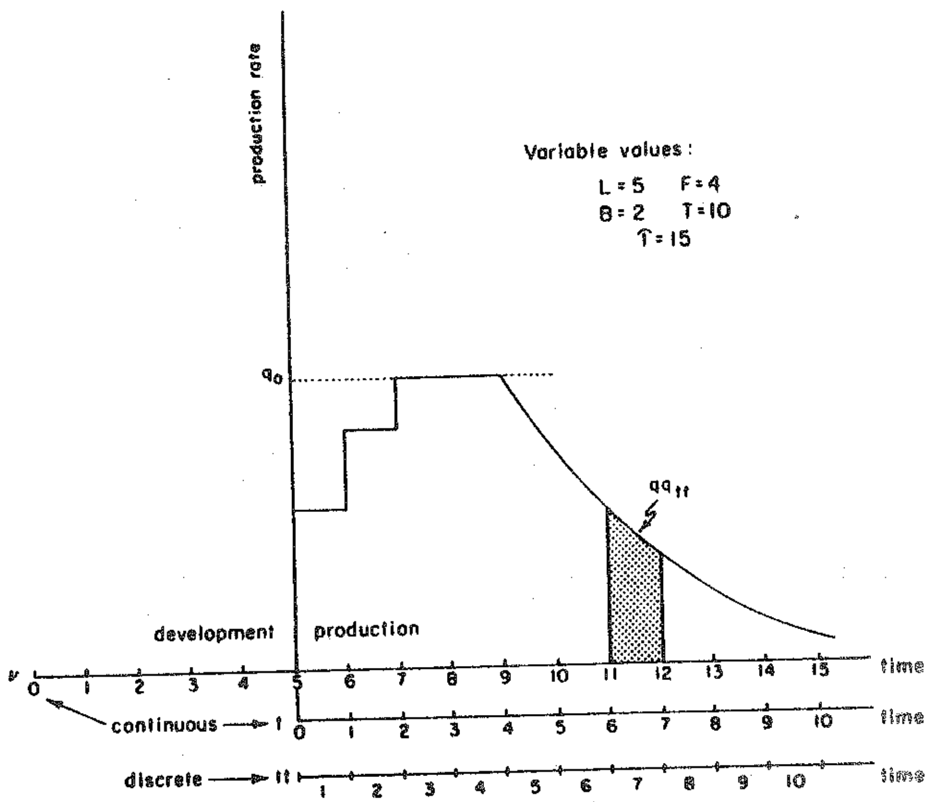

Figure 3. Illustration of Time Scales Used in the Model

TABLE 3

INTEREST RELATED VARIABLES

Variable Symbols				Computer Code Name	Definition
i				RINFL	Inflation Rate
$\emptyset$				PHI	Marginal federal tax rate
t				STAXR	Marginal state tax rate
u				EQ	Proportion of investment capital from equity (equity fraction expected over the life of the investment)
Pre-tax Real	Pre-tax Nominal	After-tax Real	After-tax Nominal		
$\boxed{d'^*}$	$\boxed{d'}$	d^*	d	RINT	Debt interest rate
e'^*	e'	$\boxed{e^*}$	$\boxed{e}$	REQ	Required rate of return on equity
r'^*	r'	$\boxed{r^*}$	$\boxed{r}$	R	Cost of capital

NOTE: Either real or nominal interest rates may be used in GEN3 depending on how the control variables for inputs (KINPT) and outputs (KOUTP) are set. However, the debt interest rate (RINT) must be input as a pre-tax rate and the equity return (REQ) as an after tax rate. Model discounting is done using an after tax rate (R).

The forms used in GEN3 are indicated above by □.

symbols, definitions, and computer code name for all interest and inflation related variables. The formula for calculating the nominal after-tax cost of capital is provided in equation (1):

$$r = eu + (1 - u)(1 - \phi)d' \quad (1)$$

When state taxes are relevant, equation (2), the actual equation in GEN3, is used:

$$r = eu + (1 - u)\ (1 - \phi - \phi t - t)d' \quad (2)$$

The user specifies the after-tax required rate of return on equity (e) and the pre-tax cost of debt (d') as effective annual rates (see pages 16-17 on specification of interest and growth rates for more detail). However, the model uses continuous discounting. Therefore, the discount rate used in the model is given by equation (3):

$$R = \ln(1 + r) \quad (3)$$

All of the variables on the right hand side of equation (2) are user specified inputs which are used to calculate the (real or nominal) discount rate. The user may input either all real or all nominal values.

Inflation

Inflation potentially affects four categories of variables: the cost of capital, annual costs and returns, the tax benefits from depreciation and cost depletion, and the terminal value of assets. Through these variables, the level of inflation can affect the net present value of an investment.[4] The model has been programmed such that inflation is properly incorporated whether the user chooses to run the model in real or nominal terms. However, the user needs to understand the effects of inflation to enter data consistently, particularly if the rate of inflation is being varied for sensitivity tests. The following sections explore the effects of inflation in this context.

The Cost of Capital

The weighted average after-tax nominal cost of capital (r) (ignoring state taxes) is given by equation (4):

$$r = e'(1 - \phi)u + d'(1 - \phi)(1 - u) \quad (4)$$

---

[4] Present values in the section on inflation are shown using discrete discounting. In actuality the model uses continuous discounting. The general effects of inflation are identical when derived from a continuous time model.

which is identical to equation (1) with $e'(1-\phi)$ substituted for e.[5]

Since e' and d' are both nominal (vs. real), the user should consider the level of inflation to be assumed. When a user varies the level of inflation, one of two assumptions relating interest rates and inflation are usually made. One, the real pre-tax values can be held constant, or two, the real after-tax values can be held constant.

Under the assumption of constant real pre-tax rates, the appropriate equations to calculate the nominal pre-tax rates at a given rate of inflation are:

$$(1 + e') = (1 + e'^*)(1 + i), \text{ therefore} \quad (5)$$
$$e' = e'^* + i + ie'^*, \text{ and} \quad (6)$$
$$(1 + d') = (1 + d'^*)(1 + i), \text{ therefore} \quad (7)$$
$$d' = d'^* + i + id'^* \quad (8)$$

Use of these equations assumes that capital markets for debt and equity adjust for inflation but not for the taxes on the inflation portion of interest or return to equity. Evidence suggests that this is a reasonable assumption (Tanzi, 1981, 1982). Assuming one uses these equations to determine the value to input for the required return on debt and equity capital, the effective annual discount rate (r) can be expressed as a constant real pre-tax rate (r*) adjusted for taxes and inflation:[6]

$$r = (r'^* + i + ir'^*)(1 - \phi) \quad (9)$$

From this relationship, it can be seen that the real after-tax discount rate (r*) decreases when the assumed rate of inflation increases. This can be deduced from the following equations which first define r*, then examine the change in r* as the rate of inflation increases:

---

[5] Remember that the after-tax cost of equity [$e = e'(1 - \phi)$] is entered into the data set rather than the pre-tax rate used above. The reader should note that the rates entered are effective annual rates (see page 16). The discussion in this section implies that the user has chosen to enter data in nominal form. The results with respect to the effect of inflation are equally applicable to the situation in which the user selects real inputs.

[6] This statement holds regardless of the proportions of debt and equity and does not require that e and d be equal.

$$1 + r^* = \frac{1 + r}{1 + i} \tag{10}$$

$$1 + r^* = \frac{[(1 + r'^*)(1 + i) - 1](1 - \phi) + 1}{(1 + i)} \tag{11}$$

$$1 + r^* = (1 + r'^*)(1 - \phi) - \frac{(1 - \phi)}{(1 + i)} + \frac{1}{1 + i} \tag{12}$$

$$r^* = (1 + r'^*)(1 - \phi) + \phi(1 + i)^{-1} \tag{13}$$

$$\frac{\partial r^*}{\partial i} = -\phi(1 + i)^{-2} < 0 \tag{14}$$

Thus, the effect of holding constant the real pre-tax rates for debt and equity as the inflation rate is increased is to reduce the real after-tax discount rate. In turn, this would increase the present value of cash flows that remain constant in real value as the inflation rate is increased.

The second method of maintaining a logical relationship between the costs of equity and debt and inflation is expressed in equations (15) and (16). When using this second pair of equations, the real after-tax costs of debt (d^*) and equity (e^*) are held constant, then the input data needed for the model (e and d') computed for a given level of inflation.

$$e = e^* + i + ie^* \tag{15}$$

$$d' = \frac{d^* + i + id^*}{(1 - \phi)} \tag{16}$$

This method assumes that capital markets adjust for inflation and the resulting increased taxes. While this assumption has some appeal, it has the problem of assuming that the tax component of the market adjustment is at the firm's tax rate. If the user specifies both debt and equity costs holding the real after-tax rate constant, then the real discount rate (r^*) is unaffected by the inflation rate. Therefore, the present value of constant real cash flows is unchanged as the rate of inflation is increased.[7]

Depreciation and Cost Depletion

The present value of the reduced income-tax generated by depreciation (P) decreases as the rate of inflation increases. This is due to the fact that the amount of depreciation (D_t) is fixed (based upon the initial basis of the asset) and does not increase over the depreciable life of the asset as shown in equations (17) and (18):

$$P = \sum_{t=1}^{n} \phi D_t (1 + i)^{-t} (1 + r^*)^{-t} \tag{17}$$

$$\frac{\partial P}{\partial i} = \sum_{t=1}^{n} -t \phi D_t (1 + i)^{-t-1} (1 + r^*)^{-t} < 0 \tag{18}$$

Here it is assumed that the real after tax discount rate (r^*) remains constant as the inflation rate increases.

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[7] When this method is used for both the cost of equity and debt, the real pre-tax required rate of return of debt and equity increases with the inflation rate. The real pre-tax cost of debt (d'^*) is defined below and the derivatives with respect to inflation taken to derive the positive sign.

$$1 + d' = 1 + \frac{(1 + d^*)(1 + i) - 1}{(1 - \phi)}$$

$$1 + d'^* = \frac{1 + d'}{1 + i} = \frac{1}{1 + i} + \frac{1 + d^*}{1 - \phi} - \frac{1}{(1 + i)(1 - \phi)}$$

$$d'^* = \frac{1 + d^*}{1 - \phi} - (1 + i)^{-1} \left[\frac{1}{1 - \phi} - 1\right]$$

$$\frac{\partial d'^*}{\partial i} = (1 + i)^{-2} \left[\frac{1}{1 - \phi} - 1\right] > 0, \text{ for } \phi > 0$$

Equations for e'^* and $\frac{\partial e'^*}{\partial i}$ are identical, simply substitute e'^* for d'^*.

Asset Terminal Values

In general the present value of an asset terminal value is decreased as the rate of inflation increases. This is due to an increase in the taxes on the terminal value that results from the fact that the ending basis (B) remains constant as inflation increases. Assuming that the real asset terminal value V_n^* remains constant, that gains are treated as long-term capital gains and taxed at the rate $.4\phi$, and that the real discount rate remains constant, we see from equations (19), (20), and (21) that the present value (P) decreases as the inflation rate increases:

$$P = [V_n^*(1+i)^n - .4\phi(V_n^*(1+i)^n - B)]\ (1+i)^{-n}(1+r^*)^{-n}, \text{ or} \qquad (19)$$

$$P = \left[V_n^* - \phi\left(V_n^* - \frac{B}{(1+i)^n}\right)\right] (1 + r^*)^{-n} \qquad (20)$$

$$\frac{\partial P}{\partial i} = -n\phi B(1 + i)^{n-1}\ (1 + r^*)^{-n} < 0 \qquad (21)$$

The real gain is $V_n^* - B$, since B is the basis when there is no inflation. As the inflation rate increases the amount of real taxable income increases (equation (20)), thus more than the real gain is taxed. This is sometimes referred to as an inflation tax on an asset. If V_n^* is equal to B, there is no tax without inflation.

Gains from the sale of depreciable assets are taxed as ordinary income up to the amount of depreciation taken. If the asset is sold for more than its initial basis the amount of the the sale over the initial basis is treated as a long-term capital gain (i.e. 60 percent is excluded from taxation under current rules).

Annual Real Costs or Returns

The model has been programmed assuming that real growth rates in costs or returns are independent of the level of inflation.[8] Thus, an annual cash flow (C_0) that increases at real rate g^* has a present value as shown in equations (22-26):

---

8 However, a user could circumvent this assumption by changing the expected real growth rates as the inflation rate is changed, since all of these variables are exogenous and user specified.

$$P = \sum_{t=1}^{n} C_t(1 - \phi)(1 + r)^{-t} \qquad (22)$$

Since,

$$C_t = C_0(1 + g^*)^t \, (1 + i)^t \qquad (23)$$

and

$$(1 + r) = (1 + r^*) \, (1 + i) \qquad (24)$$

we find that:

$$P = \sum_{t=1}^{n} C_0(1 + g^*)^t \, (1 + r^*)^{-t}, \text{ and} \qquad (25)$$

$$\frac{\partial P}{\partial i} = 0. \qquad (26)$$

This assumes that r^* (the real after-tax discount rate) is constant as inflation increases. As discussed earlier, the validity of this assumption depends upon the behavior of capital markets, and there may be reason to assume that r^* decreases as the inflation rate increases.

Summary

Clearly, the net present value of an investment is affected by inflation. If one holds the real after-tax discount rate constant as inflation is increased, the net present value decreases. The decrease is through the decreased present value of the depreciation income-tax shield and an inflation tax on asset terminal values. If capital markets do not fully compensate for inflation and the taxes on the inflation component of returns to capital, then there is an effect in the opposite direction. That is, the real after-tax discount rate is reduced and the net result of inflation on the net present value is unclear. However, the net result can be determined for specific cases by using the model.

Specification of Interest and Growth Rates

All interest and growth rate data entered by users of the model are to be effective annual rates. An effective annual rate is a compounded rate when the year has multiple periods. Thus, if there are n periods within a year (they may vary in length) and i_j is the rate for the jth period the effective annual rate is given by equation (27):

$$(1 + p) = \prod_{j=1}^{n} (1 + i_j) \tag{27}$$

In the case of equal periods and equal rates for each period the equation simplifies to:

$$(1 + p) = (1 + i)^n \tag{28}$$

Often the interest rate on a loan that has within year installment payments is stated on an annual basis, but has not been compounded. For example, the effective annual interest rate on a loan with monthly payments and an 18 percent annual rate is 19.56 percent.[9]

In many cases data reported on an annual basis are already effective annual rates. For example, yields on securities of less than one year in maturity and the yield on money market certificates are usually effective annual rates. Also, inflation rates are usually reported in terms of an effective annual rate. For example, the GNP implicit price deflator for the second quarter 1980 was 175.28 and was 171.23 in the first quarter. Thus the deflator increased 2.365 percent for the quarter.[10] This is reported as a 9.8 percent annual rate of increase.[11] One may have a daily interest rate, say 50¢ per $1000 for a one day period. This is an effective annual rate of 20.02 percent.[12]

The effective annual interest rate is the appropriate rate for discrete end-of-year discounting or compounding. The model uses continuous discount factors. Equations are contained within the model which convert the user provided effective annual rates into the equivalent rate for use with continuous factors. Where p is the user specified rate and ρ is the rate used in the model:

$$\rho = \ln (1 + p) \tag{29}$$

For example, if a growth rate (effective annual) of 20 percent is entered. The model uses 18.23216 percent as the continuous growth factor.[13]

---

9 $(1+i)^n = (1.015)^{12} = 1.1956.$

10 175.28/171.23 = 1.02365.

11 $(1.02365)^4 = 1.098.$

12 $(1.0005)^{365} = 1.200159.$

13 ln 1.2 = .1823216.

Firm Versus Project Financing

Two approaches to the use of debt financing are possible. The first, and the one used in GEN3, excludes all debt financing flows (inflows and outflows of principal as well as interest) and uses a weighted average after-tax cost of debt and equity capital. The weights used are the portions of debt and equity the firm expects to have in the future on its cost basis balance sheet. This approach implicitly assumes that the project being considered is financed in the same proportions as the firm. The primary justification for this assumption lies in the fungible nature of funds. It is simply not accurate to attribute specific loans to the financing of specific investments. The total of liabilities and owners' equity provides the financing for all of the firm's assets. The only way of identifying specific project financing, the second alternative, would be through allocating increments in debt at the time an investment is made to that specific investment. This is often argued as being reasonable since the lender providing the additional debt will take the assets associated with the new investment as security. There are pitfalls to this approach.[14] First, it is a piecemeal method which results in a different profitability criterion for each investment. The differences arise from factors such as how much cash the firm happens to have at the time the particular investment is made. Thus, the dependencies in financing of investments that result from desired or targeted firm debt to equity ratios of the firm's management and external creditors are ignored with the piecemeal method. The weighted average cost of capital method assumes that, since the firm is being financed with a desired or target debt to equity ratio, each asset is financed in the same manner.

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[14] This method of including debt flows assumes that the investment is financed according to the principal balance on the loans and cost basis of the investment over the life of the investment. That is, the implicit assumption for a given year in the investment life could be determined by computing the cost basis of the investment in that year and then computing the principal balance on loans at that time. In general, the percentages of debt and equity would change from year-to-year, remaining constant only if assets depreciated at exactly the same rate that principal is repaid. If the loan repayment schedule were carefully chosen such that the proportion of debt on the balance remained constant at the same value assumed in the computation of the weighted average cost of capital, then the method of including debt flows would give exactly the same net present value as the method excluding debt flows. Otherwise, differences will result. Assuming the after-tax cost of debt is less than the required after-tax rate of return on equity, the method with the greatest implicit leverage gives the highest net present value.

Uncertainty and the Monte Carlo Analysis

For both policy and investment analysis, it is important to determine the potential effects on private decisions of uncertainty with respect to future prices, production costs, and reserves. Using the mean (average) values of probability distributions is inadequate because only outputs resulting from these mean values are produced. No measure of the spread (variance) of potential outcomes is obtained. In other words, in the absence of some type of simulation, no measure of the potential riskiness of the final outcome is derived. For policy purposes, it is desirable to learn not only how the mean output values are affected by various policy options but also how the variance or range of the outcomes is changed.

For example, suppose two policy options have identical effects on the means of relevant policy objectives (model outputs), have identical costs (in whatever terms cost is measured), but have differential effects on the expected outcome variances. Naturally, the policy maker would want to consider the difference in variance in his policy decision. Also, because of non-linear transformations in some of the model operations, the simulation results may differ from the analysis using mean input values. For example, revenue depletion is legally constrained not to exceed 50 percent of net income before taxes. This constraint is one type of non-linear transformation which can cause the simulation means to differ from outputs using mean input values. Trade-offs between changes in means, differences in relative cost, and variances must be weighed by the decision makers.

Monte Carlo simulation is one technique for handling the problem of uncertainty in input values and estimating the variance in potential outcomes. Rather than using point estimates of uncertain input variables an assumed probability distribution is provided from which samples are taken to be used as inputs for the analysis. The process of sampling each variable from its unique probability distribution and performing the model calculations is repeated many times to produce a range of model output values. A frequency distribution of these output values can be derived and the mean and variance of the expected outcomes determined. In performing this type of simulation, we replace the unknown actual population of future prices, costs and reserves by an assumed probability distribution from which samples are drawn. By sampling many times it is possible to generate many possible combinations of prices, costs, and reserves that together produce outcomes, each in the appropriate proportion (King).

Any type of probability distribution may theoretically be specified for the uncertain variables. For this model we have used the normal, triangular, and lognormal distributions which are depicted in Figures 4, 5, and 6, respectively. The uncertain variables used in this mode and the type of distribution which is used for each variable are listed in Table 4.

After the variables are read in and stored as necessary, the first solution step is to run through the model once using mean values for all input variables. This step determines the after tax net present value (ATNPV) if all mean values are used and converts that value into a bonus bid payment to be used in subsequent calculations.[15] This conversion is assumed to be linear according to equation (30):

$$\text{BONUS} = B_o + B_1 * \text{ATNPV} \qquad (30)$$

where B_o and B_1 are input values.[16] If Monte Carlo simulation is not being used, the mean values produce the model results. In this case, the model description that follows is only relevant to the use of this single set of values.

The Exploration Phase of Resource Development

The next step in the model solution is to determine the exploration cost for the lease tract or area in question. For example, gross exploration costs (EC) are a function of the number of wells to be drilled per acre, the number of acres in the tract, and the cost per well as illustrated in equation (31):

$$\text{EC} = \text{Wells/acre} * \text{acres} * \text{dollars/well} \qquad (31)$$

A portion (exogenously specified) of exploration expenses (intangible investment) can be expensed immediately. Since no revenues are produced during exploration and development, expensed investment and other tax losses are written off and entered into the tax vector as negative taxes. Exploration expenses each year minus the savings from tax write-offs are entered into the after tax value vector as negative values. These vectors contain the time

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[15] The amount of the bonus bid is necessary for tax calculations. The use of mean input values to calculate the bonus serves to approximate the actual value. This can then be used in subsequent calculations where uncertainty is considered. Optionally, the bonus may also be recalculated after any number of Monte Carlo iterations for use in subsequent iterations.

[16] If B_o and B_1 are set equal to 0 and 1, respectively, the bonus will equal ATNPV. The values of B_o and B_1 depend on the bidding strategy employed. To eliminate the bonus (for some types of investment analysis), set $B_o = B_1 = 0$.

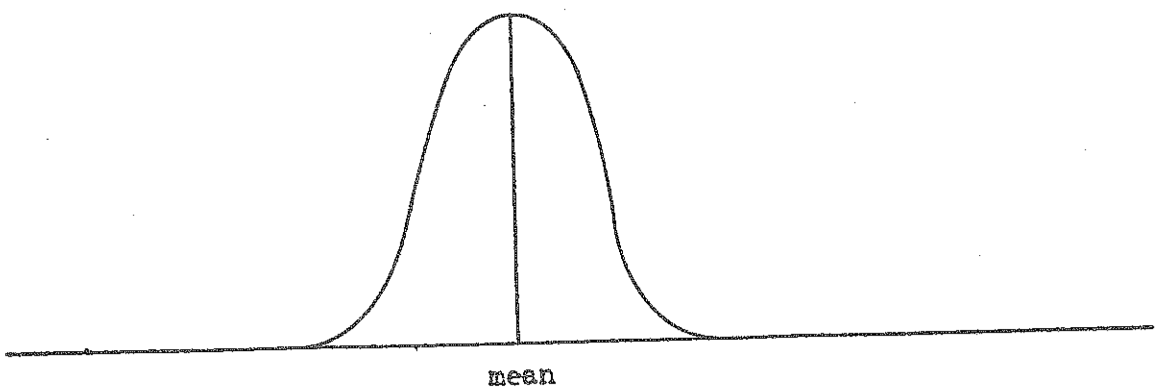

FIGURE 4
NORMAL DISTRIBUTION

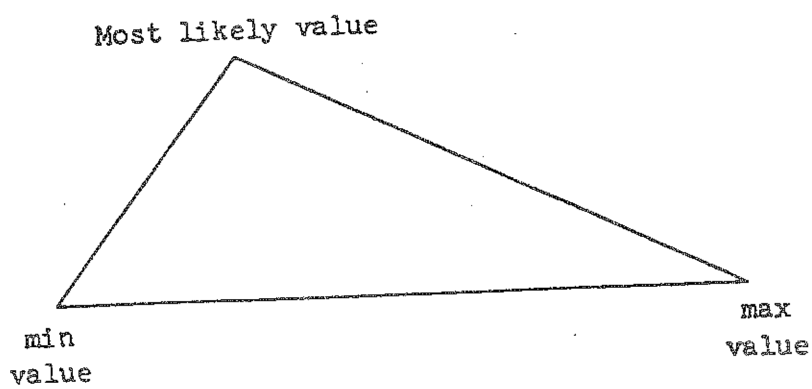

FIGURE 5
TRIANGULAR DISTRIBUTION

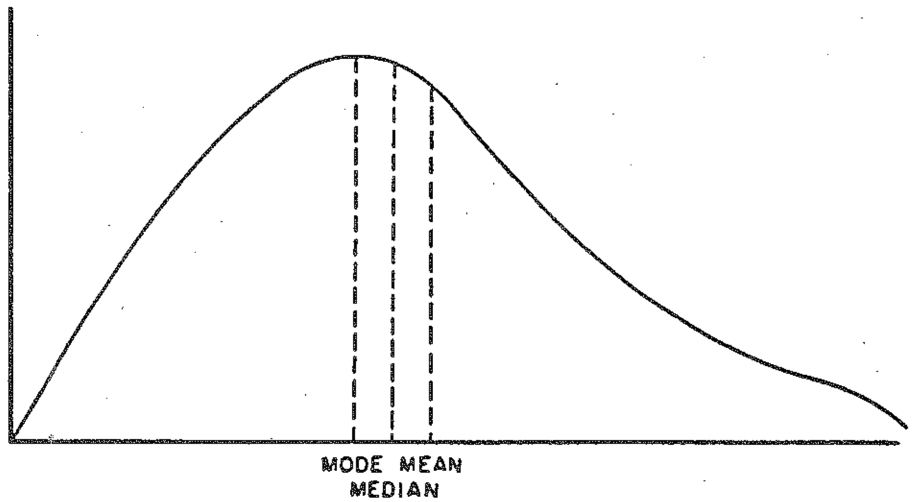

FIGURE 6
LOGNORMAL DISTRIBUTION

TABLE 4
DISTRIBUTIONS USED FOR UNCERTAIN VARIABLES

Variable	Distribution
Annual price change	Normal or truncated normal
Investment cost contingency factor	Triangular
Operating cost contingency factor	Triangular
Amount of reserves	Lognormal or normal

stream of taxes, after tax value, and other economic output variables.

In addition to calculating the total net expenses of exploration, the potential tax write-off available to the company if the lease is not developed is also calculated at this time. This potential tax write-off is the bonus payment and the remaining depreciable base of exploration expenses multiplied by the tax rate. This value is compared later in the program with the potential present value of the lease if developed to decide whether or not it is advantageous to develop the lease.

Exploration is the last aspect of the model solved outside the Monte Carlo simulation. Once exploration cost is determined, the Monte Carlo simulation begins. A number of the variables used in this model are considered uncertain and are subject to Monte Carlo analysis. Any or all of the following variables may be selected randomly in the Monte Carlo simulation: annual change in price, contingency factor for investment cost, contingency factor for operating cost, and the amount of resources actually present. In addition, the presence or absence of resources is an independently risked variable.

Future Resource Prices

Uncertainty in future resource prices is handled by randomly selecting the annual change in price each year from a normal distribution with a specified mean and variance. This vector of sample annual price changes together with the resource price at the beginning of the lease, $P_o(1)$, is used to create a vector of initial prices for each year of potential lease duration. Equation (32) illustrates this process:

$$P_o(tt + 1) = P_o(tt)e^{P_1(tt)} \qquad (32)$$

$P_o(tt)$ is the initial resource price in year tt, $P_1(tt)$ is the rate of change in price during year tt (from the vector of price change samples), and $P_o(tt+1)$ is the initial resource price in year (tt+1). The vector of initial prices for each year and the vector of price changes during each year are used in the model computations to determine gross revenue for each year of production. Since this procedure is repeated independently for each Monte Carlo iteration, a separate price distribution emerges for each year of the production period. Because the annual price change has a compound effect upon the initial price, the mean and variance of these annual price distributions would also change through time.

If desired, more than one price change distribution may be used in generating the price change vector. The model allows for as many as four unique price change distributions to be input for up to four specified time periods. For example, price could be expected to rise at an annual rate of 8 percent for three years, fall at a rate of 3 percent for six years, remain relatively constant for eight years, and then rise at a rate of 4 percent through the end of production. Each of the expected price change values could have a unique variance, so that the variance as well as the expected value of annual price change can vary through time. The price change vector is created by utilizing the distribution appropriate for each year in the vector.

It is important to understand that the price change used in this analysis is the expected price change in excess of general inflation. It is not the total expected change in price of the resource; rather, it is the difference between the expected change in price of the resource and the expected general rate of inflation in the economy. This same principle applies to investment and operating cost factors. Thus, the relative inflation rate between revenues expected from the resource and cost to obtain the resource is a derivative of the inputs to the model. Because both cost and revenue inflation factors are keyed to general inflation, relative inflation between costs and revenues for a particular investment is automatically accounted for using this procedure (given the subjective distributions pertaining to the input variables).

Specification of Variance for Growth in Prices

The annual rate of real price change in year t (P_1tt) is distributed normal with a user specified mean P_1 and variance r^2. Thus, the real price in time 1 is:

$$P_0(1) = P_0(0)e^{P1} \tag{33}$$

Since P_1 is distributed normal, $P_0(1)$ is distributed lognormal with mean $P_0(0)e^{P}{}_1$ and variance $P_0(0)^2\sigma^2$. When the model is being run with nominal values an inflation component is added, i.e.:

$$P_0(1) = P_0(0)e^{Pt}\ e^{i} = P_0(0)e^{Pt+i}, \text{ where} \tag{34}$$

$P_0(1)$ = the nominal price at time 1

i = the annual inflation rate

When the inflation rate is a constant and p_1 is distributed as before, the nominal price is distributed lognormal with mean $P_0(0)e^{P_1+i}$ and variance $P_0(0)^2e^{2i}\sigma^2$.

The real price in time period t can be written as the product in equation (35):

$$P_0(t) = P(0) \prod_{j=1}^{t} e^{P_1 j} \tag{35}$$

$$= P_0(0)e^{p_1(1)+p_2(2)+...+p_1 t}$$

Each $P_1(t)$ is distributed $N(p_1, \partial^2)$; therefore the sum is distributed $N(tp_1, t\sigma^2)$ and it follows that $P_0(t)$ is distributed $LN(P_0(0)e^{tp}, P_0(0)^2t\sigma^2)$. Similarly, the nominal price is:

$$P_0(t) = P_0(0)e^{it} \prod_{j=1}^{t} e^{p_1} \tag{36}$$

The distribution of $P_0(t)$ is $LN(P_0(0)e^{it}e^{tp}, P_0(0)^2e^{2it}t\sigma^2)$. The reader may note that the probability density function of nominal price has a greater variance than the density function for real price. This occurs even though the probability distribution of the rate of price change is identical in both the real and nominal cases. Thus the user should specify the same variance in price growth when inputing data for the model to be run with real or nominal output.

Expected Value and Variance of Net Present Value

Next we should consider the variance of net present value when the model is run in nominal and with real values. The revenues involve the product of two independent random variables, price and quantity (Q). Assuming the real input and output option is being used, the present value of revenue (R) at a given point in time (ignoring changes in price over the time period) is given by equation (37):

$$R = (1 - \phi)\ P_0(t)Q\ e^{-dt}, \text{ when} \tag{37}$$

ϕ = the marginal income tax rate,

d = the real after-tax discount rate.

The expected present value revenue is given by equation (38):

$$E(R) = (1 - \phi)e^{-dt}\, E(P_0(t))\, E(Q). \tag{38}$$

The variance of revenue is provided by equation (39):

$$V(R) = [E(P)^2V(Q)+E(Q)^2V(P)+V(Q)V(P)]\ (1-\phi)^2e^{-2dt} \tag{39}$$

Assuming nominal input and output is being used, equation (40) is for the revenue at time t:

$$R_n = (1 - \phi)P_n(t)Qe^{-(d+i)t}, \text{ where} \tag{40}$$

i = the annual inflation rate.

The expected value of R_n is given in equation (41):

$$E(R_n) = (1 - \phi)e^{-(d+i)t}\, E(P_n)\, E(Q) \tag{41}$$

Since $E(P_n) = E(P)e^{it}$, equations (38) and (41) are equal. Hence, each revenue has the same expected present value, whether the model is run in real or nominal.

Equation (42) is for the variance of present value:

$$V(R_n) = [E(P_n)^2\, V(Q)+E(Q)^2V(P_n)+V(Q)V(P)]\ (1-\phi)^2e^{-2(d+i)} \tag{42}$$

Since $E(P_n) = E(P_0)e^{it}$, and $V(P_n) = V(P_0)e^{2it}$, equations (39) and (42) are equal. Hence, the variance of the present value of revenue using nominal data is equal to the variance of present value using real data.

These results can be extended to the present value of each cash flow. Since net present value is a sum of cash flows each having the same expected value and variance whether the model is run in nominal or real, the user should expect to get very similar net present value results with the two modes of model operation. The present value results are not exactly the same because the process used in the model to determine nominal flows and calculate their present value is not mathematically identical to the process for discounting real flows. The difference in all cases should be less than 0.5 percent.

Investment Cost Contingency Factor

Investment costs are uncertain for at least three reasons, and a cost contingency factor is used to handle this uncertainty. The contingency factor is a percentage of the estimated investment cost and is selected from a triangular distribution with an input minimum, maximum, and most likely value.

One of the most important reasons for a contingency factor in investment cost is that inflation in construction costs in recent years has taken place at a rate higher than the rate of general inflation. Although this experience will not necessarily continue, it is uncertain what the rate will be in the future. Since the construction and start-up period may be five years or more, the rate of inflation can have a significant effect on total construction costs. Second, investment costs may be uncertain because technology for extracting and refining some resources is relatively new. For example, coal gasification technology and costs are highly uncertain. Unforeseen engineering and technical problems could raise investment costs substantially. A third reason for an investment cost contingency factor is that if facilities of the type and scale required have not been constructed previously, the length of the development and construction period required cannot be known with certainty. Changes in the assumed period will have a significant impact on the present value of investment costs.

As is evident from the discussion of these factors, the distribution of investment cost uncertainty tends to be one-sided. In other words, the risk is mainly on the high side, so the distribution would be expected to be skewed in that direction, although any desired distribution may be used.

Operating Cost Contingency Factor

The two factors affecting annual operating costs in the model are θ, the annual increase in cost per unit, and K_o, the initial operating cost per unit. For purposes of analysis, θ was assumed to be known and constant throughout the production period, and a triangular distribution of K_o values was utilized. Uncertainty in initial operating cost arises from the same sources as investment cost (future relative inflation and unforeseen technolgoical difficulties) plus uncertainty in the future cost of environmental protection. Since future government regulations are unknown or are subject to modification, it is difficult to forecast the environmental control costs which must be borne by the private sector. However, once production has begun with technological problems solved and environmental control equipment in place, future changes in operating cost should be subject to less uncertainty. Therefore, the initial operating cost, K_o, was assumed to be uncertain with risk mainly on the high side.

In addition to the factors K_o and θ, unit operating costs are also affected by the rate of decline in production (especially for resources like oil). Since total operating costs are determined by the factors described above, unit operating costs may rise as production falls. This point is discussed further at a later point in the text (see footnote 20, p. 33).

Presence or Absence of Resources

This variable is particularly relevant for oil and natural gas or other resources for which there is a significant chance that no resource will be discovered on the lease. Almost any distribution may be used for the chance of no resources being present. The distribution mean and standard deviation are model inputs. Table 5 displays the mean, variance, and characteristics of a number of distributions which could be used for the chance of no resource being present. Using the formulae in Table 4, the mean and variance can be calculated from the distribution parameters and input into the model. For example, if a triangular distribution is selected, the mean is the average of the minimum value (a), most likely value (b), and maximum value (c) as shown in Table 5. The standard deviation is the square root of the triangular distribution variance as calculated using the formulae in Table 5.

Rather than directly sampling from the distribution as for the other Monte Carlo variables, the risk that no resource is present is handled by combining the distributions of value with resources present and value (loss) with no resources being found. This step requires the linear combination of the product of random variables as shown in equation (43):

$$V = N * X + (1 - N) * R \qquad (43)$$

where N is the random variable representing the chance that no resource will be found (a Bernouli variable), X is the loss from exploration, (1 - N) is the variable representing the chance that resources will be found, and R is the variable representing the value of the lease given that resources are present. The expected value of the value distribution (V) is the sum of the products of expected values as shown in equation (44):

$$E(V) = E(N) * E(X) + E(1 - N) * E(R) \qquad (44)$$

where E(N) is the input mean of the chance of no resources being found distribution, E(X) is the exploration loss calculated in the exploration subroutine, E(1 - N) is the probability that resources will be found, and E(R) is the mean of the sample distribution of value calculated from the Monte Carlo simulation. In other words, the expected value of a lease is the probability of no resource being found multiplied by the expected value if no resources are found plus the probability that resources are found multiplied by the expected value if resources are present.

The variance of the total expected value distribution is somewhat more complicated. The random variable sets N and X and N and R are each independent, but clearly the two products are not

TABLE 5
SELECTED DISTRIBUTIONS

Distribution	Symbol	Domain	Restriction	Mean	Variance
Beta	$\beta\ (a,h)$	$0 \leq x \leq 1$	$a,\ b > 0$	$\frac{a}{a+b}$	$\frac{ab}{(a+b+1)(a+b)^2}$
Binomial	$\beta(n,p)$	$0 < p < 1$, $n = 1,2\ \ldots$	$x = 0,\ 1, \ldots\ n$	np	$np(1-p)$
Chi-square	$\chi^2(n)$	$0 \leq x \leq \infty$	$n = 1,\ 2, \ldots$	n	$2n$
Exponential	$E(\beta)$	$0 \leq x \leq \infty$	$\beta > 0$	β	β^2
Gamma	$G(\alpha,\beta)$	$0 \leq x \leq \infty$	$\alpha,\ \beta > 0$	$\alpha\beta$	$\alpha\beta^2$
Lognormal	$LN(\mu,\sigma^2)$	$0 \leq x \leq \infty$	$\sigma^2 > 0$	μ	σ^2
Negative Binomial	$NB(r,p)$	$0 < p < 1$, $r > p$	$x = 0,\ 1, \ldots,\ \infty$	$\frac{rp}{1-p}$	$\frac{rp}{(1-p)^2}$
Normal	$N(\mu,\sigma^2)$	$-\infty \leq x \leq \infty$	$\sigma^2 > 0$	μ	σ^2
Poisson	$P(\beta)$	$\beta > 0$	$x = 0,.,\ \ldots,\ \infty$	β	β
Triangular	$T(a,b,c)$	$a \leq x \leq c$	$c > b > a$	$\frac{a+b+c}{3}$	$\frac{a(a-b) + c(c-a) + b(b-c)}{18}$
Uniform	$U(a,b)$	$a \leq x \leq b$	$b > a$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Weibull	$W(\alpha,\beta)$	$0 \leq x \leq \infty$	$\alpha,\ \beta > 0$	$\beta\Gamma\left[\frac{1}{\alpha}+1\right]$	$\beta^2\left[\Gamma\frac{1}{\alpha} + 2 - \Gamma^2\frac{1}{\alpha} + 1\right]$

Source: Fishman, George S. Concepts and Methods in Discrete Event Digital Simulation, pp. 201, 218.

independent because both contain N. Hence, the products can be taken assuming independence, but the linear combination must include the covariance term. The resulting variance of total expected value is given in equation (45):[17]

$$\sigma_V^2 = (E(N))^2 * \sigma^2 + (1-E(N))^2\sigma^2_R + (E(R)-E(X))^2\sigma^2_N + (\sigma_X^2+\sigma_R^2)\sigma_N^2 \quad (45)$$

This term can be further simplified for this analysis because the variance of X is zero. None of the variables in the exploration calculations are ranged, so a point estimate is calculated (with no variance). Therefore equation (45) can be reduced to the form shown in equation (46):

$$\sigma_V^2 = (1-E(N))^2 * \sigma_R^2 + (E(R)-E(X))^2 * \sigma_N^2 + \sigma_R^2 * \sigma_N^2 \quad (46)$$

Since all the terms on the right side of equation (46) are either inputs or are calculated in the simulation, the variance of total expected value can be calculated directly. By using this process, uncertainty regarding the presence or absence of resources can be handled outside the Monte Carlo simulation yet allowing for maximum flexibility in the input distribution.

If resources are certain to be found, the mean and variance of the chance of no resources distribution can be specified as zero. In that event equation (46) reduces to the variance when resources are present (σ_R^2) which is the appropriate value for total variance.

Amount of Reserves

For some resources such as oil and natural gas, a major source of uncertainty is the amount of reserves present. For almost all resources some degree of uncertainty about the total quantity of resources in place exists.

Relating to petroleum exploration, a number of researchers have found that the lognormal distribution provides a good fit for experimental data on the size of petroleum deposits. (Uhler and

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[17] This variance calculation assumes that the parameters of the chance of no resources distribution (E(N), σ_N^2) are known with certainty. If the expected value for the chance of no resources being found is uncertain, equations (45) and (46) would actually understate the total expected variance. The authors are indebted to Don Bieniewicz and Mike LeBlanc for their assistance in calculating the variance.

Bradley; U.S. Geological Survey, 1975; Kaufman, 1962). Therefore, the lognormal distribution is used for the size distribution of petroleum resources and in other situations where deemed appropriate.

For resources which are not distributed lognormally (such as coal), the normal distribution also may be used in the simulation program. In either case, the mean, standard deviation and distribution selection are model inputs.

Model Description with Monte Carlo Simulation

Once the Monte Carlo simulation begins, each of the procedures is repeated for each iteration of the simulation. The results of each iteration are stored and used to calculate the mean and other statistics on selected output variables.

The number of Monte Carlo iterations actually run by the model, NLØØP, represents only the number of iterations for which resources are found. The effective number of iterations is equal to NLØØP divided by one minus the mean of the no resource risk distribution. For example, if NLØØP is two hundred and the mean of the no resource risk distribution is .6, the effective number of iterations is five hundred.

Once the amount of resources on the tract is determined, the next step in the process is to make a random selection of factors to be used in determining total investment and operating costs. A choice of four methods is allowed in making this selection of factors. First, the investment and operating cost input values may be used without any random component added. In this case, random selection is bypassed. Second, a cost adjustment factor may be randomly selected from the triangular cost distributions supplied for both investment and operating costs. For both investment and operating cost the minimum adjustment factor, the most likely adjustment factor and the maximum adjustment factor are inputs determining the shape of the triangular density function. Third, the mode (most likely value) of the triangular distribution may be used directly. Fourth, the mean of the triangular distribution (the average of the three varieties) may be selected.

The cost adjustment factor is then multiplied by the base cost, and the result added to the base cost. In essence, the random cost component which results from the cost adjustment factor is a contingency. The actual amount of the contingency may be zero (if the base value is used), equal to the mean or mode of the triangular distribution, or randomly selected from the distribution. Normally, the random selection method is used because contingency is considered a random component of total cost. Hence, the random selection method is considered to better reflect actual operating conditions.

The next two steps in the model simulation vary depending upon whether installed capacity is input or determined within the model. If installed capacity is internally determined, the random factors for investment and operating cost are immediately used to determine the investment and operating cost values that will be used for each installed capacity. If installed capacity is an input, associated investment and operating cost values are also inputs unique to each installed capacity. The same random factor is applied to each of the investment and operating cost values for each installed capacity to determine a unique investment and operating cost.

If installed capacity is an input to the model, that capacity, together with reserves and other input variables, is used to determine the maximum production time horizon which can be used given the installed capacity and the amount of reserves. On the other hand, if installed capacity is solved within the model, a time horizon is determined internally and the corresponding (maximum) installed capacity is calculated within the model. Since each of these procedures represent a different solution to the same basic structural relationship, we will develop that relationship carefully and explain the correlation between the two procedures.

Economic, Engineering and Geologic Relationships

We begin with the relationship between reserves and production. Reserve estimates enter the calculus of profitability both as a basis for the investment and as a constraint on the production from an investment. The production constraint is represented in equation (47):

$$xR \geq \sum_{tt=1}^{TT} qq(tt) \qquad (47)$$

where R represents the amount of the resources in place, x the recoverable fraction with a given technology, qq(tt) the amount of annual production, and TT, the production time horizon. This equation merely states that the sum of production through time can be no greater than the recoverable portion of the resource in place (with a given technology). Given this constraint, the producer attempts to select an initial plant capacity which will maximize his return through time. In other words, the producer attempts to select the investment which maximizes his after tax net present value of revenue subject to the production constraint.

Assume for the moment that production declines exponentially through time. Annual production may then be expressed as a function of initial installed capacity as in equation (48):

$$qq(tt)_i = \int_{t-1}^{t} q(o)_i e^{-at} dt \tag{48}$$

where $q(o)_i$ represents initial installed capacity of the ith plant which is one of a group of possible initial capacities. While this simple relationship between installed capacity and annual production may be adequate for resources such as oil, it is not adequate for other resources or for oil resources during the early production phase. A typical resource production pattern includes a production build-up period during which production is increasing each year as installed capacity is coming on stream followed by a flat production period which continues indefinitely (as for coal) or is followed by a declining production period as shown in Figure 3. Under this scenario, total production during the lease life is given by equation (49):

$$PROD = q_o \left[\sum_{j=1}^{B} h_j + (F-B) \right] + \int_{o}^{T-F} q_o e^{-at} dt \tag{49}$$

where the build-up period covers production year one through year B, the flat production period is year (B+1) through year F, and the declining production period (perhaps at a zero rate) is the period from the beginning of year F+1 through T; T being the production life of the lease as determined below.[18] Equation (49) gives the sum of production during each of the three phases of production.[19] Production during the build-up period is equal to the sum over the build-up period of the annual factors, h_j, times installed capacity; production during the flat period is

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[18] The integral for the decline period goes from zero to T-F rather than F to T because this integral properly measures the sum of production over the decline period.

[19] The notation, $q(o)_i$, is here changed to q_o representing one potential investment, but the reader should be aware that the optimization process covers all available investment opportunities.

simply the number of years in which production is constant times installed capacity; and production during the decline period is equal to the integral over the number of years production is declining. Recalling from equation (47) that total production must be less than or equal to recoverable reserves, we may now combine equations (47) and (49) to yield the relationship between recoverable resources and installed capacity as in equation (50):[20]

$$xR - \beta q_o e^{-a} \geq q_o \left[\sum_{j=1}^{B} h_j + (F-B) \right] + \int_{o}^{T-F} q_o e^{-at} dt \quad (50)$$

By assuming that recoverable reserves are exhausted, we may change equation (50) from an inequality to an equality and solve for either q_o or T. Equation (51) represents the solution for equation (50) for T which is used in the case of input q_o:

$$T = \left[\ln \left[1 + a(-xR/q_o + \beta e^{-a} + \sum_{j=1}^{B} h_j + F - B)\right] \right] / -a + F \quad (51)$$

Equation (52) represents the solution when installed capacity, q_o, is solved within the model:

$$qo = \frac{axR}{[1 + a(\beta e^{-a} + \sum_{j=1}^{B} h_j + F - B) - e^{-a(T-F)}]} \quad (52)$$

Production Period Constraints

Given that q_o and T have been determined either by input or within the model, the production time horizon, T, must be subjected to two constraints before it can be employed. These constraints are the physical and economic lifetimes of the proposed investment. The production time horizon for a given investment can be no greater than the actual physical lifetime of the initial

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[20] β is a geologic variable applicable to resources like oil which relates total recovery to the rate of recovery. (The faster the oil is produced, the lower is total recovery.) If β equals zero, recoverable reserves, xR, are greater than or equal to production as defined in equation (49).

plant.[21] Nor can the production time horizon exceed the time at which the variable unit cost of producing the product exceeds the revenues per unit obtained from marketing it. In other words, when the steadily increasing unit costs of production (assuming a rising MC curve) exceed the revenues per unit of production, production would cease.

The first constraint is simply expressed as an exogenously determined constraint $T \leq Tp$, where Tp equals the maximum physical lifetime of the investment. The second constraint is the limit obtained when marginal cost equals marginal revenue. Equation (53) states that the economic limit occurs when operating costs plus taxes exceed or equal revenue minus royalties and severance taxes:[22]

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[21] If substantial resources still remained on the leasehold beyond the lifetime of the initial plant, a new plant investment could be initiated. This situation could be handled by resimulating the leasehold using the remaining resources and production conditions. An example of this type of situation is enhanced recovery of oil using tertiary production techniques.

[22] The form of the economic time constraint shown in equation (53) corresponds to the operating cost option which makes total operating cost in each period a function of peak production:

$$q_o K_o e^{\theta t}$$

Thus, unit costs become:

$$q_o K_o e^{\theta t} / q_o e^{-a(t-F)} = K_o e^{[(\theta+a)t-aF]}$$

The denominator of this equation represents the production rate at point t. This form for operating cost is appropriate when most of the operating cost is determined by the fixed installed equipment such as for offshore oil operations where the pumps, separators, platforms and other equipment must be operated regardless of the rate of production.

For other resources or in other production situations, total operating cost may be assumed to be a function of actual production:

$$q_o e^{-a(t-F)} K_o e^{\theta t} / q_o e^{-a(t-F)}$$

Unit operating costs in this case become $K_o e^{\theta t}$.

$$(1-\lambda)(1-s)P_o e^{P_1(t+L)} \leq K_o e^{[(\theta+a)t-aF]} + \qquad (53)$$

$$\phi\ (1-\lambda)(1-s)P_o e^{P_1(t+L)} - z(1-\lambda)(1-s)P_o e^{P_1(t+L)} - K_o e^{[(\theta+a)t-aF]}$$

Solving equation (53) for the time constraint yields:

$$T_e \leq \left[\ln\left[\frac{(1-\phi)K_o}{(1-\phi+\phi z)(1-\lambda)(1-s)P}\right] - aF - P_1 L \right] / (P_1 - \theta - a) \qquad (54)$$

Note that this equation may be negative or undefined when the rate of change in price is greater than or equal to the decline rate plus the rate of change in unit operating cost ($P_1 \geq \theta+a$). The negative sign occurs because the marginal revenue-marginal cost curve intersection is in the negative quadrant to the left of the origin as shown in Figure 7. The correct interpretation for this negative sign is that the economic time constraint is infinite.

We now have each of the equations and relationships necessary to determine the production time horizon. The production time horizon is that T determined in the model either by equation (51) or through the q_o, T optimization procedure, subject to the maximum physical lifetime and the economic lifetime constraint given by equation (54). Hence, the production time horizon is the minimum of the resource exhaustion time period, maximum physical life of the plant, or the economic production time constraint.[23]

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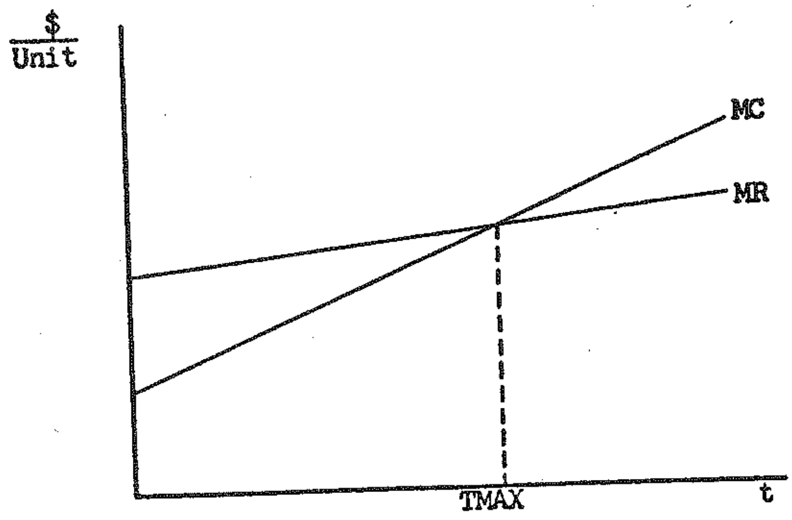
[22] (continued)

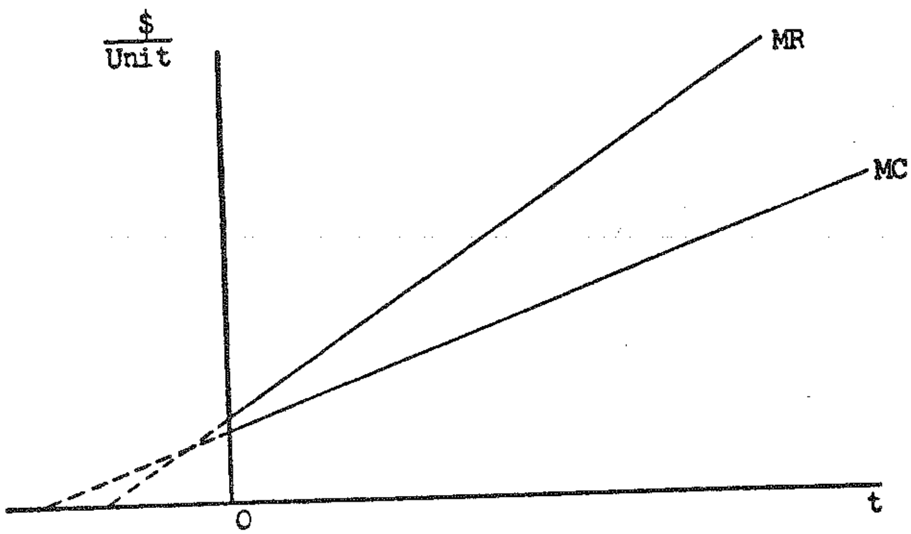
A third option programmed into the model is the average of these two assumptions:

$$\text{Unit cost} = \left[K_o e^{\theta t} + K_o e^{[(\theta+a)t-aF]} \right] / 2$$

[23] Clearly, the form chosen for operating cost affects the economic lifetime calculation. Separate economic time constraint solutions are provided for each of the operating cost assumptions.



Finite Solution



Infinite Time Horizon

FIGURE 7

SOLUTION TO THE ECONOMIC TIME HORIZON

Mechanically, these equations are determined slightly differently depending on whether installed capacity is input or determined by the model as explained above and as outline in Figures 1 and 2.

Prices

For the first q_o-T set to be evaluated in each Monte Carlo iteration, the next step is to create a vector of prices covering each year in the production period. The first step in this process is to generate a vector of annual price change. Price change values are generated for sixty years which is the maximum lease life allowed in the model. This vector may be created by randomly sampling from a normal distribution of price change with an input mean and standard deviation as explained above. Alternatively the mean annual price change may be used for each year in the vector.

The next step is to create a vector of prices from the lease time until the end of production using the initial input price P_o along with the vector of price changes. As described above, the vector is created by multiplying the price at the beginning of each year by the exponential price change during that year to get the price at the beginning of the next year.

Investment Costs

Investment for each year of the construction and development period and the discounted value of total capital investment is calculated next. Total capital investment is determined by multiplying installed capacity, q_o, by the investment cost per unit of installed capacity, C, as determined in the cost subroutine for each resource. To determine the discounted value of total investment, the total investment figure must be multiplied by the percentage of total investment occurring in each year of the development period, and the investment value for each year discounted back to the beginning of the lease. Both development costs and exploration costs for each year are summed together and discounted to the beginning of the lease. In functional form this relationship is expressed in equation (55):

$$PVI = \sum_{i=1}^{L} (q_o * C * f_i + EX_i) * DSC \qquad (55)$$

where PVI represents the present value of investment, f_i the factor used to determine the proportion of investment in each year of the lag (L), EX_i the exploration expense during each year of the lag, and DSC the continuous discount factor given in equation (59). The values for total annual investment are then used to calculate depreciation streams for the production period and to calculate expensed investment and investment tax credit.

Deferred Investment

In some situations, a substantial amount of capital investment takes place during the production period to replace equipment which has worn out. This investment normally takes place in regular cycles. For example, trucks used to haul coal may be replaced at regular intervals over the life of the mine. For this model up to three categories of deferred investment are permitted. For each category, the physical lifetime, depreciation lifetime, amount of deferred capital investment for each cycle, and salvage value are model inputs. Separate depreciation streams and salvage values are maintained for each category of deferred investment as well as for that portion of original investment which is not replaced during the mine life. The present value of the stream of deferred investment is added to the initial investment cost (and subtracted from after tax net present value). Investment tax credit is calculated for each year of deferred investment.

Depreciation and Taxes

The model allows any of the following forms of depreciation to be used:

1. No depreciation
2. Sum of years' digits (SYD) depreciation with input depreciation lifetime (N)
3. Double declining balance (DDB) with automatic conversion to straight line (SL) at the appropriate time -- using input depreciation lifetime
4. Straight line using input depreciation lifetime
5. Unit of production depreciation--at the same rate as the resource is depleted (annual production/total production) using the production horizon (T) as depreciation lifetime
6. SYD with the production horizon as depreciation lifetime
7. SL with the production horizon as depreciation lifetime
8. DDB with the production horizon as depreciation lifetime
9. Current U.S. regulations -- the ACRS method.

According to IRS regulation, capital investment cannot be depreciated until it is placed in service. Therefore, all tangible investment during the development period is depreciated beginning with the first year of production.

Tax savings during the exploration and development periods result from expensed (intangible) investment, rental payments during exploration (RENT), and the investment tax credit (at the beginning of production). Tax savings during development are entered as negative values in the tax vector (and consequently as positive values in the after tax value vector).

Production Period Calculations

Working capital is calculated as a fraction of total operating cost during a year of peak production. Once this calculation is complete, calculations for the production period begin. Annual and total production, gross revenue, operating cost, royalty, severance tax, depletion, profit share or production share, and taxes are calculated. Because many of the equations are in integral form, yet many of the values are needed on an annual basis, integral solutions are obtained over each year of production and then summed over the production period. For example, production is obtained from point zero to the end of year one and then from the beginning of year one to the end of year two and so on through the beginning of the last year of production to the end of the last year of production. These values are then summed to determine total production. In this way both annual and total values can be obtained for variables such as production, profit share, and royalty. A form of continuous discounting is utilized for variables such as gross revenue and operating cost.

The methods used to determine annual production in each year of the production period are described in detail above. In addition to calculating production for the basic resource, production is also calculated for any associated resource such as associated gas with petroleum production. The ratio of production between the major resource and the secondary resource is assumed to be a constant factor. In other words, to determine the production of associated natural gas in each period, the production of oil is multiplied by the factor (AGFAC) to determine the production of natural gas. In the equations that follow, the annual production of the major resource will be denoted by qq(tt) and production of the secondary resource will be denoted by gg(tt).

A number of equations are used in calculating the economic variables for each year of the production period. So that this process may be clearly understood, the equation for gross revenue is presented below in two forms:

1. The integral form divided into annual periods

2. The computational form actually used in the model

For simplicity of exposition, the values of F and B are assumed equal to zero. Hence, equations (56) and (57) represent the two forms of the gross revenue equation during the period of production decline:[24]

$$GR_{tt} = qq(tt)P_o(tt) \int_0^1 e^{P_1 t} dt + gg(tt)GP_o(tt) \int_0^1 e^{GP_1 t} dt \quad (56)$$

$$GR = qq(tt)P_o(tt) \left[\frac{e^{P_1 t} - 1}{P_1}\right] + gg(tt)GP_o(tt) \left[\frac{e^{GP_1 t} - 1}{GP_1}\right] \quad (57)$$

Calculation of annual operating cost (OC) proceeds in the same manner, as shown in equation (57):

$$OC = qq_c K_o \int_{t-1}^{t} e^{\theta t} dt = qq_c K_o \left[\frac{e^{\theta t} - e^{\theta(t-1)}}{\theta}\right] \quad (58)$$

where qq_c may be set equal to production in year tt, peak production (q_o), or the average of the two. The marginal cost of extracting the second resource is assumed to be zero, <u>or</u> included in the cost of extracting the primary resource.

The model contains three annual revenue calculations in addition to the gross revenue calculation in equation (57). The first net revenue value (NTREV1) is calculated by subtracting royalty, operating cost, depreciation, severance tax, and rent from gross revenue. In the next step the depletion allowance is calculated. Currently, in the United States only cost depletion is allowed for oil but for other resources such as coal, revenue depletion, cost depletion, or the maximum of the two may be selected. Cost depletion is allowed on the bonus payment and other lease acquisition costs in proportion to the depletion of reserves. The second revenue value is the annual profit share base (NTREV2). To obtain this value, depletion is subtracted from NTREV1. The third annual revenue value is taxable income (NTREV3) which is the profit share base minus profit share payments.

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[24] Actually P_1 and GP_1 are also time indexed variables as explained above, but they are written here in unindexed form for clarity of exposition. $P_o(tt)$ and $GP_o(tt)$ represent prices at the beginning of year tt, and qq(tt) and gg(tt) represent production during year tt.

After tax income in each year is simply taxable income minus state and federal taxes. After tax value and other output variables in each year are discounted using the continuous discounting factor shown in equation (59):

$$DSC = \int_{\nu-1}^{\nu} e^{-r\nu} d\nu = \frac{e^{-r\nu} - e^{-r(\nu-1)}}{-r} \qquad (59)$$

where ν is the index beginning with the lease date.

After tax net present value (ATNPV) is calculated by subtracting the present value of investment and lease acquisition cost from the discounted stream of annual after tax values and adding the discounted value of salvage and working capital. The after tax net present value represents the net worth of the lease and the residual economic rent to the resource.

Once the after tax net present value is determined for a particular q_o, other output variables associated with that ATNPV are stored. The model then checks to determine if all q_o or T values have been evaluated. If not, the model returns to the beginning of the q_o-T loop and repeats the procedure outlined above. If all possible T values or all input q values have been evaluated, the model then selects the optimal q_o-T combination for each Monte Carlo iteration; the optimal set being the one with the highest ATNPV.

This optimal ATNPV is then compared with the potential tax write-off calculated earlier during the exploration phase. If the ATNPV is greater than the potential tax write-off, the optimal ATNPV value is stored as the result for this iteration. If the potential tax write-off from not developing the lease is greater than the potential gain from developing the exploration loss is entered into the after tax net present value register. A zero is entered into the register for other output variables such as production, production time horizon, profit share, royalty, and tax. This result corresponds to the real world situation in which some quantity of resource is discovered during the exploration phase, but the economics dictate that the quantity is so small that it is not commercial and the lease is not developed.

Monte Carlo Results and Model Outputs

With the final values of all output variables determined for this Monte Carlo iteration, the model then checks to see if all Monte Carlo iterations specified have been completed. If not, the model returns to the beginning of the Monte Carlo simulation and repeats the entire process. If all the Monte Carlo iterations have been completed, then the mean, standard deviation, and other statistics on each output variable are calculated. If desired, histograms can be constructed for after tax net present value (ATNPV) and reserves. The histograms illustrate the distribution of output for these two variables. The distribution of after tax net present value provides the range of potential outcomes and the frequency with which each outcome occurs.

In the above described model, economic rent is composed of royalty and profit share payments, tax payments, and the after tax net present value (ATNPV). These rent components can be manipulated to determine expected bidding behavior and associated impacts for various leasing policy alternatives. For example, in a bonus bidding system with a fixed royalty rate, the expected bonus bid is a function of after tax net present value.[25] The sum of the bonus bid, royalty income, and taxes is equal to total government revenue.

<u>Expected Value and Variance of ATNPV</u>

The data input for lease exploration may require more explanation. Two items are required, the mean and standard deviation of no resource being present.

Exploring a lease should be considered an experiment with events, a wet lease (w) and a dry lease (ω). The random variable of interest in the data input is the number of dry leases (X). The variable X has a binomial distribution. The expected value of X is simply the number of experimental trials (n) times the probability of dry (p). Therefore, E(X) = np, and the variance is V(X) = np(1-p). In our case only one lease is explored, thus n is equal to 1.

The income from the experiment (I) is the number of dry leases (X) times the value of a dry lease (V_1) plus the number of wet leases times the value of a wet lease (V_2). Thus,

$$I = XV_1 + (n-x)V_2 \tag{60}$$

Expected income (considering that n is 1 and X and V_2 are independent) is:

$$E(I) = E(X)V_1 + (1-E(X))\ E(V_2) \tag{61}$$

which is:

$$E(I) = pV_1 + (1 - p)\ E(V_2) \tag{62}$$

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[25] Actual bonus bids are a result of bidding strategies formulated from game theoretic approaches combined with bidders estimates of lease values.

The variance of income is:

$$V(I) = V_1^2 V(X) + n^2 V(V_2) - [E(X)^2 V(V_2) + E(V_2)^2 V(X) + V(X)V(V_2)] \quad (63)$$

This reduces to:

$$V(I) = V_1^2 p(1-p) + V(V_2) - p^2 V(V_2) - E(V_2)^2 p(1-p) - p(1-p)V(V_2) \quad (64)$$

Social Value

Occasionally model users are interested in adding to the net present value of the investment the present value of the net payments to the government. Two related questions arise in this context. What is the appropriate interest rate to discount cash flows with in this analysis? How is the calculation affected by taxation of the returns to capital?

If one wishes to use a market related interest rate as an indicator of the social time preference rate, one often looks at pre-tax market rates. However, the pre-tax rates arise in a market that has taxation. In a market without taxation a new (and lower) equilibrium interest rate would result. One ad hoc method of approximating this rate would be to multiply the current interest by one minus the average of the marginal tax rates of market participants. If the tax rate entered by the model user for the firm is close to the rate reflected by the market, then the discount rate used in the model may be close to the desired approximation of the social time preference. The reader may note that the major question here relating to the taxation of capital lies not in the aspect of transfers to the government but rather in the appropriate discount rate.

When debt flows are explicitly included, the tax deductable aspect of the flows is explicitly entered. To be consistent with the above discussion (which assumed debt flows were being excluded) the tax deductability of interest, a benefit or reduced cash outflow, should not be netted out of the transfers to the government. Rather it should be viewed as the appropriate reduction of the interest rate to implicitly adjust to the desired social time preference rate.

Policy Options

A number of policy options are available for use with the generalized policy evaluation model. This section describes the major options and how they are utilized within the model.

The first set of options are called loop options in that each requires the model to be run iteratively to solve for the desired output variable. The first of these options is termed the delayed development (lag) loop and determines the optimum development time in situations where rising prices provide an incentive to delay development. The other three loop options involve determining the price, profit share, or royalty rate which would equate after tax net present value to zero. In addition to the loop options, a number of profit share, royalty, and variable rate options are available. Also, several advance royalty, depletion, and deferred bonus options may be used. These and other options are described below.

Delayed Development Loop

The delayed development loop option is most often used in conjunction with evaluation of advance royalty policy options. The effectiveness of advance royalty policies in deterring premature purchasing of leases can be evaluated by simulating lease development with expectations of rising prices and advance royalty policies in place. The development delay can be as long as eighteen years; hence, the model would be run iteratively with delays ranging from zero through the maximum delay to determine the year in which after tax net present value is highest--which is the year the lease would be developed. Lease delays can also be evaluated without utilizing the loop option by specifying a fixed length of delay before development of the lease begins.

Other Loop Options

The price loop is used to determine that price which is just sufficient to produce the resource at a specified rate of return (discount rate). This option can be utilized to determine the minimum price to produce given a set of cost inputs and expected production. One potential leasing policy which can be evaluated using this process is price bid leasing. In this approach, the government would lease resources to the private sector based on sealed bids of price to produce a specified amount of the resource. This option could be particularly useful in stimulating development of resources which have not been produced on a large scale in the past such as gas production from coal because price uncertainty may be a major factor inhibiting resource development.

The royalty and profit share loops determine the royalty or profit share rate at which after tax net present value is approximately equal to zero. These options can be used to determine the royalty or profit share bid for leasing systems in which the bid variable is the contingency rate. The profit share bid options can be used with any of the profit share systems which are included in the model.

The input variable V1 is used to calibrate the approximations to zero for the price, royalty, and profit share loop options. In other words, this variable sets the upper limit of an acceptable "zero" value. Hence, when ATNPV falls between zero and V1, the model accepts this value as an appropriate approximation to zero. The default value for V1 is one million (used only when V1 is input as zero).

Profit Share Options

Five different profit share systems have been included in the model. These are termed the taxable income profit share system, the annuity capital recovery profit share system, the fixed capital recovery profit share system, the British system, and the modified net income profit share system. In the taxable income profit share system, the profit share base is specified as taxable income as defined by the United States Internal Revenue Service.

The annuity and fixed capital recovery profit share systems allow a return to capital to be subtracted from the taxable income base to calculate a revised profit share base. For the annuity capital recovery system, the initial investment plus interest during construction is converted to an annuity with a prespecified interest rate and length of recovery period. The value of this annuity is subtracted from the profit share base in each of the capital recovery years before a government profit share is taken.

For the fixed capital recovery system, the initial capital investment is multiplied by some prespecified factor and this capital recovery amount is subtracted from the taxable income base before any profit share is taken. In other words, the capital recovery amount is credited against taxable income in the early years of production until the capital recovery amount is exhausted. At that point, the full government profit share is taken until the lease is terminated.

The British profit share system is more complicated than the above systems. It involves both a royalty and a profit share (called a petroleum revenue tax, PRT) as well as corporate income tax.[26] The system also includes participation by the UK

[26]The British system may be evaluated with the following variable values: KPFS = 4, MDEPL = 0, MROYL = 1, NDEPR = 0, Omega = 0.0, TAXF = 0.0, STAXR = 0.0, PART = .51, RØYRT = .125, PFSHRT = .45, and BPF = 1.75. In addition, all investment for both exploration and development is considered tangible. These values are not current for 1982 as the system has changed considerably in recent years.

The UK government is planning to establish depletion controls to lengthen the production time horizon from the conventional 10 to 15 years to a 20-30 year period. Hence, users may want to set the variable ITMIN higher than the usual case.

government in offshore leases. Although the government participation rate is a variable (PART), the UK intends to establish majority participation of 51 percent in most cases. All costs and revenues are shared at this rate. A royalty (currently 12.5 percent) is collected on all production. At the discretion of the government, this royalty rate may be returned (tax free) on marginal or uneconomic fields. In this model, the royalty is returned in any year in which after tax value is negative. In addition to the royalty, a petroleum revenue tax (currently 45 percent) is assessed on net revenues. However, a number of exemptions or limitations on the tax are allowed. The first 7.3 million barrels (1 million tons) of production are exempt for the first ten years of production. In the early years of production, a fixed multiple (currently .75) of initial capital investment is subtracted from the PRT base as in the fixed factor capital recovery profit share system described above. In any year in which PRT causes pre-tax net income to fall below 30 percent of accumulated capital investment, the PRT is forgiven. Also, PRT is constrained not to exceed 80 percent of the amount by which annual pre-tax net profits exceed 30 percent of capital expenditure. All of these features are modeled for the British system. PRT calculations are modeled as profit share payments. Participation is modeled somewhat differently. Participation expenditures during exploration and development are included as negative taxes. Participation receipts are added to profit share (PRT) receipts. These procedures were adopted to avoid major increases in the size of the program for this option.

The fifth profit share system uses a modified net income base. It uses taxable income plus depletion and depreciation as the profit share base each year. It allows the deduction of all capital investment plus interest from the profit share base before a profit share is taken. Interest is also added to any carry-over of the capital recovery amount. The system is modeled using the annuity capital recovery framework with the annuity period set equal to one.

Royalty Options

Two different royalty options may be used. First, a fixed rate royalty system such as that currently being used by the United States may be specified. Second, a constrained royalty system such as that in use in India and Indonesia may be selected. This system is more commonly (and accurately) called a production sharing system. It allows cost to be recovered from a portion of the oil produced (which is termed cost oil) with the remainder of the oil (termed profit oil) divided between the government and the private contractor. When this option is specified, input variables needed to define the system are determined exogenously, including production sharing rates and cost recovery schedules.

Variable Rate Options

For both royalty and profit share systems, a variable rates (rather than fixed rates) may be specified. In particular, variable profit share rate may be specified with any of the three profit share systems. Variable royalty rates based on both production and value may be specified. In each case a minimum rate, maximum rate and rate adjustment factor are input. In addition the value of production, production level, or net income level for which the minimum rate applies is also input. The minimum rate applies to production or income levels up to the prespecified amount, and the rate is adjusted according to the adjustment factor up to maximum rate for levels above this amount.

Advance Royalty

Another royalty option which may be specified is advance royalty. Advance royalty may be calculated in one of three forms: (1) a specific advance royalty calculated using a rate per unit of resource produced, (2) a specific advance royalty calculated using the initial resource price and the advance royalty rate throughout the life of the lease, and (3) an ad valorem advance royalty using the resource price for each year of the lease. The advance royalty rate may be the same as or different from the basic royalty rate. The advance royalty option involves a complex set of sums of advance royalty payments and basic royalty payments which are checked against each other in the royalty calculation for each year of the lease. Advance royalty is payable only through the prespecified lease life. The production for advance royalty payments is calculated from the prespecified lease life and reserves found on the tract.

Depletion

Another policy option relates to the method used for depletion. Three options are allowed for depletion: (1) cost depletion, (2) revenue depletion, and (3) the highest of cost or revenue depletion in each year. In addition no depletion may be selected. Revenue depletion is currently not allowed for oil and natural gas for major producers but is allowed for coal and other resources. Cost depletion is calculated for the lease acquisition cost and the lease bonus. Revenue depletion is calculated as a percentage of gross revenue not to exceed 50 percent of net income in each year.

Deferred Bonus

Another policy option allows for deferred payment of the bonus. This option changes the absolute value of the bonus because the payments are spread out into the future. For example, suppose the normal initial bonus is estimated to be $1000. The absolute value of that bonus paid in five equal installments at 10 percent interest (one at the beginning of the lease and four annual installments) is $1199.08 with five installments of $239.82. This conversion of the bonus amount is made and adjustments are also provided in the depletion calculations to compensate for the deferred payment schedule. The length of the bonus deferral period is an input.

Other Policy Options

Another possible use of the model is to evaluate public and private sector revenues with different discount rates. The model may be run with the same rate used for all calculations or with one rate used for private sector calculation of after tax net present value and another rate used in calculating the present value of royalty, profit share, and taxes which are components of government revenue.

Another option which may be of interest is the price guarantee. With this option, the market price is simulated independently and if the price falls below a prespecified rate, the government guarantees the price at this prespecified rate. In other words, the government guarantees to purchase all production at the prespecified price or to subsidize production to achieve at least that price for all sales. Both the effects on development of resources and the cost to the government of this option can be calculated.

Another price related option is the price subsidy. With this option, the subsidy value is added to the market price to obtain total revenue per unit for the producer.

The price subsidy and minimum price policies conceptually could be either nominal or real. In practice this would mean that a guaranteed minimum price or price subsidy could be specified at a fixed nominal level or indexed to change with the rate of inflation. In this model, both are assumed to be fixed nominal values. In other words, a $30 price minimum stays at $30 regardless of what happens to oil price.[27]

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27 To change to indexed price subsidy and minimum price use a dummy control variable (KDUM1 or KDUM2) and make the following change at line 81 in the PRICES subroutine:

```
IF (KDUM1.EQ.1) PSUB = PSUB * EXP(RINFL)*(YEAR-1.))
IF (KDUM1.EQ.1) PMIN = PMIN * EXP(RINFL)*(YEAR-1.))
```

An investment subsidy option also is provided. With this option the government guarantees to pay a certain percentage of the investment cost. Again, both the effects on development and the cost to the public of this option can be calculated.

For all the policy options and for the normal tax policies not discussed above any rates desired may be specified. Additional policy options which involve changes in rates of existing or proposed policies can be evaluated. For example, the effects of changing the investment tax credit from 10 to 15 percent could be easily evaluated. Similarly, the effects of changing the royalty rate from .1667 to .3333 could be simulated.

In addition to the above options, a number of options for determining investment cost, operating cost,installed capacity and the form in which output and input variables are to be specified are provided. For example, operating costs may be calculated as a function of installed capacity, annual production, or the average of the two.

Output Options

With respect to outputs, histograms may be specified for after tax net present value and reserves. A number of print options are allowed with varying degrees of detail. Annual output values are printed with the results for the mean values case. The model may be run with or without Monte Carlo simulation.

Internal Rate of Return

Instead of using an input discount rate to calculate present values, the internal rate of return (IRR) may be calculated. Internal rate of return is that interest rate which equates the stream of revenues and costs to zero as shown in equation (65):

$$o = \sum_{\nu=1}^{T} \frac{ATV\nu}{(1+r)\ (\nu-.5)} \qquad (65)$$

where ATV is after tax value in year ν and r is the internal rate of return.[28] For the mean values case, one IRR is calculated, and

[28] For some ATV streams, more than one interest rate may solve equation (65). The model selects the first solution as the IRR. For a more complete discussion of the internal rate of return see Mishan, pp. 215-57.

Also note that the model uses the continuous rather than the discrete form of the IRR equation shown in equation (65).

for the Monte Carlo simulation, the distribution of IRR is determined and a histogram produced.

Annual Value Distributions

Another output option which may be selected is annual distributions of gross revenue, taxable income, taxes, royalty, profit (or production) share, and production. For this option, the annual outputs for the optimal installed capacity are saved for each Monte Carlo iteration. After the simulation is completed, distributions of the above output variables are obtained for each year of development and production.

Model Summary

Clearly a wide range of leasing policy options including bonus bidding systems, profit share systems, the Indian production sharing system and a number of combinations of these systems and their many variants may be analyzed with the generalized leasing model. In addition to the wide range of leasing policy options, a number of other policy options such as tax policies, price subsidies, purchase guarantees, price supports, investment subsidies and other policy options designed to increase certainty for private subsidies and other policy options designed to increase certainty for private investors are included in the model. Furthermore, other tax policy, general policy, or leasing policy options can easily be incorporated in the model framework.[29] Hence, the model is ideally suited for analysis of a wide range of government alternatives dealing with the disposition of publicly owned natural resources.

Outputs of the basic model include statistics on the following variables: production time horizon, installed capacity, present value of royalty payments, present value of depletion, present value of taxes and taxable income, present value of profit share or production share payments, production, reserves, present value of investment and operating cost, after tax net present value, and the present value of gross revenue. Additional outputs are provided for specialized leasing or other policy options such as the royalty bidding system.

The use of Monte Carlo simulation with uncertain variables provides an additional dimension to government policy analysis. Not only can the change in expected value of model outputs be determined when a policy variable is changed, but also the change in variance of the model outputs can be determined. The simulation process more closely approximates the decision making procedure used in the private sector when evaluating potential resource investments.

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[29] Since the model is well structured with numerous subroutines, additional policy options beyond those described above can be added.

APPENDIX

GENERALIZED RESOURCE POLICY MODEL--PROGRAM INPUT SEQUENCE (VERSION GEN3)

This appendix contains operating instructions for the Generalized Resource Policy Evaluation Model. It is designed to provide the basic information needed to successfully utilize the model for resource policy analysis.

The model has been successfully implemented on Control Data, IBM, and Honeywell computers. It is written in standard FORTRAN and has been compiled on two different optimising compilers. Any user having difficulty implementing the model on a different system should contact one of the authors.

Control variables

The control variables determine the options which are used in analyzing the data and calculating and printing the results. Although the definitions of the control variables are provided in the program input sequence (see below), it may be helpful to review the meanings of some of them in more detail. The first card contains the number of cases to be evaluated -- up to 99 cases per run are allowed. A complete set of data cards including the control card must be included for each case.

Print options

The first control variable determines what output will be printed. Acceptable values for this variable range from one to five with more output being printed at higher numbers. If the control is set at one, only the ATNPV mean and standard deviation are printed. When the control is set at two, the mean and standard deviation of all output are printed. At a value of three, the mean, standard deviation, and coefficients of skewness and kurtosis are printed for all output. For a value of four, all of the above plus the detailed results for the mean values case are printed. At a level of five, all detailed output for each Monte Carlo iteration plus all output statistics are printed. Users are cautioned that if a high number of iterations are specified, this option involves a large volume of printing.

Internal rate of return

IDISC, the discount control variable may be used to specify the internal rate of return calculation. When internal rate of return is calculated, a discount rate must still be provided for the installed capacity (Q_O) optimization procedures. In other words, the optimal installed capacity is determined using the input discount rate (R), and the undiscounted stream of values for after tax value are fed into the internal rate of return calculation. If desired, this process could be repeated with the internal rate of return from the first trial used as a discount rate to determine if any significant difference results in the optimization procedure.

Annual output statistics

The IVEC control variable when set equal to one stores annual outputs for each iteration and calculates statistics on output variables for each year of the lease. Because this procedure requires a large amount of core, two versions of the GEN3 model are used. The basic version contains a dummy subroutine for the vector statistics and can be used only when IVEC equals zero. GEN3 Extended contains the complete IVEC subroutine and should be used only when IVEC equals one to minimize cost. The extended version may be used for both values, however, if computer cost is not a concern.

Deferred investment with production change

The INVC equal two option for deferred investment is inoperative in this model version. It has been provided for future changes which will allow deferred investment to change the rate of resource recovery and level of operating cost.

Number of iterations

The GEN3 model allows a maximum of 200 Monte Carlo iterations. These iterations determine the value of a lease given that resources are found. The number of effective iterations is NLØØP/(1 - DTRSK). If DTRSK equals zero, the number of iterations is NLØØP. If DTRSK equals .5, the effective number of iterations is 2*NLØØP. If a greater or smaller number of iterations are desired, the STAT and IRR common areas and the dimension statements for R, A, RIR2, and LQ in SETAR, STATIC, IRSHFT, and STDMN subroutines would have to be changed to increase the vector sizes.

Potential problem areas

As with any complex computer program, there are a number of possible problems what may be encountered by program users. Three such potential problem areas are untested options, illegal variable values or combinations, and the random number generators.

Untested options

First of all, it should be stressed that, as of this writing, all possible options have not been thoroughly tested. The most commonly used options such as the royalty and profit share system options have been tested, but some of the new options such as overlapping production and development have not been throoughly evaluated under all conditions.

Illegal variable values

Also, there may be variable values or combinations of values that will cause the program not to work. Some of these values have been determined and corrections included in the program. For example, the program will not accept a discount rate of minus one. The program changes any minus one input for discount rate to zero. Similarly, if depreciation is specified (NDEPR > 0), the depreciation lifetime must be greater than zero. If depreciation lifetime is input as zero, the model sets depreciation method (NDEPR) to zero also.

The input variables for chance of no resource being present should be set carefully. Normally, the chance of no resource being present would be considered a Bernoulli variable because the condition in nature can take only two values: (1) resources are present, or (2) no resources are present. The mean of this distribution is the probability of resources being present, p, and the standard deviation is p(1-p). From Table 4 in the body of the text, it is clear that this is the binomial distribution with n = 1.

If other distributions are used, users should remember that this variable represents a probability so it must take a value between zero and one. Hence, the mean of the chance of no resources distribution must have a value between zero and one. The standard deviations should be calculated directly from the parameters of the assumed input distribution. For example, the mean and standard deviation of an assumed triangular distribution should be calculated from the distribution parameters (min, mode, and max values). If the triangular distribution has a min of 0, mode of .1 and max of .5, the mean is .2 and the standard deviation is .11. The triangular distribution is often chosen because it is easily bounded. Other distributions (especially those for which the variable can take on negative values) could be more of a problem. For example, if the normal distribution were used, the standard deviation should be small enough that the mean plus or minus three times the standard deviation falls within the range zero to one. For example, if the mean is .5, the standard deviation should be no larger than .16. For any distribution chosen, the input values should provide a high probability that any value selected from the distribution would fall within the range zero to one. Otherwise, the distribution is conceptually inappropriate.

Random number generators

It is hoped that past problems with random number generators have been largely solved with this version. We have included three different normal random variate generators (GGNØR, NØRM2, and NØRM3). Each of the generators may be used for both prices and reserves. At least two of these generators should work on all computer systems. All the generators give identical or near identical results on CDC and IBM computers. In addition, the uniform generator, GGUB, should provide consistently good results on most systems. Generators which do not work on a given system may be replaced or simply not used. Also, the random number generators may produce biased distributions when used with some seeds. Seeds which have produced good results in the past are included as defaults in the program.

Inputs

The inputs consist of a case card (number of cases), control card, plus seven to twenty data cards depending on the options being used. Variables are arranged by general topic. The first data card (card 3) is the case description which is printed at the top of the output heading. Card 4 contains price variables, and card 5 contains geologic inputs. Tax variables are contained on card 6, economic factors on card 7, and cost information on card 8. Most of the optional cards numbered 9 relate to the various royalty and profit share options. Optional card 9d is used if random number generator seeds other than the defaults are desired. Cards 10 and 11 are the Q_o (installed capacity) cards -- one is used for each capacity to be evaluated (up to ten). If installed capacity is determined endogenously, only one q_o set is used. Cards 10-11a are used for deferred investment, and optional cards 12 are used if annual operating cost is input.

For all data and control variables, a blank is read as a zero (on some computer systems blanks are printed as -0). Variables which are not used need not be punched.

Outputs

As described above, five different print levels may be specified. Three of these pertain to the amount of simulation output statistics which are calculated and printed, a fourth prints all output statistics plus detailed results of the mean values case and the fifth specifies printing results of each M.C. iteration. This section describes the print for each iteration and the highest level of print for simulation statistics.

Heading

The heading contains input variables for each case. The case title is printed first followed by the resource name. Control options and policy options in use for the case are printed next. Input variables are printed in the same groupings that were used for card inputs. If the Indian system is used, input data for this system is printed next. Finally, the input data for each installed capacity is printed including deferred investment if that option is exercised.

Iteration print

The first item printed for each iteration is the iteration number (or mean run heading). Next, prices for the primary and secondary resource (if SPØ is greater than 0) are printed. The remainder of the interation print is repeated for each installed capacity (whether input or determined endogenously).

The actual print differs for the Indian system and other systems. In both cases, a series of annual values are printed followed by a summary of present value results. For the Indian system the following annual outputs are printed: gross revenue, cost oil, profit oil, foreign tax, government share, private share, income tax, and after tax value. For other systems, the following annual outputs are printed: gross revenue, depletion, depreciation, state severance and income tax, royalty or profit share payments, net revenue (taxable income), federal tax, and after tax value.

The same summary results are printed for all systems as follows:

RC = investment cost per unit
RK1 = operating cost per unit
TAXW = amount of the potential tax write-off if the lease is not developed
EXX = exploration expenses (PV)
RIVTC = investment tax credit
RNTLS = rent payments during exploration (PV)
XLØSS = exploration loss (PV)
RCAP = reserves found
NYMAX = maximum economic lifetime
FTAX = federal tax payments (PV)
YEAR = lease life for this installed capacity (lag plus production period)
Q = installed capacity
RNPVLM = royalty payments (PV)
PFSRAT = profit share rate (for variable profit share systems, the rate in the last year)
PVDEP = depletion (PV)
TAXINC = taxable income (PV)
SSTAX = total state income and severance taxes (PV)
ATNPV = after tax net present value
PRØD = total production
TRANIV = investment cost (PV)
TCØST = total investment plus operating cost (PV)
TDEPPD = total depreciation (PV)
SALVG = salvage value
PRFSHR = profit share payments (PV)
ØPCØ = total operating cost (PV)
ØPC = operating cost in the last year of production
SALVAL = present value of salvage during production period (for deferred investment)
VALUE = gross revenue (PV)
IT = production time for input installed capacity
FCAP = annuity value for the annuity capital recovery profit share system
WPTAX = windfall tax payments (PV)

Simulation output

Output distribution statistics are calculated for total expected value and for value given that resources are found. For total expected value, the mean, standard deviation, and standard error of the mean are calculated and printed (if desired). For value given that resources are found, these statistics plus the coefficient of skewness and coefficient of kurtosis may be calculated. Total expected value is calculated for after tax net present value (ATNPV), taxes (PV), royalty (PV), depletion (PV), profit or production share (PV), reserves, gross value (PV), and production. Value if resources are found for these variables includes both cases in which the resource is developed and situations in which resources are found but no development ococurs (for economic reasons).

Statistics are also calculated on a number of output variables for the cases (iterations) in which development occurs. Development statistics are calculated on reserves, production, installed capacity (Q_O), investment cost per unit of installed capacity (B), operating cost per unit (K_1), production time horizon (T), production of the second resource (such as associated gas), and the present value of operating cost, investment cost, and taxable income. If no development occurs, these statistics are deleted.

Following the above output statistics, the exploration loss, tax write-off if not developed, and net value from exploration are printed. The number of iterations developed and the development percentage are also printed. ATNPV and reserve histograms are the last output.

Conclusion

With these operating instructions, users of the generalized resource policy model should encounter little difficulty. However, the model is not simple; users should thoroughly review the model description before attempting to utilize it. The authors are available for consultation should any problem arise.

The table that follows provides the program input sequence for the generalized resource policy model. It contains the card spaces, format, variable name, and brief definition for each variable used in the program.

Generalized Resource Policy Model--Program Input Sequence (version GEN3)

Card Spaces	Format	Variable Name	Data Element
Case card (card 1):			
1-2	I2	NCASE	Number of cases
Control card (card 2):			
1-2	I2	NPRINT	Print and statistics options-- 1=ATNPV mean and S.D. only 2=mean and S.D. of all output 3=all statistics on all output 4=all statistics on all output plus results for the mean values case 5=all output including results for each M.C. iteration
3-4	I2	LPØVER	Royalty, price, profit share, or lag loop options-- 0=no loops 1=lag loop 2=price bid loop 3=royalty bid loop 4=profit share bid loop
5-6	I2	NMRES	Resource code-- 1=offshore oil 2=offshore natural gas 3=onshore oil 4=onshore gas 5=oil shale 6=coal 7=uranium 8=geothermal
7-8	I2	NHIST	Histogram options-- 0=none 1=ATNPV distribution only 2=ATNPV and reserve distributions 3=cumulative and the standard ATNPV 4=cumulative and standard for ATNPV and reserves

Card Spaces	Format	Variable Name	Data Element
9-10	I2	NPMETH	Price generation method-- 0=mean price change used 1=random selection of annual price change-GGNØR 2=random selection with NORM2 3=random selection with NORM3
11-12	I2	NCM	Investment cost (contingency) method-- 0=mean of contingency distribution 1=random selection 2=most likely investment cost (mode) 3=base investment cost (contingency=0)
13-14	I2	NKM	Operating cost contingency method--same as for NCM
15-16	I2	KRS	Reserve distribution-- 0=lognormal (GGNØR) 1=normal-GGNOR 2=normal-NORM2 3=normal-NORM3 4=mean value used
17-18	I2	NQØ	No. of installed capacities to be evaluated-- 0=variable installed capacity (Q_o) 1-10=no. of Q_o cards
19-20	I2	MDEPL	Depletion method-- 0=no depletion 1=cost depletion 2=revenue depletion 3=select best depletion method each year
21-22	I2	MRØYL	Royalty method-- 0=no royalty 1=basic royalty 2=Indian-Indonesian type constrained royalty

Card Spaces	Format	Variable Name	Data Element
23-24	I2	KPFS	Profit share method-- 0=no profit share 1=taxable income profit share 2=annuity capital recovery profit share 3=fixed capital recovery profit share 4=British PRT system 5=modified net income profit share 6=modified annuity with no interest on buildup of capital account; NCAP=1 & RI=U may be used 7=new U.S. profit share regulations
25-26	I2	KVAR	Variable rate control-- 0=no variable royalty or profit share 1=variable profit share 2=variable royalty-production 3=variable royalty-value 4=reciprocal V..R.-value 5=semi-log V.R. 6=sliding scale V.R.
27-28	I2	KSEED	Seed Input Control-- 0=use seeds in program 1=read in seeds
29-30	I2	MBØN	Bonus method-- 0=normal bonus calculations other=deferred bonus years
31-32	I2	KCRL	Operating cost control-- 0=cost times Q_o 1=cost times PRØD 2=cost times $(Q_o + PROD)/2$ 3=input operating cost
33-34	I2	INVC	Investment cost control-- 0=inoperative 1=deferred investment 2=deferred investment with change in operating cost and production rate (enhanced recovery)

Card Spaces	Format	Variable Name	Data Element
35-36	I2	IDISC	Discount control variable-- 0=one private discount rate used 1=private and public rates used 2=IRR calculated
37-38	I2	IVEC	Vector storage control-- 0=no output vectors 1=annual output vectors
39-40	I2	KINP	Input control for discount rates, and price and cost increases: 0=real inputs 1=inputs are nominal
41-42	I2	KOUTP	Output control: 0=analysis is performed and outputs produced in real terms 1=analysis is performed and outputs produced in nominal terms
43-44	I2	KPROD	Production decline control: 0=exponential decline at rate A 1,2,3=reserved for other decline approaches to be added
45-46	I2	KFIN	Financial accounting method: 0=corporate (constant debt/equity) 1=project (declining debt/equity ratio as the debt on the project is retired)
47-48	I2	KDEV	Input control for development time values: 0=read in new values 1=use default development time values (for 5 year development period)
49-50	I2	KDUM1	Dummy control variable not presently in use, but which can be used in user modifications to GEN3

Card Spaces	Format	Variable Name	Data Element
51-52	I2	KDUM2	Same as for KDUM1
53-56	I4	NLØØP	Number of Monte Carlo iterations
57-60	I4	MCR	Number of M.C. iterations used for further bonus approximation if desired (if MCR=0, bonus after mean run is used)

Data cards:

Card 3 - title card

1-80	20A4	TITL	Case description (blank card may be used)

Card 4 - price card

1-5	F5.2	PPO	Initial primary resource price
6-10	F5.2	SPO	Initial secondary resource price
11-15	F5.3	P1M(1)	Primary resource-price change mean
16-20	F5.3	PSD(1)	Primary resource-price change std. dev.
21-25	F5.3	S1M(1)	Secondary resource-price change mean
26-30	F5.3	SSD(1)	Secondary resource-price change std. dev.
31-32	I2	NPP	Number of primary resource price change distributions
33-34	I2	NPS	Number of secondary resource price change distributions
35-36	I2	LPMIN	Length of time primary resource minimum price is valid
37-38	I2	LSMIN	Length of time secondary resource minimum price is valid
39-43	F5.2	PMIN	Minimum price for primary resources
44-48	F5.2	SMIN	Minimum price for secondary resource
49-53	F5.2	PSU8	Price subsidy for primary resource

Card Spaces	Format	Variable Name	Data Element
Card 4 - optional price card			
1-2	I2	NP(1)	End of period for 1st P1M distribution
3-4	I2	NP(2)	End of period for 2nd P1M distribution
5-6	I2	NP(3)	End of period for 3rd P1M distribution
7-8	I2	NP(4)	End of period for 4th P1M distribution
9-10	I2	NS(1)	End of period for 1st S1M distribution
11-12	I2	NS(2)	End of period for 2nd S1M distribution
13-14	I2	NS(3)	End of period for 3rd S1M distribution
15-16	I2	NS(4)	End of period for 4th S1M distribution
17-20	F4.2	P1M(2)	Mean of second primary resource price change distribution
21-24	F4.2	PSD(2)	Std. dev. of second primary resource price change distribution
25-28	F4.2	P1M(3)	Mean of third primary resource price change distribution
29-32	F4.2	PSD(3)	Std. dev. of third primary resource price change distribution
33-36	F4.2	P1M(4)	Mean of fourth primary resource price change distribution
37-40	F4.2	PSD(4)	Std. dev. of fourth primary resource price change distribution
41-44	F4.2	S1M(2)	Mean of second secondary resource price change distribution
45-48	F4.2	SSD(2)	Std. dev. of second secondary resource price change distribution
49-52	F4.2	S1M(3)	Mean of third secondary resource price change distribution
53-56	F4.2	SSD(3)	Std. dev. of third secondary resource price change distribution
57-60	F4.2	S1M(4)	Mean of fourth secondary resource price change distribution
61-64	F4.2	SSD(4)	Std. dev. of fourth secondary resource price change distribution

Card Spaces	Format	Variable Name	Data Element
Card 5 - geologic			
1-4	F4.2	A	Production decline rate (%)
5-6	I2	IFLATP	Length of flat production period
7-8	I2	ITMIN	Minimum production period (not including development)
9-10	I2	ITMAX	Maximum production period (not including development)
11-20	E10.2	RMEAN	Mean of reserve distribution
21-30	E10.2	RSTD	Std. dev. of reserve distribution
31-40	E10.2	ACRES	Numbers of acres in lease area
41-45	F5.3	DTRSK	Mean chance of no resource being present
46-50	F5.3	DTSD	Std. dev. for no resources distribution
51-55	F5.3	GEO	Geologic factor 1 -- for oil, GEO (1) = AGFAC = associated gas factor -- for coal, GEO (1) = SULF = coal sulfur content (%) -- for gas GEO (1) = AGFAC = NGL factor
56-60	F5.3	GEO (2)	Geologic factor 2 -- for oil, GEO (2) = WELLS = number of exploratory wells/ 1000 acres -- for coal GEO (2) = ASH = coal ash content (%)
61-65	F5.3	GEO (3)	Geologic factor 3 -- for oil, GEO (3) = BETA = geologic decline factor -- for coal, GEO (3) = H2Ø = coal water content (%)
66-70	F5.0	GEO (4)	Geologic factor 4 -- for oil GEO (4) = H2OD = water depth -- for coal, GEO (4) = COHT = coal height (seam thickness)
71-75	F5.0	GEO (5)	Geologic factor 5 -- for coal, GEO (5) = ØBHT = height of overburden
76-80	F5.0	GEO (6)	Geologic factor 6 -- for coal, GEO (6) = BTU = coal energy content (BTUs/pound)

Card Spaces	Format	Variable Name	Data Element
Card 6 - tax related			
1-2	I2	N	Depreciation lifetime
3-4	I2	NDEPR	Depreciation method-- 0=no depreciation 1=sum of year's digits (SYD) 2=double declining balance with switch over to straight line (DDB) 3=straight line (SL) 4=depreciation based on the rate of resource depletion (annual prod./total prod.) 5=SYD with N=production lifetime 6=DDB with N=production lifetime 7=SL with N=production lifetime 8=ACR method (U.S.)
5-8	F4.2	SLVGPC	% investment salvagable
9-12	F4.2	ØMEGA	Investment tax credit rate
13-16	F4.2	PHI	U.S. income tax rate
17-20	F4.2	TAXF	Foreign tax rate
21-24	F4.2	STR	Severance tax rate
25-28	F4.2	STAXR	State income tax rate
29-32	F4.2	TX(1)	Proportion of exploration expense occuring each year which is tangible
33-36	F4.2	TX(2)	Proportion of exploration expense occuring each year which is tangible
37-40	F4.2	TX(3)	Proportion of exploration expense occuring each year which is tangible
41-44	F4.2	WPRATE	Windfall profits tax rate
45-48	F4.2	WPPADJ	Annual adjustment factor for windfall profits tax base price
49-53	F5.2	WPBASP	Windfall profits tax base price
54-58	I5	LASTWP	Last year of windfall profits tax
59-62	F4.2	Z	Depletion rate for gross depletion

Card Spaces	Format	Variable Name	Data Element
Card 7 - economic factors			
1-5	F5.3	REQ	After tax return on equity
6-10	F5.3	RINT	Borrowing (debt) rate of interest
11-15	F5.3	RINFL	Inflation rate
16-19	F4.2	EQ	Fraction of investment which is equity capital
20-23	F4.2	RR	Public sector discount rate
24-27	F4.2	BFAC	Bonus factor for ATNPV
28-37	E10.2	BØNUS	Initial bonus
38-47	E10.2	BCØN	Bonus calculation constant
48-55	F8.0	V1	Price, royalty, and profit share loop calibration factor
56-59	F4.2	SUBINV	Investment subsidy--% of investment is subtracted from investment
60-63	F4.2	BYPRCD	By-product credit--value of production of additional resources is credited as a percent of primary resource price
64-65	I2	LEXLOR	Exploration time period
66-67	I2	LAGD	Maximum length of development period including any development delay (years)
68-71	I4	ISALYR	Year of lease sale (calendar year)
72-73	I2	KP1	No. of years from the beginning of investment to the beginning of production - if KP1=0, the program sets production to begin in LAGD +1 year - KP1 must be less than or equal to LDEV +1 - if, for example, KP1=3 and LDEV=5, production begins <u>during</u> year 3 of a 5 year development period
Card 8 - cost related			
1-4	F4.2	CMIN	Minimum value for investment cost contingency distribution
5-8	F4.2	CMAX	Maximum value for investment cost contingency distribution
9-12	F4.2	CMØDE	Most likely value for investment cost contingency distribution

Card Spaces	Format	Variable Name	Data Element
13-16	F4.2	KMIN	Minimum value for operating cost contingency distribution
17-20	F4.2	KMAX	Maximum value for operating cost contingency distribution
21-24	F4.2	KMØDE	Most likely value for operating cost contingency distribution
25-28	F4.2	THETA	Rate of change in operating cost
29-32	F4.2	WCF	Working capital factor--multiplied by first year's operating cost to determine working capital value
33-34	I2	LCLM	Investment cost control variable (climatic region for OCS costs)
35-44	E10.2	EXPWLL	Cost per exploratory well
45-49	F5.2	RENT	Annual rent per acre
50-53	F4.2	FX(1)	Proportion of exploration expense occurring each year
54-57	F4.2	FX(2)	Proportion of exploration expense occuring each year
58-61	F4.2	FX(3)	Proportion of exploration expense occuring each year
62-70	F9.0	AQCST	Lease acquisition cost (excluding the bonus)

<u>Card 9a</u> - royalty policy card (used only if MROYL > 0 & MROYL ≠ 2)

Card Spaces	Format	Variable Name	Data Element
1-6	F6.4	RØYRT	Royalty rate (or minimum royalty rate for variable royalty)
7-14	F8.6	RØYFAC	Factor used to change royalty rate with annual production or value levels (variable royalty) --the factor is divided by 100,000 in the program
15-18	F4.2	RRTMAX	Maximum royalty rate (variable royalty)
19-28	E10.2	PRØDF	Production (or value) allowable for minimum royalty rate (variable royalty)
29-30	I2	MADROY	Advance royalty method 0=No advance royalty 1=specific advance royalty calculated using CNT 2=specific advance royalty calculated using the <u>initial</u> resource price and the advance royalty rate 3=<u>ad valorem</u> advance royalty

Card Spaces	Format	Variable Name	Data Element
31-32	I2	LLIFE	Lease life--used to calculate required production level for advance royalty
33-34	I2	LAGR	Allowable development period (used for advance royalty)
35-38	F4.2	FR	Fraction of total reserves used in calculating production requirement for advance royalty
39-42	F4.2	CNT	Rate for specific advance royalty ($/ton)
43-46	F4.2	ALAMB	Advance royalty rate
47-50	F4.2	CHALMB	Change in advance royalty rate for each year production is delayed (LAGR+1)
51-54	F4.2	SCF	Scale factor

Card 9a-01 - optional royalty card 1 (Used when KVAR=6)

Card Spaces	Format	Variable Name	Data Element
1-5	F5.3	ROY1	Royalty rate for the separate step function variable royalty
6-10	F5.3	ROY2	Royalty rate for the separate step function variable royalty
11-15	F5.3	ROY3	Royalty rate for the separate step function variable royalty
16-20	F5.3	ROY4	Royalty rate for the separate step function variable royalty
21-25	F5.3	ROY5	Royalty rate for the separate step function variable royalty
26-30	F5.3	ROY6	Royalty rate for the separate step function variable royalty
31-35	F5.3	ROY7	Royalty rate for the separate step function variable royalty
36-40	F5.3	ROY8	Royalty rate for the separate step function variable royalty
41-45	F5.3	ROY9	Royalty rate for the separate step function variable royalty
46-50	F5.3	ROY10	Royalty rate for the separate step function variable royalty
51-55	F5.3	ROY11	Royalty rate for the separate step function variable royalty
56-60	F5.3	ROY12	Royalty rate for the separate step function variable royalty

Card Spaces	Format	Variable Name	Data Element
Card 9a-02 - optional royalty card 2 (used when KVAR=6)			
1-10	E10.2	RL1	Value production limit for the separate steps in step function variable royalty
11-20	E10.2	RL2	Value production limit for the separate steps in step function variable royalty
21-30	E10.2	RL3	Value production limit for the separate steps in step function variable royalty
31-40	E10.2	RL4	Value production limit for the separate steps in step function variable royalty
41-50	E10.2	RL5	Value production limit for the separate steps in step function variable royalty
51-60	E10.2	RL6	Value production limit for the separate steps in step function variable royalty
61-70	E10.2	RL7	Value production limit for the separate steps in step function variable royalty
71-80	E10.2	RL8	Value production limit for the separate steps in step function variable royalty
Card 9a-03 - optional royalty card 3 (used when KVAR=6)			
1-10	E10.2	RL9	Value production limit for the separate steps in step function variable royalty
11-20	E10.2	RL10	Value production limit for the separate steps in step function variable royalty
21-30	E10.2	RL11	Value production limit for the separate steps in step function variable royalty
31-40	E10.2	RL12	Value production limit for the separate steps in step function variable royalty

Card Spaces	Format	Variable Name	Data Element
Card 9b - profit share (used only when KPFS>0)			
1-4	F4.2	PFSHRT	Profit share rate (or minimum profit share rate for variable profit share)
5-8	F4.2	PRTMAX	Maximum profit share rate (variable profit share)
9-18	E10.2	PRTFAC	Factor used to change profit share rate with the level of net annual income (variable profit share)--the factor is divided by 100,000 in the program
19-28	E10.2	PSBASE	Allowable net income level for the minimum profit share rate (variable profit share)
29-32	F4.2	RI	Interest rate used for annuity capital recovery
33-36	F4.2	BPF	Capital recovery factor used when a British type profit share system is employed
37-38	I2	NCAP	Capital recovery payback period used with annuity capital recovery
39-42	F4.2	PART	British system participation rate
43-44	I2	KDEF	Control variable for using default 0=use default values for profit share 1=read in profit share variables 2=standard default values (not for profit share)
45-52	F8.0	PRODEX	Production exemption for U.S. profit sharing system
53-54	I2	IYEXMT	Last year of production exemption for U.S. profit sharing system
Card 9b-01 - optional (used only if KDEF equal 1)			
1-4	F4.2	CPQUAL (1,1)	Fraction of investment cost in development year i (i=1, 8) qualifying for overhead and ACR. Default values are .50 before production begins (LGD1+KP1-1) and .32 once production begins

Card Spaces	Format	Variable Name	Data Element
5-8	F4.2	CPQUAL (1, 2)	
9-12	F4.2	CPQUAL (1, 3)	
13-16	F4.2	CPQUAL (1, 4)	
17-20	F4.2	CPQUAL (1, 5)	
21-24	F4.2	CPQUAL (1, 6)	
24-28	F4.2	CPQUAL (1, 7)	
29-32	F4.2	CPQUAL (1, 8)	
33-36	F4.2	CPQUAL (2, 1)	Fraction of investment cost in development year i qualifying for ACR only. Default values are .50 before production begins and .64 once production begins
37-40	F4.2	CPQUAL (2, 2)	
41-44	F4.2	CPQUAL (2, 3)	
45-48	F4.2	CPQUAL (2, 4)	
49-52	F4.2	CPQUAL (2, 5)	
53-56	F4.2	CPQUAL (2, 6)	
57-60	F4.2	CPQUAL (2, 7)	
61-64	F4.2	CPQUAL (2, 8)	
65-68	F4.2	EXQUAL (1)	Fraction of exploration costs qualifying for overhead and ACR. Default value is .30.
69-72	F4.2	EXQUAL (2)	Fraction of exploration costs qualifying for ACR only. Default value is .65.
73-76	F4.2	OVRHD (1)	Allowable overhead during capital recovery period. Default is set equal to .04.
77-80	F4.2	OVRHD (2)	Allowable overhead after capital recovery period. Default is set equal to .10.

Card 9b-02 - optional (used only if KDEF equal 1)

Card Spaces	Format	Variable Name	Data Element
1-2	I2	IECP	Control for end of the capital recovery period 1=LAGD 2=LGD1 + KP1-1 (beginning of production) 3=when revenue exceeds costs (determined by the model). Default is 3.
3-6	F4.2	PCTDPR	Fraction of depreciation included in profit share base. Default equals 1.0.

Card Spaces	Format	Variable Name	Data Element
7-10	F4.2	PCTDPL	Fraction of depletion included in the profit share base. Default is set equal 1.0
11-14	F4.2	OPQUAL (1)	Fraction of operating cost qualifying for overhead and ACR. Default value is 0.8
15-18	F4.2	OPQUAL (2)	Fraction of operating cost qualifying for ACR only. Default value is 0.95.

Card 9c - Indian-Indonesian system (card is used only when MRØYL equals 2)

Card Spaces	Format	Variable Name	Data Element
1-2	I2	NYCT	Minimum time for cost recovery (years)
3-6	F4.2	CSTFAC	Maximum proportion of cost which may be recovered each year
7-10	F4.2	DOMRQ	Domestic requirement of profit oil
11-15	F5.3	PI(1)	Profit share rates (for private contractor) for each level of production
16-20	F5.3	PI(2)	Profit share rates (for private contractor) for each level of production
21-25	F5.3	PI(3)	Profit share rates (for private contractor) for each level of production
26-30	F5.3	PI(4)	Profit share rates (for private contractor) for each level of production
31-35	F5.3	PI(5)	Profit share rates (for private contractor) for each level of production
36-40	F5.3	PI(6)	Profit share rates (for private contractor) for each level of production
41-46	F6.0	PIL(2)	Production level for each rate step
47-52	F6.0	PIL(3)	Production level for each rate step
53-58	F6.0	PIL(4)	Production level for each rate step
59-64	F6.0	PIL(5)	Production level for each rate step
65-70	F6.0	PIL(6)	Production level for each rate step

Card Spaces	Format	Variable Name	Data Element
Card 9d - optional (used only when KSEED equal 1)			
1-5	I5	NPSEED	Seed for primary resource price change generation such as 47906
6-10	I5	NSSEED	Seed for secondary resource price change generation such as 34579
11-15	I5	NCSEED	Investment cost seed such as 77777
16-20	I5	NKSEED	Operating cost seed such as 23787
21-25	I5	NRSEED	Seed for reserve value generation such as 41687
Card 10 - Q_o card (repeated for each input Q_o; if NQØ=0, one card only)			
1-10	E10.2	Q	Installed capacity
11-16	F6.2	C	Cost per unit of installed capacity
17-21	F5.3	K	Initial operating cost
22-23	I2	LDEV	Length of the development period
24-27	F4.2	FD(1)	Fraction of development cost incurred in first year
28-31	F4.2	FD(2)	Fraction of development cost incurred in second year
32-35	F4.2	FD(3)	Fraction of development cost incurred in third year
36-39	F4.2	FD(4)	Fraction of development cost incurred in fourth year
40-43	F4.2	FD(5)	Fraction of development cost incurred in fifth year
44-47	F4.2	FD(6)	Fraction of development cost incurred in sixth year
48-51	F4.2	FD(7)	Fraction of development cost incurred in seventh year
52-55	F4.2	FD(8)	Fraction of development cost incurred in eighth year
56-59	F4.2	TD(1)	Fraction of development cost first year which is tangible
60-63	F4.2	TD(2)	Fraction of development cost second year which is tangible
64-67	F4.2	TD(3)	Fraction of development cost third year which is tangible
68-71	F4.2	TD(4)	Fraction of development cost fourth year which is tangible
72-75	F4.2	TD(5)	Fraction of development cost fifth year which is tangible
76-79	F4.2	TD(6)	Fraction of development cost sixth year which is tangible

Card Spaces	Format	Variable Name	Data Element
Card 11 - Q_o card two (Q_o cards 1 and 2 are repeated for each input Q_o; if NQØ=0, use one card set)			
1-4	F4.2	TD(7)	Fraction of development cost seventh year which is tangible
5-8	F4.2	TD(8)	Fraction of development cost eighth year which is tangible
9-10	I2	LBP	Length of production build-up period
11-14	F4.2	BP(1)	Fraction of initial (peak) production during each year of build-up period
15-18	F4.2	BP(2)	Fraction of initial (peak) production during each year of build-up period
19-22	F4.2	BP(3)	Fraction of initial (peak) production during each year of build-up period
23-26	F4.2	BP(4)	Fraction of initial (peak) production during each year of build-up period
Card (10-11)a - deferred investment (used only when IVC=1)			
1-2	I2	NNL(1)	Physical life -category 1
3-4	F2.0	AND(1)	Depreciation life-category 1
5-14	E10.2	ADB(1)	Amount of deferred inv.-category 1
15-18	F4.2	ASV(1)	Salvage value (%)-category 1
19-20	I2	NNL(2)	Same as above-category 2
21-22	F2.0	AND(2)	
23-32	E10.2	ADB(2)	
33-36	F4.2	ASV(2)	
37-38	I2	NNL(3)	Same as above-category 3
39-40	F2.0	AND(3)	
41-50	E10.2	ADB(3)	
51-54	F4.2	ASV(3)	
55-56	I2	NNL(4)	Same as above-category 4
57-58	F2.0	AND(4)	
59-68	E10.1	ADB(4)	
69-72	F4.2	ASV(4)	
Cards 12o - input operating cost (used only when KCRL = 3)			
1-8	40F8.0	OPCST(I)	Annual operating cost

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