LECTURES ON BIOECONOMICS
AND THE
MANAGEMENT OF RENEWABLE RESOURCES

by

Jon M. Conrad

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PREFACE

This paper contains five lectures prepared as part of a short course on mathematical ecology held at the International Centre for Theoretical Physics, Trieste, Italy, November 16 through December 10, 1982. The lectures review the relatively recent advances in dynamic modelling which attempt to synthesize both the biological and economic aspects of commercially exploited renewable resources. The basic bioeconomic model and the two empirical studies in Lecture V are concerned with the management of marine fisheries. Time precluded a discussion of similar models that might be used for managing other resources, such as timber, wildlife, and water.

In these lectures, time is partitioned into discrete intervals and the necessary conditions for dynamic optimization are obtained through an extension of the method of Langrange multipliers. It is possible (and perhaps easier) to formulate the optimization problems in continuous time and solve them using the maximum principal. The extension of the Lagrangian technique from static to dynamic problems is perhaps more comprehensible for students who have not been exposed to the calculus of variations or modern control theory. A thorough continuous-time treatment of the concepts in these lectures may be found in Mathematical Bioeconomics: The Optimal Management of Renewable Resources by Colin C. Clark.

The author wishes to thank Tom Hallam for his invitation to present these lectures and Si Levin for his comments and encouragement. Both are absolved of any errors or incomprehensibilities that remain.

Jon M. Conrad

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LECTURES ON BIOECONOMICS AND THE MANAGEMENT OF RENEWABLE RESOURCES

I. BIOECONOMICS

Within the past decade many economists have become interested in natural resource models which simultaneously consider economic flows, (such as cost and revenue), and population dynamics. Resource management is often cast as a problem in dynamic optimization where the management objective may be to maximize the present value of net benefits subject to the stock adjustments which result from growth, natural mortality, and man's harvesting activities. When the resource in question is a plant or animal, capable of regeneration, these resource models are called <u>bioeconomic models</u>.

The basic bioeconomic model assumes that the renewable resource in question can be adequately described by a single (state) variable measuring biomass; for example, metric tons of fish or cubic board feet of timber. While such models have the advantages of simplicity and mathematical tractability, they cannot take into account age or sex related attributes, nor multispecies interactions. In spite of such shortcomings, the basic model is a useful vehicle to introduce various biological and economic concepts.

With the resource stock described by a single state variable we could characterize the <u>change</u> in the resource by a differential or difference equation. Because of the discrete nature of data encountered in most empirical studies we will choose to partition time into uniform intervals or periods and the change in the stock of an unharvested resource will be given by

$$X_{t+1} - X_t = F(X_t), \tag{1}$$

where X_t denotes the resource stock (biomass) in period t. Equation (1) implies that the change in the stock from period t to period t+1 is dependent on the stock in period t. The function $F(X_t)$ will reflect factors affecting net growth of the resource and environmental carrying capacity. A famous specification for $F(X_t)$ is the logistic growth function which takes the form

$$X_{t+1} - X_t = rX_t\left(1 - \frac{X_t}{K}\right) \qquad (2)$$

where r is a positive constant referred to as the intrinsic rate of growth and K, also a positive constant, is the environmental carrying capacity. The logistic function is a symmetric function with roots at $X_t = 0$ and $X_t = K$, and with a maximum sustainable yield at $X_t = K/2$ (see Figure 1).

In a "pristine" system, undisturbed by man's harvesting activities, a species subject to the dynamics of the logistic growth function would tend toward the stable equilibrium at K. This is evident by examining (2) and noting

$$(X_{t+1} - X_t) > 0 \quad \text{for} \quad 0 < X_t < K$$

$$(X_{t+1} - X_t) = 0 \quad \text{for} \quad X_t = K$$

$$(X_{t+1} - X_t) < 0 \quad \text{for} \quad X_t > K$$

If X_t were not equal to K initially it would approach K asymptotically as is shown in Figure 2 where time path (a) is an approach path from an initial X in excess of K and time path (b) is an approach path from an initial X less than K.

There are many alternative specifications for the function $F(X_t)$. The logistic function belongs to a family of functions that is said to be "purely compensatory" and which generate a smooth and continuous yield response when the species is subject to commercial exploitation by man. The alternatives to purely compensatory models are models which exhibit depensation or critical depensation. These models will not yield smooth yield functions and admit the possibility of collapse and species extinction. Space precludes a discussion of these models and the interested reader is directed to Clark (1976).

In summary, difference equation (1) describes the change in the resource stock for a species not commercially exploited by man. It is used to characterize the population dynamics of the species in its pristine, natural state. Commercial exploitation by man requires a modification of equation (1) to account for man's harvesting activities.

A production function defines the maximum level of output obtainable from a given bundle of inputs. It is the economist's way of characterizing the technology whereby inputs produce output. In a single species fishery the output from commercial fishing would be catch or yield denoted by Y_t. The bundle of manmade inputs utilized in catching fish are assumed capable of aggregation into a single input variable called "effort" and denoted by E_t. Fishing effort is directed at the fish stock X_t and results in a yield Y_t according to

$$Y_t = H(E_t, X_t) \tag{3}$$

where $H(E_t, X_t)$ is the production function for the fishery. The partial derivatives of $H(\cdot)$ are referred to as marginal products and assumed positive. If catch per unit effort is proportional to the fish stock one obtains the

FIGURE 1. THE LOGISTIC GROWTH FUNCTION

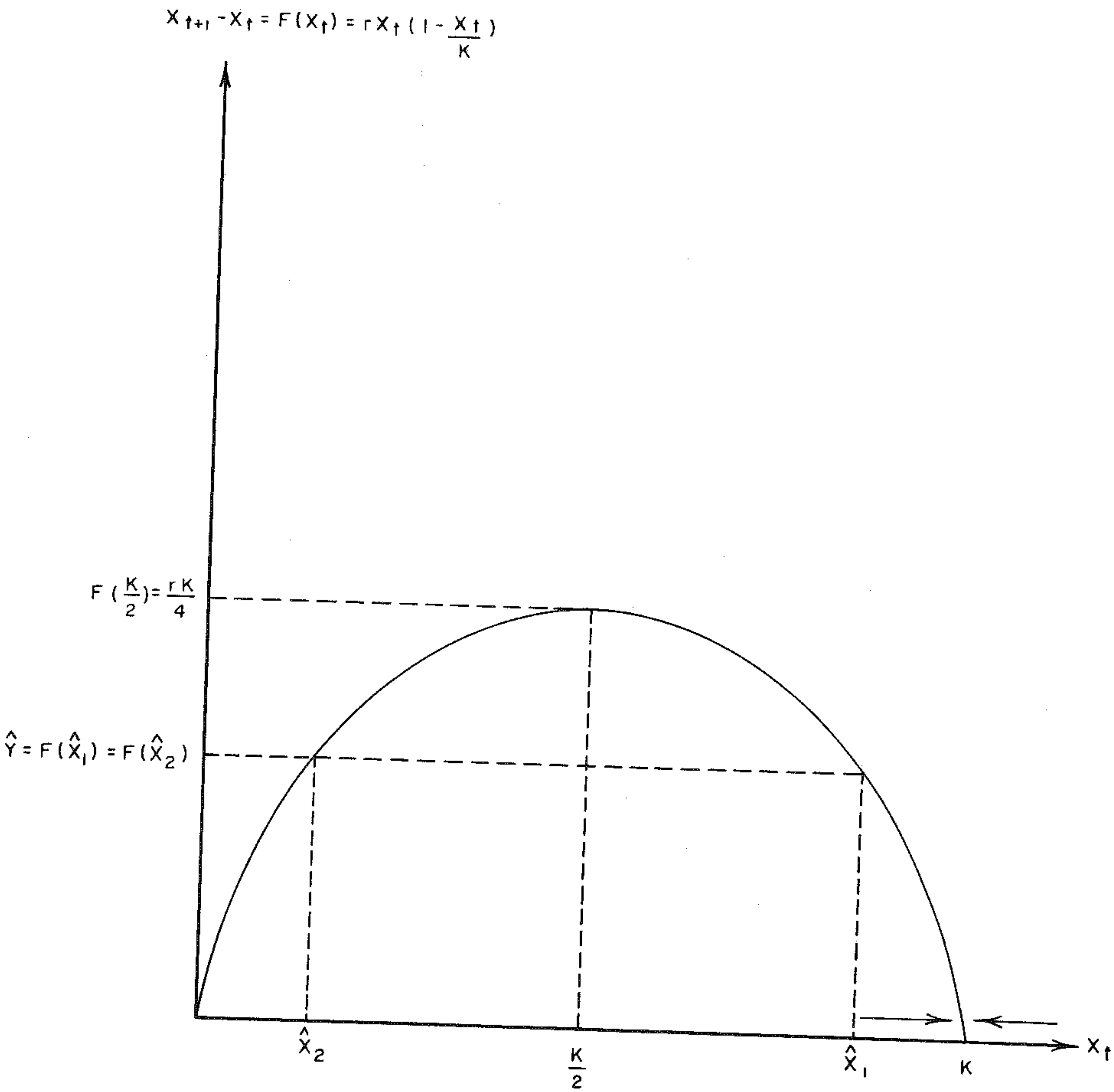
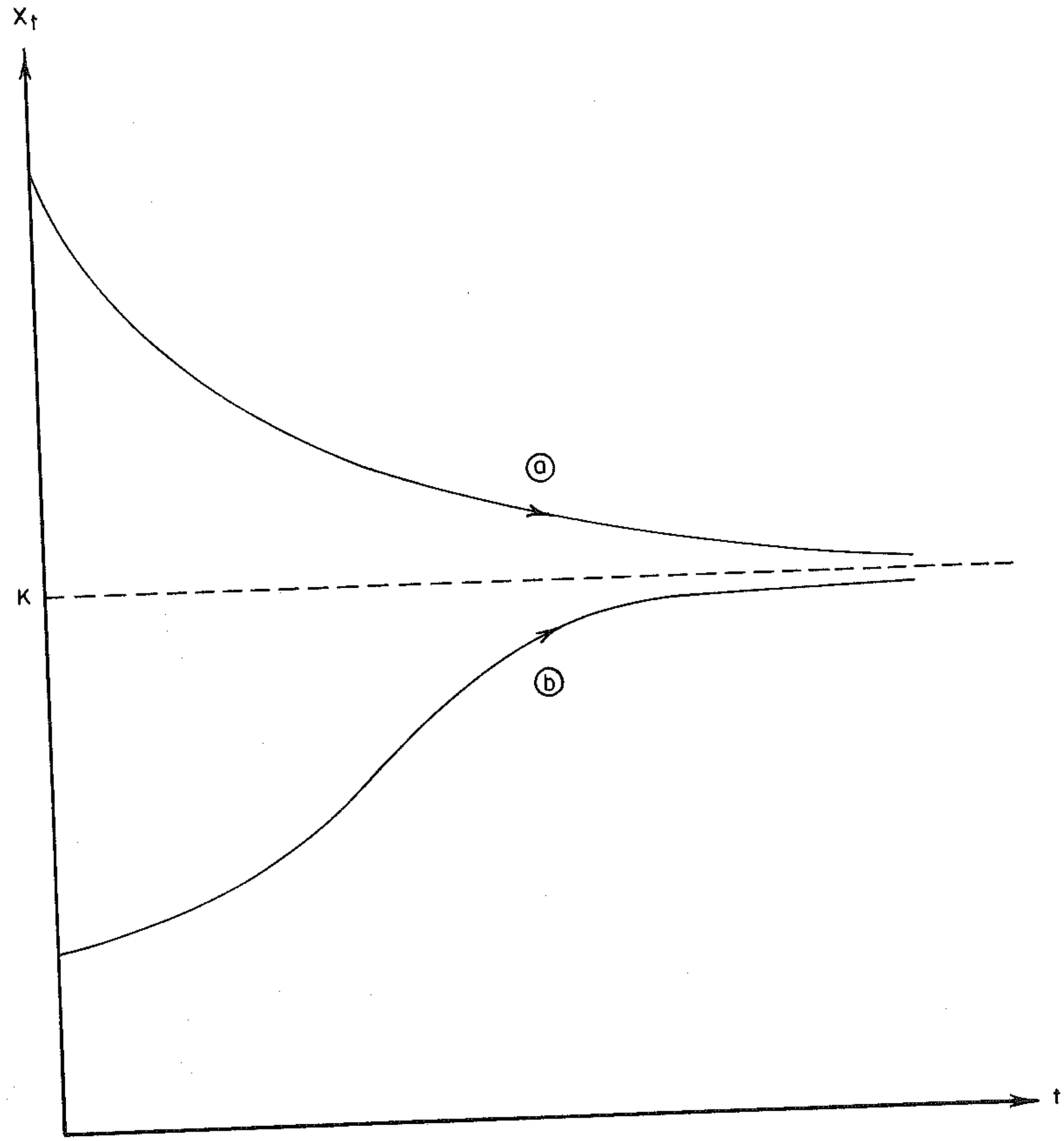


FIGURE 2. APPROACH PATHS FOR THE LOGISTIC GROWTH FUNTION



production function

$$Y_t = qE_tX_t \tag{4}$$

where q is called the catchability coefficient. This production function underlies much of the early work in fisheries economics and has been studied extensively by Schaefer (1957).

When a fishery comes under commercial exploitation the equation describing population dynamics is modified to

$$X_{t+1} - X_t = F(X_t) - H(E_t, X_t) \tag{5}$$

In words: yield is deducted from the natural change in biomass to determine the net change in the fish stock. Attention is often focused on harvesting regimes that are sustainable in perpetuity. This will usually require attainment of a <u>steady state equilibrium</u> where $X_t = X$, $E_t = E$, and $Y_t = Y$ for all future periods. In such an equilibrium the left-hand-side of equation (5) equals zero and

$$H(E_t, X_t) = F(X_t) \tag{6}$$

For the logistic function and the catch per unit effort production function, (collectively referred to as the Gordon-Schaefer model), this implies:

$$qEX = rX(1 - \frac{X}{K}) \tag{7}$$

If one were to solve equation (7) for X as a function E, and multiply both sides by qE one would obtain a <u>yield-effort curve</u> expressing catch as a function of effort which for the Gordon-Schaefer model becomes

$$Y = Y(E) = qKE(1 - \frac{qE}{r}) \tag{8}$$

For our purposes we can arbitrarily set $q = 1$ and graph the resulting yield-effort curve in Figure 3. It is also a symmetric curve with roots at $E = 0$ and $E = r$ and a maximum yield at $E = \frac{r}{2}$. As noted earlier, the compensatory nature of the logistic function results in a smooth continuous yield-effort curve where incremental changes in effort result in incremental changes in yield. This is in contrast to yield-effort curves where an incremental increase in effort may result in a fishery collapse with yield plummeting to zero.

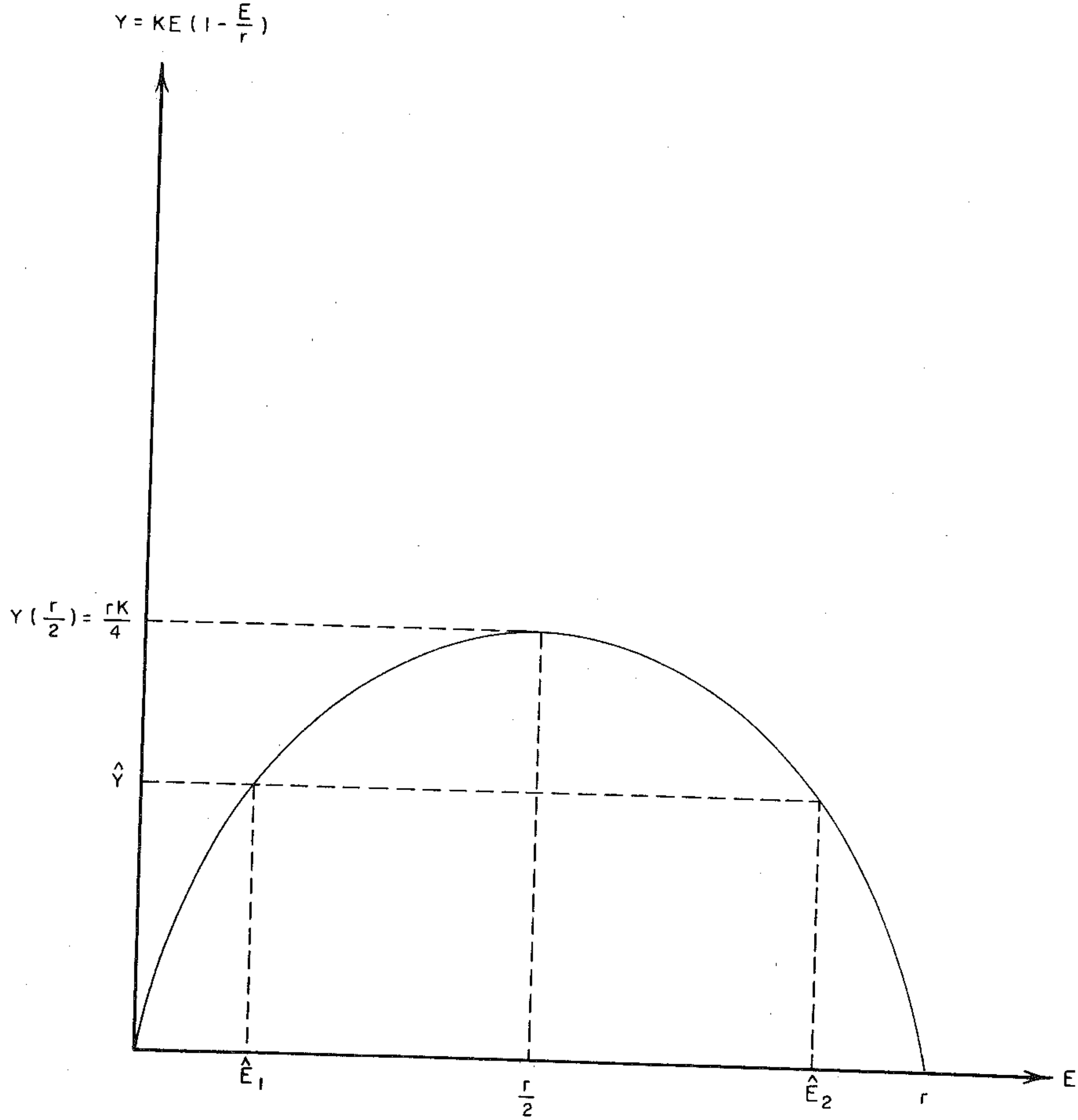
To summarize, the yield-effort curve was derived for a fishery in steady-state equilibrium and was determined by the growth and production functions that underlie the fishery. The reader should note in Figure 3 that any positive yield level less than $Y(\frac{r}{2}) = \frac{rK}{4}$ can be generated by two levels of effort. For example $\hat{Y}$ can be obtained with effort $\hat{E}_1$, and stock $\hat{X}_1$ or with effort $\hat{E}_2$ and stock $\hat{X}_2$ where $\hat{E}_2 > \hat{E}_1$ and $\hat{X}_1 > \hat{X}_2$. (See Figure 1 as well). The level of effort, stock, and yield which emerge in an open access fishery and under an optimally managed fishery are discussed in the next two sections.

II. THE OPEN ACCESS FISHERY

In an open access fishery, where the fish stock is treated as a common property resource, fishermen and vessels will enter the industry until profits are driven to zero. Suppose the cost of a unit of fishing effort is constant and denoted by the letter c. If E_t units of effort are directed at the fish stock in period t then the total cost in period t would be given by

$$C_t = cE_t \tag{9}$$

FIGURE 3. THE YIELD-EFFORT CURVE FOR THE GORDON-SCHAEFER MODEL WITH q=1



Suppose further that the price per unit received by fishermen upon landing their catch is denoted by the letter p. Then total revenue would equal $pY(E_t)$ and for the Gordon-Schaefer model with q = 1 we obtain:

$$R_t = pY(E_t) = pKE_t(1 - \frac{E_t}{r}) \tag{10}$$

With p a given positive constant the revenue curve would look identical to the yield-effort curve; only the scale of the vertical axis will have changed to measure revenues in dollars.

Profits or fishery rents are defined as

$$N_t = R_t - C_t = pY(E_t) - cE_t \tag{11}$$

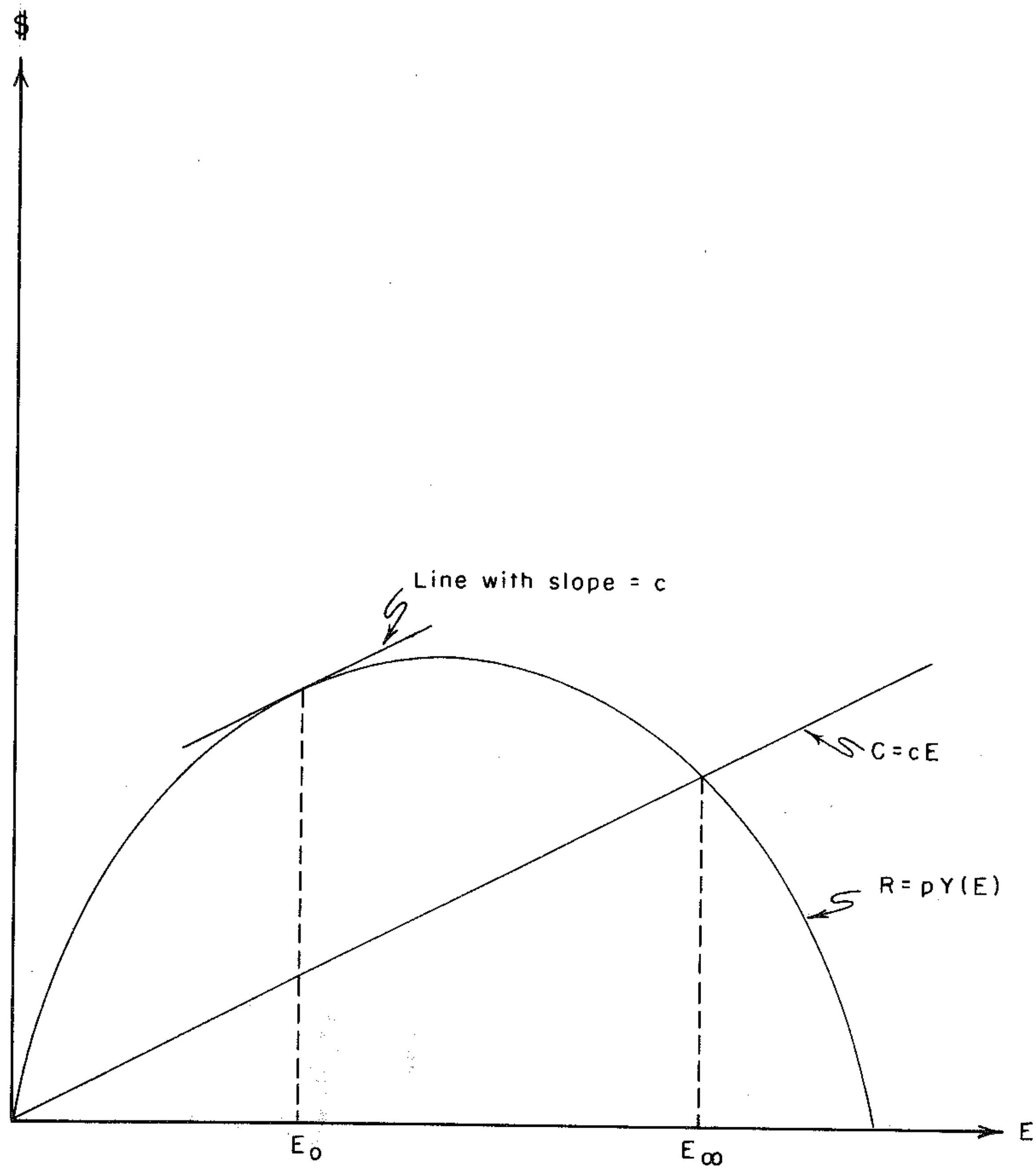
Under open access fishermen will enter until fishery rents are "dissipated" or driven to zero at which point $R_t = C_t$ or

$$\frac{Y(E_t)}{E_t} = \frac{c}{p} \tag{12}$$

That is, yield per unit effort is equal to the cost/price ratio. Graphically this situation is portrayed in Figure 4. The cost curve is shown as a ray from the origin with slope c while the revenue curve, similar in shape to the yield-effort curve, is shown as a quadratic R = pY(E). Revenue equals cost, resulting in zero profits, at E_∞. The significance of the subscript ∞ will be explained shortly.

In an open access fishery E_∞ will often be associated with excessive capital, low fish stocks, and low yield. Open access stock denoted X_∞, may be determined for the Gordon-Schaefer model by noting that when $Y_t = qE_tX_t$ and $C_t = cE_t$ that

FIGURE 4. OPEN ACCESS EQUILIBRIUM AND MAXIMUM SUSTAINABLE ECONOMIC RENT



$$C_t = \frac{cY_t}{qX_t} \quad (13)$$

Equation (13) is referred to as a cost function, which in this instance is linear in Y_t. The zero profit or rent conditions may be restated as

$$N_t = R_t - C_t = pY_t - \frac{cY_t}{qX_t} = 0 \quad (14)$$

and for $Y_t > 0$ this implies

$$X_\infty = \frac{c}{pq} \quad (15)$$

After explaining how a competitively exploited open access fishery will tend toward overcapitalization, depleted stocks, and low yields, early fisheries economists suggested that the fishery should be managed so as to maximize sustainable economic rent; that is, set effort so as to

$$\max N_t = pY(E_t) - cE_t \quad (16)$$

This situation is achieved when

$$pY'(E_t) = c \quad (17)$$

or in words, when marginal revenue equals marginal cost, (note: $Y'(E_t)$ is the derivative of the yield effort curve and $pY'(E_t)$ is marginal revenue). This situation is shown graphically in Figure 4, where the level of effort which maximizes sustainable economic rent is denoted by E_0 and is determined by the tangency of a line with slope c and the revenue curve $R = pY(E)$.

As it turns out neither E_0 nor E_∞ will be optimal for a society with a positive but finite rate of time preference. The social rate of time preference, or society's discount rate, is the result of individuals' preferences for income now as compared to the same amount of income at a later date. To get a typical individual to give up (invest) current income today you usually need to repay that income plus a premium or interest payment at a later date. Society, comprised of many such individuals, will typically reveal a collective preference for income (or fish) today as compared to an equivalent amount of income (or fish) tomorrow.

The concept of social time preference or discounting raises numerous technical and ethical issues beyond the scope of these lectures. For our purpose it will be assumed that some positive but finite rate of discount is appropriate when society is considering how to allocate its resources over time. Within the simple fishery model developed thus far, the open access equilibrium level of effort will be optimal when society's rate of discount is infinite. The maximum sustainable rent level of effort will be socially optimal when society's rate of discount is zero. For a positive but finite rate of discount the optimal level of fishing effort will usually lie somewhere between E_0 and E_∞. Thus the open access and sustainable rent maximizing levels of effort, while in general not optimal, do serve to bracket the socially optimal level of effort which will depend on cost, price, the parameters of the growth and the production functions, and the discount rate. Determination of the optimal level for fish stock, yield, and effort will be examined next.

III. THE OPTIMALLY MANAGED FISHERY

A convenient persona for inquiring into the optimal management of a fishery is the sole owner. The sole owner has exclusive harvesting rights to a resource and it is possible to formulate a management problem for the sole owner which conforms to how the fishery should be managed from a social point of view.

Suppose δ represents the social rate of discount. Then $N(1+\delta)$ would be the amount of money one would have to pay an individual to obtain a loan of N dollars for one period, (say, a year). By analogous reasoning, the value of a note promising to pay N dollars one year from now would be discounted to a value $\frac{N}{(1+\delta)}$. From another point of view δ may be thought of as the opportunity cost of investment or capital funds. It is the amount of money which could be earned on a dollar invested elsewhere in the economy. A single net cash payment N_t realized at period t in the future could be expressed as a present value equal to $\rho^t N_t$, where $\rho = \frac{1}{(1+\delta)}$ is referred to as a discount factor. The present value of a stream of net cash flows over the interval [0,T] would be calculated according to

$$N = N_o + \frac{N_1}{(1+\delta)} + \frac{N_2}{(1+\delta)^2} + \ldots + \frac{N_T}{(1+\delta)^T} = \sum_{t=0}^{T} \rho^t N_t \qquad (18)$$

Suppose the sole owner is a price taker in that his level of fishing effort and yield do not affect per unit cost c or the per unit price p. In this case the cost function, (equation [13]), may be written in the more general form

$$C_t = \frac{cY_t}{qX_t} = c(X_t)\, Y_t \qquad (19)$$

where $c(X_t)$ is a stock dependent average cost function. Since larger stocks can be expected to reduce average harvest costs the first derivative of

$c(X_t)$, denoted as $c'(X_t)$, will be negative. The net revenues from harvest Y_t may be written as

$$N_t = pY_t - c(X_t)\ Y_t = [p - c(X_t)]\ Y_t \tag{20}$$

and the present value of all future net revenues becomes:

$$N = \sum_{t=0}^{\infty} \rho^t\ [p - c(X_t)]\ Y_t \tag{21}$$

A logical objective for the sole owner, (and in this instance a desirable one for society as well), would be to maximize the present value of net revenues subject to the equation describing the change in the fish stock which may be rewritten as:

$$X_{t+1} = X_t + F(X_t) - Y_t \tag{22}$$

In this form the sole owner's management problem is an example of a broader class of problems referred to as control problems. Specifically, the sole owner seeks to control the fish stock through harvesting so as to maximize the present value of net revenues. A solution to this problem may be achieved by an extension of the method of Lagrange multipliers. This method introduces a set of artificial variables, (called multipliers), and adds the product of each constraint with its multiplier to the objective functional to form a Lagrangian expression. For the management problem confronting our sole owner, the Lagrangian expression takes the form:

$$L = \sum_{t=0}^{\infty} \rho^t\ \{[p - c(X_t)]\ Y_t + \rho\lambda_{t+1}\ \ [X_t + F(X_t) - Y_t - X_{t+1}]\} \tag{23}$$

where λ_{t+1} is the Lagrangian multiplier associated with the constraint which defines X_{t+1}. Because the Lagrangian multiplier will indicate the value of an additional unit of the fish stock in period t+1, and given that we wish to maximize the present net value represented by L, λ_{t+1} is premultiplied by ρ to yield the term $\rho^{t+1}\lambda_{t+1}$ which can be interpreted as the present value of an additional unit of the fish stock in period t+1.

Necessary conditions for a maximum require that the partial derivatives of L be set equal to zero, demanding that

$$\frac{\partial L}{\partial Y_t} = \rho^t\{[p - c(X_t)] - \rho\lambda_{t+1}\} = 0 \tag{24}$$

$$\frac{\partial L}{\partial X_t} = \rho^t\{- c'(X_t)\, Y_t + \rho\lambda_{t+1}\, [1 + F'(X_t)]\} - \rho^t\lambda_t = 0 \tag{25}$$

$$\frac{\partial L}{\partial \lambda_{t+1}} = \rho^{t+1}\{X_t + F(X_t) - Y_t - X_{t+1}\} = 0 \tag{26}$$

These conditions may be simplified somewhat and rewritten as:

$$p = c\,(X_t) + \rho\lambda_{t+1} \tag{27}$$

$$\lambda_t = -c'(X_t)\, Y_t + \rho\lambda_{t+1}\, [1 + F'(X_t)] \tag{28}$$

$$X_{t+1} = X_t + F(X_t) - Y_t \tag{29}$$

Equation (27) requires that the marginal value of a fish harvested today, (p), equal the sum of marginal harvesting cost and user cost, $(c(X_t) + \rho\lambda_{t+1})$. This latter term represents the discounted value of an additional unit of the resource next period. Thus, the marginal condition governing the harvest of the fish stock requires a balancing of current market value with harvest and user costs.

The Lagrange multiplier, (λ_t), may be interpreted as the marginal value of an additional unit of the fish stock in period t. It is sometimes referred to as a shadow price. Equation (28) requires that stock be maintained so that the shadow price in period t equals the sum of marginal stock induced reductions in average costs (note $-c'(X_t) > 0$), plus the discounted future value of a unit of the resource plus growth in period t+1. Thus, the current shadow price must equal current marginal stock induced cost savings plus the discounted value of an additional unit and associated growth in the next period.

In deriving the yield-effort curve we defined a steady-state as an equilibrium for a dynamic system where variables within the system are unchanging through time. A steady-state for the fishery under sole ownership would occur when $X_t = X$, $\lambda_t = \lambda$ and $Y_t = Y$ for all future t, (note $E_t = E$ is also unchanging). While an unchanging physical or economic environment is unusual for any protracted period of time, the concept of a steady-state equilibrium is useful in identifying the long term effects of a change in important biological or economic parameters. In more complex, adaptive models of fishery management short run decisions might be based on prevailing estimates of a steady-state equilibrium, with later decisions based on updated or revised estimates of steady-state. Thus the concept is fundamental to many stochastic models as well.

In steady-state equations (27) - (29) become a system of three equations in three unknowns; specifically:

$$p = c(X) + \rho\lambda \tag{30}$$

$$\lambda(1 - \rho[1 + F'(X)]) = -c'(X)Y \tag{31}$$

$$Y = F(X) \tag{32}$$

The left-hand-side (LHS) of equation (31) can be manipulated to become:

$\rho\lambda(1 + \delta - 1 - F'(X)) = \rho\lambda(\delta - F'(X))$. From equation (30) we note: $\rho\lambda = p - c(X)$. Substituting into the LHS of (31) and solving for Y yields:

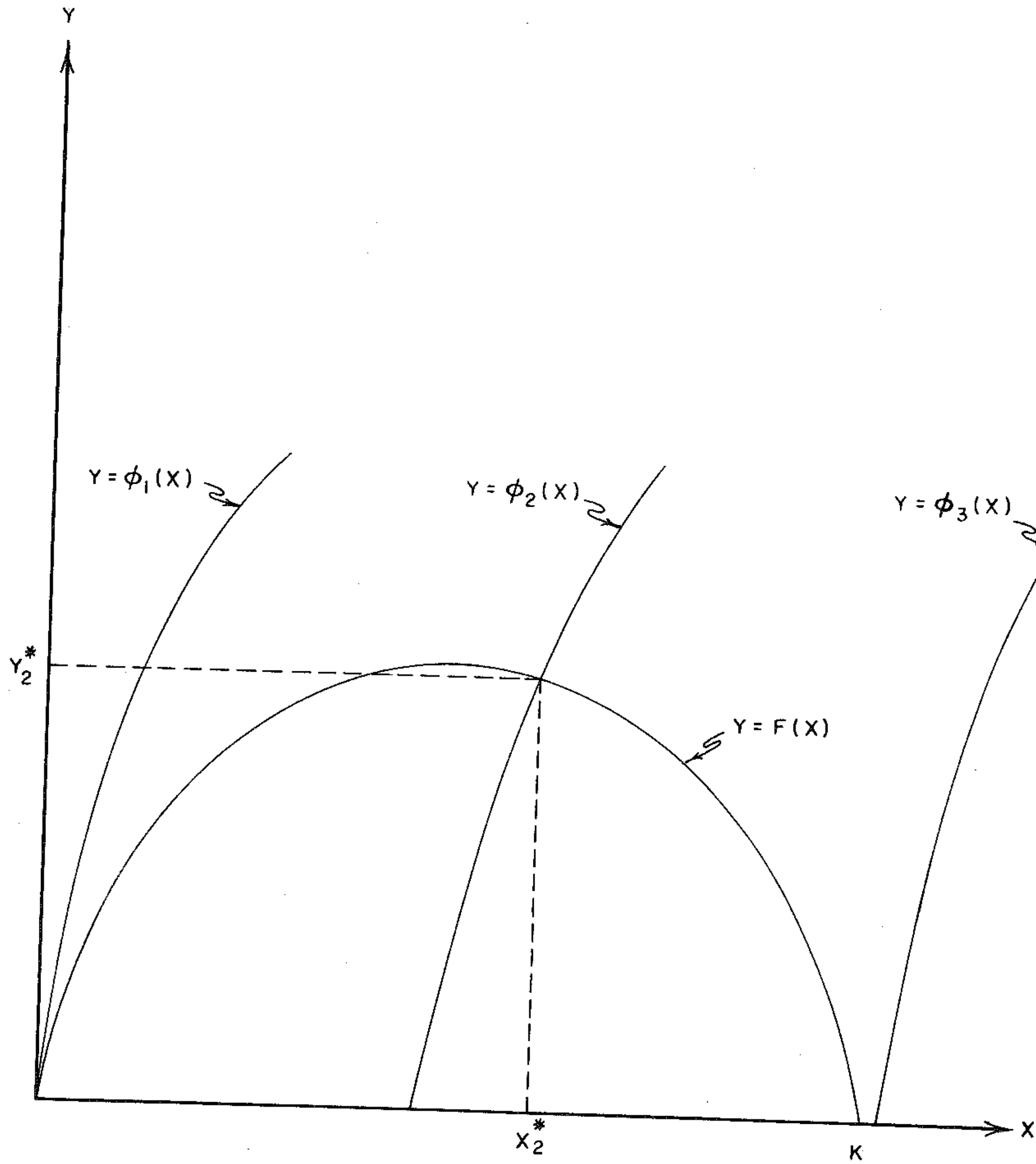
$$Y = \frac{[F'(X) - \delta]\ [p - c(X)]}{c'(X)} = \phi(X) \tag{33}$$

Equation (33) is referred to as the catch locus, (see Gould 1972). In the positive orthant, $(X > 0,\ Y > 0)$, the catch locus will typically have a positive slope, $(\frac{dY}{dX} > 0)$, and may be graphed along with the growth function $Y = F(X)$ to identify the steady-state optimum. The precise position and shape of the catch locus will depend on the various bioeconomic parameters used to specify $F(X)$ and $c(X)$ as well as p and δ. Three situations are shown in Figure 5.

The intersection of catch locus one, $(\phi_1(X))$, and the growth function occurs at $X = 0$, $Y = 0$ implying that on purely economic grounds it is optimal to harvest the resource to extinction. Such a situation might arise if the intrinsic growth rate for the species is relatively low, the discount rate is high, and the cost of harvesting the last unit of the resource is finite and less than the market price, $(c(0) < p)$. Clark (1973) discusses in greater detail the situation where commercial harvesting can lead to extinction.

In the second case the intersection of $\phi_2(X)$ and $F(X)$ occurs at (X_2^*, Y_2^*). In this situation the optimal stock actually exceeds $X_{msy} = K/2$. Such a stock

FIGURE 5. CATCH LOCI, THE GROWTH FUNCTION AND STEADY-STATE OPTIMUM



level might be justified on the basis of cost savings which result from higher stock levels.

In the third case the intersection of $\phi_3(X)$ and the X-axis occurs to the right of K (the species environmental maximum). In this instance the cost of harvesting is so high relative to the market price that it is <u>un</u>economic to commercially harvest the population, and the species goes unexploited by man. The majority of fish species would be described by this latter case.

The catch locus and growth function are simply a system of two equations in two unknowns. One could substitute Y = F(X) into equation (33) to obtain a single equation for the optimal steady-state stock. This equation is commonly written as:

$$F'(X) - \frac{c'(X)F(X)}{[p - c(X)]} = \delta \tag{34}$$

(see, for instance, Clark 1976, p. 40), and requires that the steady-state stock equate the sum of the marginal growth rate and stock effect to the discount rate. Thus, in steady-state the stock which is being maintained is providing returns in the fishery, (in the form of growth and cost savings), which precisely equal the rate of return obtainable on other capital assets elsewhere in the economy, (equal to δ). With the exception of extinction or no commercial exploitation, equation (34) can be solved for the optimal stock $X^* > 0$.

Up to this point, we have confined our discussion to steady-state equilibrium and have ignored the issue of <u>short run dynamics</u>. If the initial stock X_0 is not equal to the optimal stock X^*, what should be the optimal <u>approach path</u> for X_t? In general, the approach path will be asymptotic; that is, a more or less gradual approach to equilibrium with the $\lim_{t\to\infty} X_t \to X^*$.

Under certain circumstances, the approach to equilibrium is optimal if it is most rapid. Spence (1974) has shown that a most rapid approach path (MRAP) is optimal if the objective function may be written as a quasi-concave and separable function of X_t and X_{t+1}. The objective of the sole owner was to maximize the present value of net revenues (see equation [21]). The expression for net revenues in period t was

$$N_t = [p - c(X_t)]\, Y_t . \tag{35}$$

Solving the difference equation

$$X_{t+1} - X_t = F(X_t) - Y_t, \tag{36}$$

for Y_t and substituting into equation (35) we obtain what Spence and Starrett (1975) call the derived utility function:

$$N_t = W(X_t, X_{t+1}) = [p - c(X_t)]\, [F(X_t) + X_t - X_{t+1}]. \tag{37}$$

If $W(X_t, X_{t+1})$ is additively separable in X_t and X_{t+1}, it may be written as

$$W(X_t, X_{t+1}) = A(X_t) + B(X_{t+1}), \tag{38}$$

and the present value of net benefits may be written as:

$$N = A(X_o) + \sum_{t=1}^{\infty} \rho^t V(X_t). \tag{39}$$

If $V(X_t)$ is quasi-concave MRAP is optimal.

Spence and Starrett (1975) discuss several problems where the above separability and quasi-concavity conditions may hold. For the fishery management problem, harvesting along a MRAP would be determined by the relationship of X_t to X^* such that

$$\begin{aligned} &\text{If } X_t > X^*,\ Y_t = Y_{max} \\ &\text{If } X_t < X^*,\ Y_t = 0 \\ &\text{If } X_t = X^*,\ Y_t = Y^* \end{aligned} \tag{40}$$

Thus, if the initial stock were in excess of the optimal stock, one would wish to reduce it to X^* as rapidly as possible by harvesting fish at the maximum Y_{max}. If the initial stock were less than the optimal stock, one would impose a moratorium ($Y_t = 0$) until the stock reached X^*. If, by happenstance, the initial stock equalled the optimal stock, one would commence harvesting at $Y^* = F(X^*)$ and stay there forever.

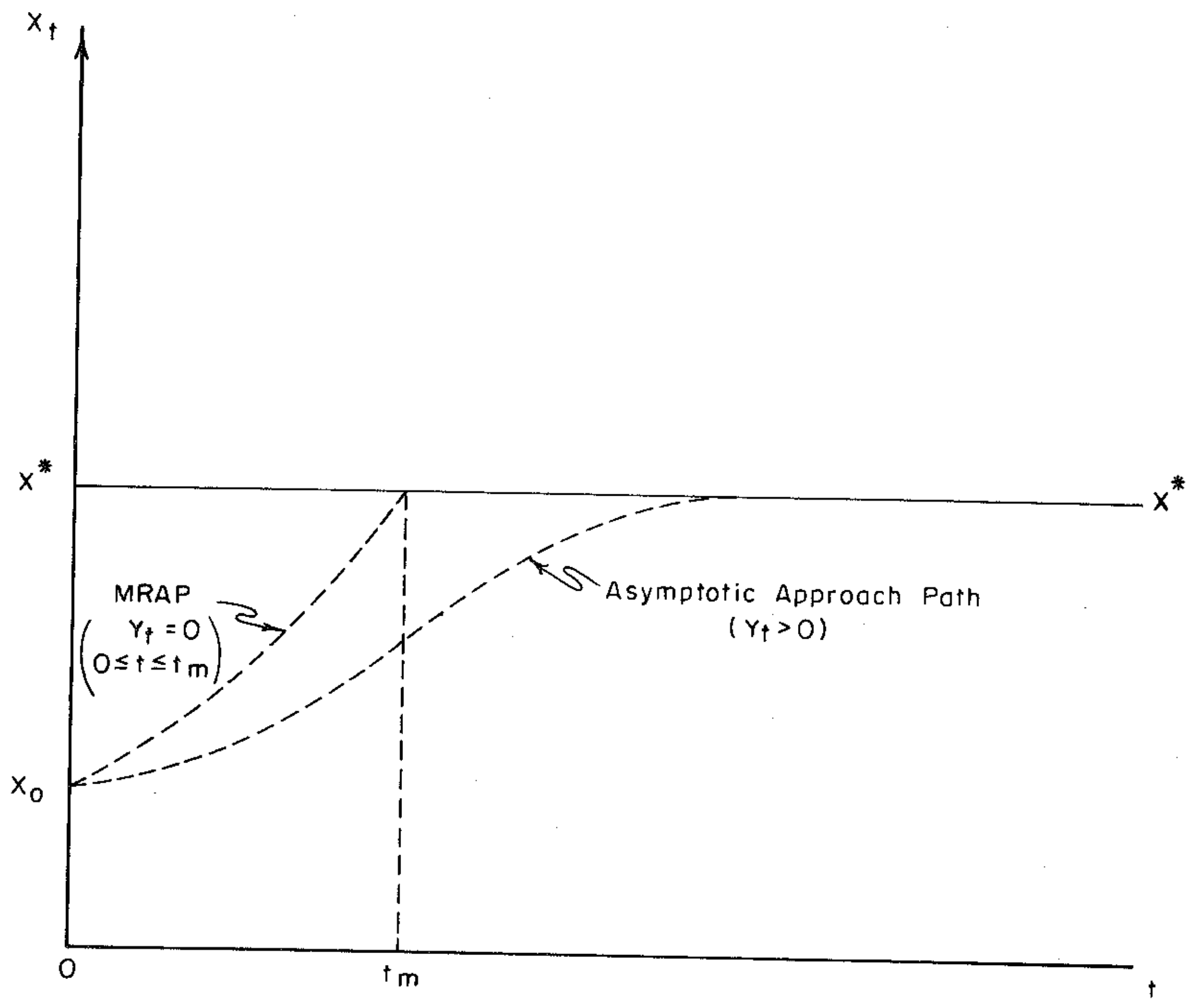
Figure 6 shows the difference between the most rapid approach path (MRAP) and the asymptotic approach path for $X_0 < X^*$. Along the MRAP, $Y_t = 0$ for $0 < t < t_m$, until $X_t = X^*$ at $t = t_m$, at which time $Y_t = Y^* = F(X^*)$ for $t > t_m$. Along the asymptotic approach path, harvest is positive ($Y_t > 0$), but less than net natural growth, allowing the stock to slowly approach X^*: thus, as $t \to \infty$, $X_t \to X^*$ and $Y_t \to Y^*$.

Most rapid approach paths are relatively easy to calculate (one calculates $X_{t+1} = X_t + F(X_t) - Y_t$, where $Y_t = Y_{max}$ or $Y_t = 0$, until $X_{t+1} = X^*$). Asymptotic approach paths are more difficult and would require the introduction of a transversality condition and solution of a two point boundary problem. A discussion of these concepts and procedures is beyond the scope of these lectures.

IV. MANAGEMENT FROM A BIOECONOMIC PERSPECTIVE

Up until the 1970s, the discussion of how commercial fisheries should be managed was almost exclusively dominated by biologists. With the development and application of dynamic optimization techniques, economists, while still in

FIGURE 6. MOST RAPID AND ASYMPTOTIC APPROACH PATHS TO THE OPTIMAL STOCK X^*



the minority, have begun to exert influence over the types of policies employed to manage marine fisheries. Let us examine some of the more traditional, often biologically based management policies, and then proceed to the policy implications of the basic bioeconomic model.

Traditional management policies have included closed seasons, gear restrictions, quotas, and limited entry.

Closed Seasons: This policy limits fishing to certain times during the year. It may be justified on a biological basis in terms of protecting the species during a critical period, such as spawning. A similar rationale might be used in prohibiting fishing in a particular area, (for example, commercial salmon fishing in rivers or streams). Closed seasons can result in idle capital, (vessels and processing equipment), and fishermen while the season is closed. A tremendous amount of fishing effort is often expended during the open season. In some commercial fisheries seasons have been limited to a few weeks. Excess capacity may be directed elsewhere; perhaps exacerbating the management of other depelted species.

Gear Restrictions: This policy attempts to reduce the effectiveness of harvesting units by prohibiting the use of certain technologies. This has the effect of raising (or maintaining) cost per unit effort and making the fishery less attractive financially. (For example, the State of Maryland restricts commercial harvest of oysters to vessels powered by sail.) Gear restrictions, while limiting the use of one input, might cause fishermen to use other inputs more intensively with little or no reduction in effective effort and yield.

Quotas: This policy establishes a maximum quantity that may be harvested per period. Quotas might be collective, setting a fleet limit, or assigned individually to each vessel. If the quota is collective, there is a strong incentive for an individual vessel to attempt to catch the largest share of the quota possible before the fishery is closed down (when the collective quota is reached). If assigned individually, each vessel has an upper limit to the amount of fish caught per period. Uniform quotas assigned individually, but based on vessel characteristics, are sometimes felt to penalize the most efficient fishermen, restricting their catch while reserving stock for harvest by higher cost (less efficiently run) vessels. (Transferable quotas will theoretically avoid this shortcoming and will be discussed in more detail.)

Limited Entry: This policy restricts harvesting to vessels which are licensed and controlled by the management agency. Licenses are often allocated to boats harvesting the resource at the time the limited entry program is introduced. The number of licensed boats may be reduced over time by attrition and subsequent licenses allocated by lottery. A yearly fee, (sometimes substantial) is charged for a license. Current holders are often allowed to sell (transfer) their license to other potential fishermen. Allowing sale of licenses would theoretically result in only the most efficient boats gaining access to the fishery (they can pay the most for a license in a competitive situation). In addition to license fees, fleet or individual quotas may be imposed.

The traditional management policies are thus oriented at protecting the resource during critical periods in its life cycle and at reducing effort and catch. Such policies are usually of limited success. While they may reduce the degree of biological overfishing (the extent to which stock is less than X_{MSY}) they may exacerbate the problem of economic overfishing (zero fishery rents). Missing from these policies is an understanding of user cost.

In the analysis of an open access fishery we saw that effort would be expended to the point where fishery rents were driven to zero. The open access equilibrium may be denoted as $(E_\infty, X_\infty, Y_\infty)$. Open access was inefficient because there typically exists a bioeconomic optimum, denoted (E^*, X^*, Y^*) such that $E^* < E_\infty$, $X^* > X_\infty$, and $Y^* > Y_\infty$, which is capable of producing income (rents) that could make some fishermen better off without making anyone worse off. In the Gordon-Schaefer model, open access occurred when

$$N = [p - \frac{c}{qX_\infty}]\, Y_\infty = 0 \qquad (41)$$

with $Y_\infty > 0$, we note $p = \frac{c}{qX_\infty}$ or $X_\infty = \frac{c}{qp}$.

The bioeconomic optimum was derived from the perspective of a sole owner who, for the Gordon-Schaefer model, would seek an equilibrium such that

$$p = \frac{c}{qX^*} + \rho\lambda^*, \qquad (42)$$

where $\rho\lambda^*$ was defined as user cost, equal to the present value of an additional fish (in the water) tomorrow. The user cost term plays a key role in defining fishery management policies that can correct for economic overfishing. Quite simply, the economist would like to establish policies which would create a real cost comparable to $\rho\lambda^*$, and induce fishermen to take it into account.

Transferable Quotas and Landings Taxes

Suppose a team of biologists and economists could estimate (or assign) values to the parameters of the Gordon-Schaefer model (p, c, δ, r, K, and q) and determine the bioeconomic optimum (E^*, X^*, Y^*). Assume that the resource stock was moved from X_0 to X^*. The management team would now allow a total effort of E^*, resulting in a yield of Y^*. The total quota, Y^*, might be assigned to an arbitrary group of fishermen in, say, metric ton units. Any initial assignee could choose to harvest the permitted number of metric tons of fish or he could sell it to another fisherman. The fact that the individual quota is transferable to another creates an opportunity cost for its initial owner. He must decide whether the net revenues from harvesting his quota exceeds the amount he could get if he would sell it to another. Presumably, a market would develop for individual quotas, with fishermen who could harvest at least cost being able to outbid less efficient fishermen. What would the going price be for the right to harvest a metric ton of fish? If prices, costs and other bioeconomic parameters were expected to remain unchanged, the economists would predict a per ton quota price of

$$P_Q^* = \rho\lambda^* \tag{43}$$

where P_Q^* is the per ton quota price and $\rho\lambda^*$ is user cost. This relationship would result because at (Y^*, X^*)

$$p - \frac{c}{qX^*} = P_Q^* = \rho\lambda^*. \tag{44}$$

That is, at the bioeconomic optimum, each metric ton harvested would yield a net revenue of $\rho\lambda^*$. In deciding whether to "fish or sell," the rational fishermen would subtract the opportunity cost of holding the permit along with the per unit harvest costs to determine the "real" value of fishing. By defining exclusive but transferable rights of harvest, the management team would create incentives that (a) lead to the efficient (least cost) harvest of the total quota, Y^*, and (b) cause fishermen to individually consider the opportunity cost (equal to user cost) of harvesting another unit of the resource.

There is another way of establishing and maintaining the bioeconomic optimum (E^*, X^*, Y^*). The management team (or authority) could simply notify all fishermen that their catch had to be landed at certain locations and a per ton tax of

$$\tau^* = \rho\lambda^* \tag{45}$$

would be levied on their catch. From a net revenue point of view, fishermen would harvest an additional metric ton until

$$p - \frac{c}{qX^*} - \tau^* = 0. \tag{46}$$

The per unit landings tax allows the introduction of an additional unit cost which in equilibrium would equal user cost. If initially $X_0 < X^*$, a tax set at $\tau^* = \rho\lambda^*$ would actually choke off fishing effort until the stock increased (thereby reducing costs) to the optimum X^*.

The landings tax has the advantage (from the management authority's point of view) of generating tax revenues equal to

$$R_\tau = \tau^* Y^* \tag{47}$$

in equilibrium. These revenues might be earmarked for administration, enforcement and research efforts by the management authority. Fishermen, of course, would much prefer freely assigned quotas to a landings tax. In theory, the management authority could employ <u>both</u>, and in equilibrium we would expect the following relationship:

$$P_Q^* = \rho\lambda^* - \tau, \tag{48}$$

that is, the price emerging from the quota market would equal net revenue ($\rho\lambda^* = p - \frac{c}{qX^*}$) less the landings tax rate. Note that as $\tau \rightarrow \rho\lambda^*$, $P_Q^* \rightarrow 0$; that is, as the landings tax is increased from zero to user cost ($\rho\lambda^*$), the market price for the right to harvest a metric ton will decline toward zero. At some tax rate $\tau < \rho\lambda^*$, the authority can generate some revenues to support their activities and still leave positive fishery rents to be captured by fishermen.

V. TWO APPLICATIONS: YELLOWFIN TUNA AND BLUE WHALES

To illustrate some of the concepts presented in the preceding lectures we will examine two empirical studies. The first is a study of yellowfin tuna in the Eastern Tropical Atlantic (ETA) and employs the Gordon-Schaefer model (see equation [7]). The second is a study of the blue whale and will employ an alternative specification attributable to Spence (1974).

Yellowfin Tuna in the Eastern Tropical Atlantic (ETA)

In seeking to optimally manage a single species fishery we formulated a dynamic optimization problem that sought to maximize the present value of net revenues subject to a difference equation describing the change in the fish stock. A Lagrangian expression was formed and the first order (necessary) conditions derived. In steady-state these conditions lead to a system of three equations in three unknowns (X^*, Y^*, λ^*). We could eliminate λ^* from the system leaving a two equation system consisting of the catch locus and grow curve (see equations [32] and [33]). Further substitution led to a single equation in X^* (see equation [34]). For the Gordon-Schaefer model with $c(X^*) = c/qX^*$ and $Y = rX^*(1 - X^*/K)$ this single equation is a quadratic in X^* and the optimal stock level will equal the positive root according to:

$$X^* = \frac{K}{4}\left[\left(\frac{c}{qpk} + 1 - \frac{\delta}{r}\right) + \sqrt{\left(\frac{c}{qpK} + 1 - \frac{\delta}{r}\right)^2 + \frac{8c\delta}{qpKr}}\,\right]. \qquad (49)$$

If one had estimates of the bioeconomic parameters r, K, q, c, p, and δ, one could estimate the optimal stock X^* as well as yield and effort according to

$$Y^* = F(X^*), \tag{50}$$

and

$$E^* = \frac{Y^*}{qX^*}. \tag{51}$$

Estimates of these parameters for the yellowfin tuna fishery in the ETA were obtained by Adu-Asamoah and Conrad (1982) based on data from the International Commission for the Conservation of Atlantic Tuna (ICCAT), the National Marine Fisheries Service (NMFS) of the U.S. Department of Commerce, and earlier economic studies of purseseiners, baitboats, and longliners. The values of the parameters used to calculate maximum sustainable yield (MSY), open access, and the bioeconomic optimum were

$$\begin{aligned} r &= 1.2883 \\ K &= 351.2244 \times 10^3 \text{ MT} \\ q &= 1.372 \times 10^{-2} \\ p &= \$1,300/\text{MT} \\ c &= \$2,000;\ \$2,500;\ \$3,000;\ \$3,500/\text{standard day at sea} \\ \delta &= 0.00,\ 0.05,\ 0.10,\ 0.15,\ 0.20. \end{aligned} \tag{52}$$

The values for stock ($X10^3$ MT), yield ($X10^3$ MT) and effort ($X10^3$ standard day at sea) for the various equilibria are shown in Table 1. Maximum sustainable yield is $Y_{MSY} = rK/4 = 113,121$ MT occurring at $X_{MSY} = 175,612$ MT and $E_{MSY} = 46,950$ SDS. Open access stock, where net revenues are zero, occurs at $X_\infty = cp/q$. The four values of c used to test sensitivity to cost produced the estimates of open access equilibria at the bottom of Table 1. Bioeconomic

TABLE 1: MSY, BIOECONOMIC, AND OPEN ACCESS EQUILIBRIA FOR YELLOWFIN TUNA IN THE ETA*

Yellowfin Parameters: p = \$1300, $q = 1.372 \times 10^{-2}$, r = 1.2883, K = 351.2244

Maximum Sustainable: X_{MSY} = 175.61, Y_{MSY} = 113.12, E_{MSY} = 46.9495

	δ	c = \$2,000	c = \$2,500	c = \$3,000	c = \$3,500
BIOECONOMIC EQUILIBRIA	0.00	X* = 231.68 Y* = 101.59 E* = 31.96	X* = 245.70 Y* = 95.10 E* = 28.21	X* = 259.71 Y* = 87.18 E* = 24.47	X* = 273.73 Y* = 77.81 E* = 20.72
	0.05	X* = 228.21 Y* = 102.97 E* = 32.89	X* = 242.81 Y* = 96.56 E* = 28.98	X* = 257.35 Y* = 88.61 E* = 25.10	X* = 271.83 Y* = 79.16 E* = 21.23
	0.10	X* = 224.85 Y* = 104.23 E* = 33.79	X* = 240.02 Y* = 97.90 E* = 29.73	X* = 255.07 Y* = 89.96 E* = 25.71	X* = 270.00 Y* = 80.44 E* = 21.71
	0.15	X* = 221.58 Y* = 105.37 E* = 34.66	X* = 237.32 Y* = 99.15 E* = 30.45	X* = 252.87 Y* = 91.23 E* = 26.30	X* = 268.24 Y* = 81.65 E* = 22.19
	0.20	X* = 218.41 Y* = 106.40 E* = 35.51	X* = 234.71 Y* = 100.31 E* = 31.15	X* = 250.74 Y* = 92.42 E* = 26.87	X* = 266.54 Y* = 82.80 E* = 22.64
OPEN ACCESS	$\delta \to \infty$	X_∞ = 112.13 Y_∞ = 98.34 E_∞ = 63.92	X_∞ = 140.17 Y_∞ = 108.51 E_∞ = 56.43	X_∞ = 168.20 Y_∞ = 112.92 E_∞ = 48.93	X_∞ = 196.23 Y_∞ = 111.56 E_∞ = 41.44

*Stocks (Xs) and Yields (Ys) are measured in 10^3 metric tons. Effort is measured in 10^3 standard days at sea per year.

equilibria for various combinations of c and δ are shown in the body of the table. Four aspects of these equilibria should be noted. First, for a given discount rate increased costs result in increased stocks. Within the context of Figure 5, increases in c cause the catch locus to shift to the right.

Second, for a given cost, increases in the discount rate result in lower optimal stocks. Increases in δ tend to shift the catch locus to the left.

Third, the optimal stock for all combinations of c and δ are in excess of X_{MSY}. The marginal stock effect (that is, the second term on the LHS of equation [34]) exceeds even the high value of $\delta = 0.20$ resulting in optimal stocks to the right of X_{MSY}. This situation is similar to the intersection of $\Phi_2(X)$ and F(X) in Figure 5 where $X_2^* > X_{MSY}$, and the stock induced cost-savings exceed the interest costs.

Finally, with the exception of the lowest cost estimate, the open access stock is in excess of X_{MSY}. For c > \$2,000/standard day the yellowfin stock would not be subject to biological overfishing (i.e., $X_\infty > X_{MSY}$) but would be subject to economic overfishing (i.e., $X^* > X_\infty$).

Blue Whales

One of the first attempts to apply control theory to a problem of renewable resource management was the bioeconomic analysis of the blue whale (Balenoptera musculus) by Spence (1974). While the techniques of analysis are similar to those encountered in previous lectures, Spence did not employ the Gordon-Schaefer specification but rather an alternative formulation for growth and production within the basic bioeconomic model. Let

X_t = the blue whale population in year t

E_t = the number of fully equipped whale boats in year t.

Spence assumes that the growth function takes the following form:

$$X_{t+1} = F(X_t) - Y_t = AX_t^a - Y_t, \tag{53}$$

where $A > 1$ and $0 < a < 1$. This results in a concave (from below) growth function similar to the curve drawn in Figure 7.

Fishing effort is directed at the stock of blue whales and yield is assumed to be some fraction of next year's stock, i.e.,

$$Y_t = F(X_t)[1 - e^{-bE_t}] = AX_t^a [1 - e^{-bE_t}] \tag{54}$$

where the term $[1 - e^{-bE_t}]$ determines the proportion of next year's stock harvested this year. Assuming $0 < b < 1$, we note

$$[1 - e^{-bE_t}] \begin{Bmatrix} =0 \\ >0 \\ =1 \end{Bmatrix} \quad \text{when} \quad E_t \begin{Bmatrix} =0 \\ >0 \\ E_t \to \infty \end{Bmatrix} . \tag{55}$$

If one substitutes equation (54) into equation (53), a simplified expression for X_{t+1} results:

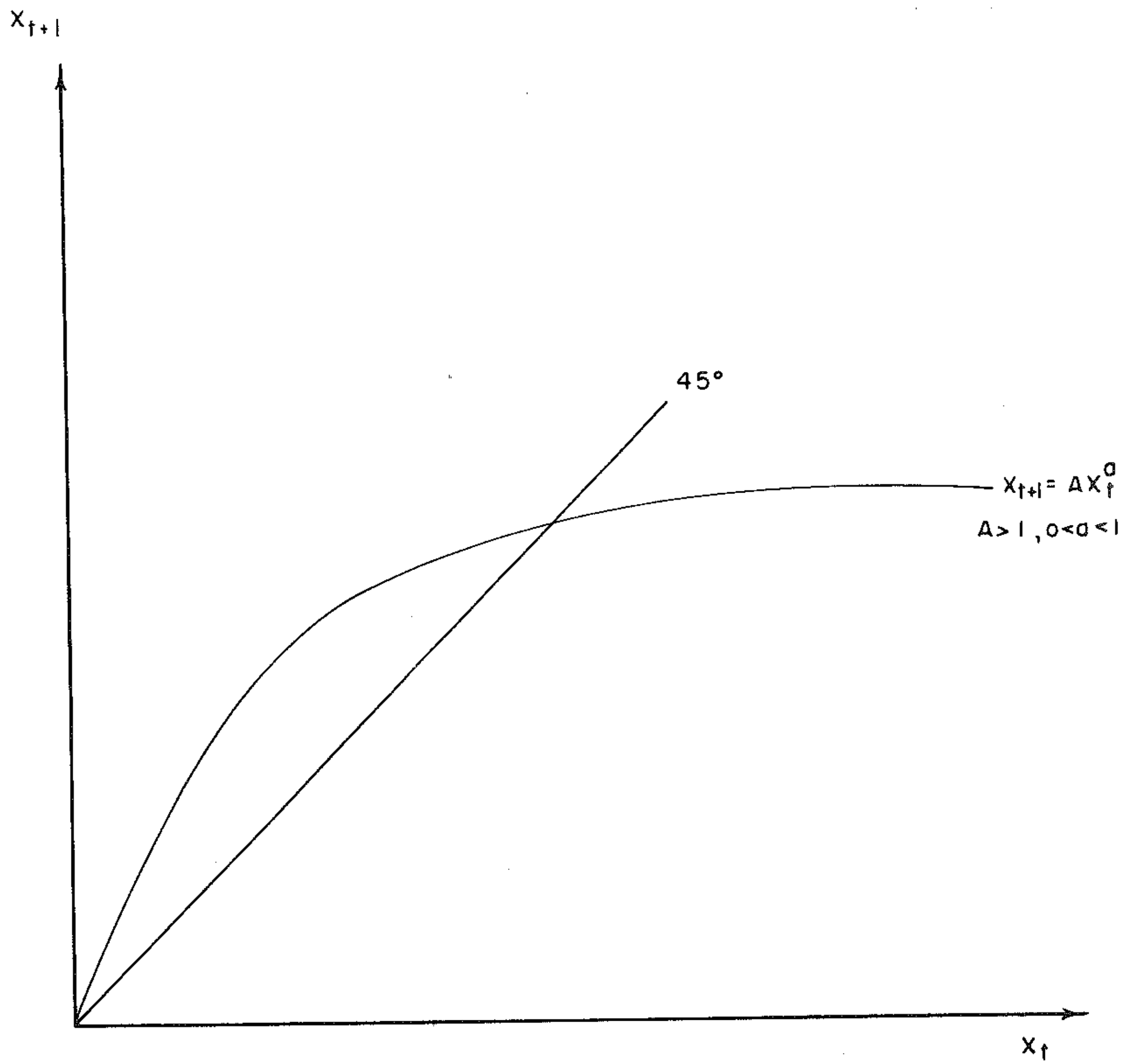
$$X_{t+1} = AX_t^a e^{-bE_t}. \tag{56}$$

Denoting the average price of a blue whale as p and the yearly cost of operating a whaling vessel as c, the net revenues from the harvest of blue whales may be written as

$$N_t = pY_t - cE_t = pAX_t^a [1 - e^{-bE_t}] - cE_t. \tag{57}$$

Maximization of the present value of net revenues subject to equation (56) can be accomplished by forming the Lagrangian

FIGURE 7. GROWTH IN SPENCE'S MODEL OF THE BLUE WHALE



$$L = \sum_{t=0}^{\infty} \rho^t \{pAX_t^a \ [1 - e^{-bE_t}] - cE_t + \rho\lambda_{t+1}[AX_t^a \ e^{-bE_t} - X_{t+1}]\} \qquad (58)$$

Recall $\rho = \frac{1}{1+\delta}$ is our discrete time discount factor, and δ is the discount rate. In the appendix to this lecture, we derive the first order necessary conditions and ultimately a single equation defining the optimal stock of blue whales. The equation takes the form

$$\frac{abAX^a - a(c/p)}{[bX - c/p]} = (1 + \delta) \qquad (59)$$

where a, A, and b are parameters of the growth and production functions which Spence estimates using data from International Whaling Statistics, c/p is the ratio of yearly cost to price per average whale, and δ is the aforementioned discount rate. Spence obtained the following estimates: $a = 0.8204$, $A = 8.3560$, and $b = 0.0019$. He set $\delta = 0.05$ and then solved for the optimal stock of blue whales as a function of (c/p). His results are shown in Table 2. Spence admits to having limited data on the costs of operating a whaling vessel and thus wanted to examine the sensitivity of the optimal stock, X^*, to the cost/price ratio. Even for low c/p ratios, it is optimal to require relatively large standing stocks, much greater than the current estimate of 5,000 blue whales.

Given the functional forms employed by Spence, it is also possible to show that the approach path is most rapid (i.e., MRAP is optimal). In 1973, the estimated stock of blue whales was $X_0 = 1,639$. Since $X_0 < X^*$ for any of the bioeconomic optima in Table 2, MRAP would call for a moratorium on commercial harvest of blue whales. How long? Spence calculates the length of time for the stock to grow from X_0 to X^* by iterating

$$X_{t+1} = AX_t^a \tag{60}$$

until $X_{t+1} > X^*$ the length of the moratorium will depend on this optimal stock, with larger stocks requiring a longer moratorium. The length of moratorium for $\delta = 0.05$ and for four values of (c/p) is shown in Table 3. Depending on the cost/price ratio, the moratorium would extend from 5 to 17 years.

As in the ETA tuna study discussed earlier, the estimates of a, A, and b should be regarded as preliminary. Spence rightfully suggests that the blue whale stock should be monitored during the moratorium to see if it is growing according to our estimates of a and A. If not, these parameters should be revised and new optima and moratoria calculated.

Spence concludes by noting that for the blue whale, extinction is not optimal on economic grounds and, given the large standing stocks at steady-state, there does not appear to be any conflict between economic and environmental objectives.

TABLE 2: Optimal Stock of Blue Whales, Boats, and Catch for $\delta = 0.05$ and Alternative Values of (c/p).*

(c/p)	X^*	E^* (boats)	Y^*	Y^*/E^* (catch/boat)	N^*/c (profits per year per $ of operating cost)
0	34,421	129	9,635	74.6	--
9	40,000	115	9,834	85.5	8.34
18	45,000	104	9,890	95.1	4.21
28	50,000	95	9,845	104	2.70
49	60,000	77	9,501	122	1.47
73	70,000	63	8,870	141	0.94
98	80,000	50	8,002	159	0.63
124	90,000	39	6,930	177	0.43
151	100,000	29	5,681	195	0.29
180	110,000	20	4,276	213	0.19
209	120,000	12	2,732	277	0.09
240	130,000	4.3	1,062	246	0.025
250	136,000	0	0	--	--

*Table modified from Spence (1974).

TABLE 3: Length of Moratorium on Blue Whales for $\delta = 0.05$, $X_0 = 1{,}639$, $p = 6{,}000$, and Four Alternative Values for (c/p).*

(c/p)	X^*	E^*	Y^*	Years with no whaling	Profit per year at X^* (millions of $)
0	34,421	129	9,635	5	57.8
28	50,000	95	9,845	7	57.4
124	90,000	39	6,930	12	41.5
209	120,000	12	2,732	17	16.4

*Table modified from Spence (1974).

Bioeconomics of the Blue Whale: Mathematical Appendix

Maximization of the present value of net revenues for the growth and production functions used by Spence (1974) to characterize the blue whale led to the following Lagrangian:

$$L = \sum_{t=0}^{\infty} \rho^t \{pAX_t^a \, [1 - e^{-bE_t}] - cE_t + \rho\lambda_{t+1}[AX_t^a \, e^{-bE_t} - X_{t+1}]\} \qquad \text{(A.1)}$$

The first order conditions require:

$$\frac{\partial L}{\partial E_t} = \rho^t\{bpAX_t^a e^{-bE_t} - c - b\rho\lambda_{t+1}AX_t^a \, e^{-bE_t}\} = 0 \qquad \text{(A.2)}$$

$$\frac{\partial L}{\partial X_t} = \rho^t\{apAX_t^{a-1}[1 - e^{-bE_t}] + a\rho\lambda_{t+1}AX_t^{a-1} \, e^{-bE_t}\} - \rho^t\lambda_t = 0 \qquad \text{(A.3)}$$

$$\frac{\partial L}{\partial \lambda_{t+1}} = \rho^{t+1}\{AX_t^a \, e^{-bE_t} - X_{t+1}\} = 0 \qquad \text{(A.4)}$$

Equations (A.1) through (A.4) may be rewritten as

$$bAX_t^a \, e^{-bE_t}[p - \rho\lambda_{t+1}] = c, \qquad \text{(A.5)}$$

$$apAX_t^{a-1} - aAX_t^{a-1}e^{-bE_t} \, [p - \rho\lambda_{t+1}] - \lambda_t = 0, \qquad \text{(A.6)}$$

$$AX_t^a e^{-bE_t} - X_{t+1} = 0 \qquad \text{(A.7)}$$

In steady state:

$$[p - \rho\lambda] = \frac{c}{bAX^a \, e^{-bE}} \qquad \text{(A.8)}$$

$$apAX^{a-1} - aAX^{a-1}e^{-bE}[p - \rho\lambda] - \lambda = 0, \qquad \text{(A.9)}$$

$$AX^a \, e^{-bE} = X. \qquad \text{(A.10)}$$

Solving (A.8) for λ and using (A.10) yields:

$$\lambda = 1+\delta \{p - \frac{c}{bAX^{a}e^{-bE}}\} = (1+\delta) \{p - \frac{c}{bX}\} \qquad (A.11)$$

Substituting (A.8) and (A.11) into (A.9) yields

$$apAX^{a-1} - aAX^{a-1} e^{-bE} \left[\frac{c}{bAX^{a}e^{-bE}}\right] - (1+\delta) \left[p - \frac{c}{bX}\right] = 0,$$

$$apAX^{a-1} - \frac{ac}{bX} - (1+\delta) \left[\frac{pbX - c}{bX}\right] = 0,$$

$$\frac{apAX^{a-1}}{\left[\frac{pbX - c}{bX}\right]} - \frac{\frac{ac}{bX}}{\left[\frac{pbX - c}{bX}\right]} = (1+\delta),$$

or finally,

$$\frac{abAX^{a} - a(c/p)}{[bX - c/p]} = (1+\delta) \qquad (A.12)$$

Given the bioeconomic parameters a, A, b, (c/p) and δ one can "iterate" (A.12) to solve for the optimal stock of blue whales. This was done by Spence for various values of (c/p) with $\delta = 0.05$ and the previously noted estimates of a, A, and b based on IWC data. This equation was used to generate the values of X^{*} shown in Tables 2 and 3 in the text of this lecture.

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