Management of a Multiple Cohort Fishery:
The Hard Clam in Great South Bay*

by

Jon M. Conrad†
Associate Professor of Resource Economics

February 1981 81-3

*This research was sponsored by the New York Sea Grant Institute under a grant from the Office of Sea Grant, National Oceanic and Atmospheric Administration (NOAA), US Department of Commerce. The US Government (including Sea Grant Office) is authorized to produce and distribute reprints for governmental purposes notwithstanding any copyright notation appearing hereon.

†The conclusions and policy recommendations in this report are made by the author and do not necessarily represent the views and opinions of Sea Grant.

Management of a Multiple Cohort Fishery:
The Hard Clam in Great South Bay

ABSTRACT

Bioeconomics to date has focused almost exclusively on lumped parameter models where the resource is described by a single state variable defined as biomass. While such models are convenient mathematically they may be inappropriate in fisheries where recruitment and fecundity are dependent on the age structure of the resource.

This paper develops a reasonably general multiple cohort model and derives conditions for optimal harvest and age structure based on a discrete time control problem which maximizes the present value of net revenues subject to recruitment and spawning constraints. The model is applied to the hard clam resource in Great South Bay which is located on Long Island, New York. The steady state optimum calls for exclusive harvesting of the younger, (and more valuable), "littleneck" cohorts; leaving the older, (and less valuable), "cherrystone" and "chowder" cohorts to specialize in regeneration.

Management of a Multiple Cohort Fishery: The Hard Clam in Great South Bay

I. Introduction and Overview

Most of the recent economic models of renewable resources have presumed that the resource in question can be adequately described by a single index or state variable. Such a view is consistent with much of the classical literature in physical biology where population growth was described using the logistic equation (see A. J. Lotka, 1956). Such models may be inadequate, however, when dealing with a resource where recruitment and fecundity are dependent on the population's age structure; or where the revenues and costs of resource exploitation vary with age at harvest.

The development of multiple cohort bioeconomic models is mathematically complex. With regard to models of commercial fishing Colin Clark has noted:

> Most fish populations, for example, consist of several different ages; both commercial value and reproductive potential generally depend on the age of the individual fish. These phenomena are often highly significant in determining optimal policies. But including age structure in the analysis introduces significant new mathematical difficulties. Indeed, the problem of the optimal harvesting of age-distributed populations remains unsolved (Clark, 1976, p. 256).

In the next section we will develop a reasonably general multiple cohort model which allows for a finite number of cohorts or year classes. Cohorts which are commercially harvested are characterized by a transition equation, average weight, price per pound, and a stock dependent cost function. The reproductive potential of adult cohorts is described by a fecundity index. Rules for optimal harvest and age structure are derived from a discrete time control problem which maximizes the present value of net revenues subject to recruitment and spawning constraints.

In section three the model is applied to the hard clam, (Mercenaria, mercenaria), in Great South Bay on Long Island, New York. The relevant economic and biological processes are specified and parameters for price, cost, mortality, and fecundity are presented. A steady state optimum is derived and its management implications discussed.

The final section summarizes the results of the general model and the application to the hard clam resource. Conclusions and management implications are discussed, and the value of applied bioeconomic research in estimating the gains from management, (vis-a-vis transactions costs to achieve those gains), is noted.

II. A Multiple Cohort Model

The model to be developed relies on the following notation:

$t = 0,1,2, \ldots, \infty$ — year index

$k = 0,1,2, \ldots, K$ — year class or cohort index, where $k = 0 =$ larval stage and $k = K =$ oldest year class.

$X_{k,t} =$ — the number of fish of the k^{th} year class in the t^{th} year.

$S_0 =$ — the survival rate for larvae, invariant to the size of the larval cohort.

$S_k(X_{k,t}) =$ — the survival function for juveniles and adults, exclusive of fishing mortality.

$\gamma_k =$ — spawning rate of the k^{th} year class, where $k = k^*, \ldots, K-1$. Thus k^* is the youngest year class to spawn and K-1 is the oldest.

$Y_{k,t} =$ — the number of fish of the k^{th} class caught in the t^{th} year; $k = \hat{k}, \ldots, K-1$. Thus $\hat{k}$ is the youngest year class to be harvested and K-1 is the oldest.

$B_k =$ — average weight for a member of the k^{th} year class.

$P_k =$ — Market price per unit of cohort k.

$C_{k,t} = C_k(Y_{k,t}, X_{k,t}) =$ — total cost of catching $Y_{k,t}$ with starting stock of $X_{k,t}$ such that $\frac{\partial C_k(\cdot)}{\partial Y_{k,t}} > 0$ $\frac{\partial C_k(\cdot)}{\partial X_{k,t}} < 0$.

$N_{k,t}$ = net revenues from the k^{th} class in the t^{th} year.

$\rho = \frac{1}{1+\delta}$ = discount factor, where δ = discount rate.

Four aspects of the model should be emphasized

1. A particular cohort is spawned and moves through a juvenile stage until it reaches maturity as year class k* where it will spawn for the first time.

2. It is assumed that there is no harvest of cohorts $k < \hat{k}$, nor of the last year class, $(k = K)$.

3. In each period it is assumed that the mature year classes spawn before natural and fishing mortality take their toll. The last year class X_K is something of a biological and economic nonentity. It does not contribute to spawning ($\gamma_K = 0$) nor is it commercially harvested ($Y_{K,t} = 0$). Including such a class in the problem, however, facilitates solution and analysis of steady state equilibrium, with no great loss in realism.

4. For juveniles and adults the survival function is such that $0<S_k(\cdot)<X_{k,t}$ for $X_{k,t}>0$ and $S_k(0) = 0$ for $k = 1, \ldots, K-1$. For the terminal cohort $S_K(\cdot) = 0$ for all $X_{K,t}$. One might expect that natural survival might be enhanced over an initial range of $X_{k,t}$, but at higher cohort densities the rate of survival would diminish. Thus $S_k'(\cdot) \gtreqless 0$.

In any year the number of larvae will be given by

$$X_{o,t} = \sum_{k=k^*}^{K-1} \gamma_k X_{k,t} \tag{1}$$

With no harvest of the larval year class, and with a constant survival rate the number of first year juveniles would be

$$X_{1,\ t+1} = S_o \sum_{k=k^*}^{K-1} \gamma_k X_{k,t} \tag{2}$$

The transition for precommercial cohorts will occur according to $X_{k+1,\ t+1} = S_k(\cdot)$. By substitution it is possible to express the stock of the harvestable cohort $\hat{k}$ in period $t+\hat{k}$ as a function of adult stocks in period t; specifically

$$X_{\hat{k},\ t+\hat{k}} = R_{t+\hat{k}} \left(\sum_{k=k^*}^{K-1} \gamma_k X_{k,t}\right) \tag{3}$$

Equation (3) will be referred to as the "spawning constraint."

For cohorts $\hat{k} < k \leq K$ the transition equation would include deductions for yield (or fishing mortality) so that

$$X_{k+1,\ t+1} = S_k(\cdot) - Y_{k,t} \qquad \text{for } k = \hat{k},\ \ldots,\ K-1 \tag{4}$$

Equation (4) is referred to as the "recruitment constraint." We will assume that $k^* \geq \hat{k}$; that is, that sexual maturity is achieved at the same age or after commercial maturity. In this case equations (3) and (4) will be functions of only the commercial cohorts.

Presuming commercial harvest is profitable for cohorts $k = \hat{k},\ \ldots,\ K-1$ the net revenues from the harvest of the k^{th} year class in period t would be given by

$$N_{k,t} = [P_k B_k Y_{k,t} - C_k(\cdot)] \tag{5}$$

Summing over all harvestable year classes we could calculate aggregate net revenues according to

$$N_t = \sum_{k=\hat{k}}^{K-1} [P_k B_k Y_{k,t} - C_k(\cdot)] \tag{6}$$

The present value of all such net revenues may be written as

$$N = \sum_{t=0}^{\infty} \rho^t \ \{ \sum_{k=\hat{k}}^{K-1} [P_k B_k Y_{k,t} - C_k(\cdot)]\} \tag{7}$$

Maximization of the present value of net revenues subject to spawning and recruitment constraints could be accomplished by formulating the Lagrangian expression

$$L = \sum_{t=0}^{\infty} \rho^t \ \{ \sum_{k=\hat{k}}^{K-1} [P_k B_k Y_{k,t} - C_k(\cdot)] + \rho^{\hat{k}} \lambda_{\hat{k},\ t+\hat{k}} [R_{t+\hat{k}}(\cdot) - X_{\hat{k},\ t+\hat{k}}] + \sum_{k=\hat{k}}^{K-1} \rho \lambda_{k+1,\ t+1} [S_k(\cdot) - Y_{k,t} - X_{k+1,\ t+1}]\} \tag{8}$$

first order necessary conditions require

$$\frac{\partial L}{\partial Y_{k,t}} = \rho^{t} \{[P_k B_k - \frac{\partial C_k(\cdot)]}{\partial Y_{k,t}} - \rho\lambda_{k+1,\ t+1}\} \qquad \text{for } k = \hat{k}, \ldots, K-1 \qquad (9)$$

$$\frac{\partial L}{\partial X_{k,t}} = \rho^{t} \{[- \frac{\partial C_k(\cdot)]}{\partial X_{k,t}} + \rho\lambda_{k+1,\ t+1} S_k'(\cdot)\} - \rho^{t}\lambda_{k,t} = 0 \qquad (10)$$

$$\text{for } k = \hat{k}, \ldots, k^*-1$$

$$\frac{\partial L}{\partial X_{k,t}} = \rho^{t} \{[- \frac{\partial C_k(\cdot)]}{\partial X_{k,t}} + \rho^{\hat{k}}\lambda_{\hat{k},\ t+\hat{k}} \frac{\partial R_{t+\hat{k}}(\cdot)}{\partial X_{k,t}} + \rho\lambda_{k+1,\ t+1} S'_k(\cdot)\} - \rho^{t}\lambda_{k,t} = 0 \qquad (11)$$

$$\text{for } k = k^*, \ldots, K-1$$

$$\frac{\partial L}{\partial \lambda_{\hat{k},\ t+\hat{k}}} = \rho^{t+\hat{k}} [R_{t+\hat{k}}(\cdot) - X_{\hat{k},\ t+\hat{k}}] = 0 \qquad (12)$$

$$\frac{\partial L}{\partial \lambda_{k+1,\ t+1}} = \rho^{t+1} [S_k(\cdot) - Y_{k,t} - X_{k+1,\ t+1}] = 0 \qquad (13)$$

$$\text{for } k = \hat{k}, \ldots, K-1$$

The last terms in equations (10) and (11) are obtained by "backing up" one term in the Lagrangian to year (t-1) and to term (k-1) within the sum of recruitment constraints and differentiating with respect to $X_{k,t}$.

To facilitate interpretation the first order conditions may be simplified to

$$P_k B_k = \frac{\partial C_k(\cdot)}{\partial Y_{k,t}} + \rho\lambda_{k+1,\ t+1} \qquad \text{for } k = \hat{k}, \ldots, K-1 \qquad (14)$$

$$\lambda_{k,t} = \frac{-\partial C_k(\cdot)}{\partial X_{k,t}} + \rho\lambda_{k+1,\ t+1} S_k'(\cdot) \qquad \text{for } k = \hat{k}, \ldots, k^*-1 \qquad (15)$$

$$\lambda_{k,t} = \frac{-\partial C_k(\cdot)}{\partial X_{k,t}} + \rho^{\hat{k}}\lambda_{\hat{k},\ t+\hat{k}} \frac{\partial R_{t+\hat{k}}(\cdot)}{\partial X_{k,t}} + \rho\lambda_{k+1,\ t+1} S_k'(\cdot) \qquad (16)$$

$$\text{for } k = k^*, \ldots, K-1$$

$$X_{\hat{k},\, t+\hat{k}} = R_{t+\hat{k}}(\cdot) \tag{17}$$

$$X_{k+1,\, t+1} = S_k(\cdot) - Y_{k,t} \qquad \text{for } k = \hat{k}, \ldots, K-1 \tag{18}$$

Equation (14) requires that harvests be arranged so the marginal revenue is equated to the sum of marginal harvest and user costs. In this instance user cost is the discounted value of an additional fish in the next oldest cohort, one year in the future. An additional fish harvested today would reduce the stock of cohort (k+1) in (t+1) with certainty.

Equations (15) and (16) provide rules for managing cohort stocks. For juveniles equation (15) requires that the stock of cohort k in year t be maintained so that its marginal imputed value ($\lambda_{k,t}$) equals the sum of stock related marginal cost ($-\frac{\partial C_k(\cdot)}{\partial X_{k,t}}>0$) and user cost which is discounted for time and survival ($\rho\lambda_{k+1,\, t+1}\, S_k'(\cdot)$). If $S_k(\cdot) = S_k X_{k,t}$ where $0 < S_k < 1$, then S_k could be interpreted as the probability that an unharvested member of cohort k will survive to become a member of the k+1 cohort. In a sense the future values in equations (15) and (16) are being discounted for time and risk; the risk being that a fish left unharvested today may die from natural causes before it can be harvested tomorrow.

Equation (15) applies to juviniles who do not contribute to spawning; that is, cohorts $k = \hat{k}, \ldots, k^*-1$. Equation (16) applies to cohorts that do spawn (i.e., $k = k^*, \ldots, K-1$). The stocks of spawners must be maintained so as to equate marginal imputed value to the sum of stock related marginal cost, user cost, and the discounted value of its progeny who reach commercial size in year $t+\hat{k}$ ($\rho^{\hat{k}}\lambda_{\hat{k},\, t+\hat{k}} \frac{\partial R_{t+\hat{k}}(\cdot)}{\partial X_{k,t}}$).

This latter term is also discounted for time and risky survival. Equations (17) and (18) are restatements of the spawning and recruitment constraints.

A steady or stationary state is characterized by unchanging values for stocks, yields, and shadow prices. Specifically

$$X_{k,t} = X_k;\ Y_{k,t} = Y_k;\ \text{and}\ \lambda_{k,t} = \lambda_k \tag{19}$$

In steady state equilibrium the first order necessary conditions become

$$P_k B_k = \frac{\partial C_k(\cdot)}{\partial Y_k} + \rho\lambda_{k+1} \qquad \text{for } k = \hat{k}, \ldots, K-1 \tag{20}$$

$$\lambda_k = \frac{-\partial C_k(\cdot)}{\partial X_k} + \rho\lambda_{k+1} S_k'(\cdot) \qquad \text{for } k = \hat{k}, \ldots, k^*-1 \tag{21}$$

$$\lambda_k = \frac{-\partial C_k(\cdot)}{\partial X_k} + \rho^{\hat{k}}\lambda_{\hat{k}} \frac{\partial R_{\hat{k}}(\cdot)}{\partial X_k} + \rho\lambda_{k+1} S_k'(\cdot) \qquad \text{for } k = k^*, \ldots, K-1 \tag{22}$$

$$X_{\hat{k}} = R_{\hat{k}}(\cdot) \tag{23}$$

$$X_{k+1} = S_k(\cdot) - Y_k \qquad \text{for } k = \hat{k}, \ldots, K-1 \tag{24}$$

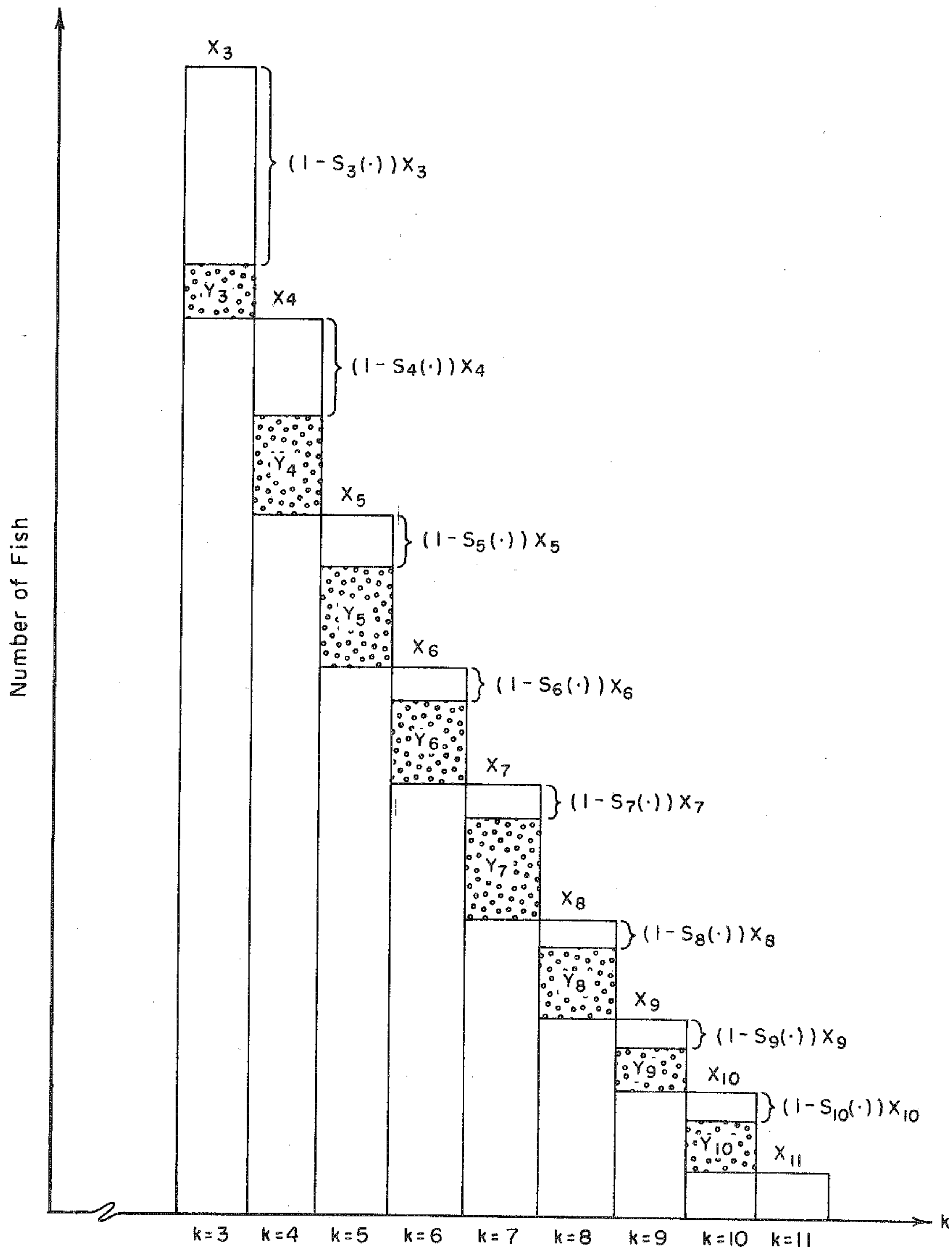
When evaluated in steady state the first order conditions comprise a system of $3(K-\hat{k}) + 1$ equations in $3(K-\hat{k}) + 1$ unknowns, where the unknowns are Y_k $(k = \hat{k}, \ldots, K-1)$, $X_k (k = \hat{k}, \ldots, K)$ and $\lambda_k (k = \hat{k}, \ldots, K-1)$. The previous assumptions regarding the fecundity and commercial value of the K^{th} cohort imply $\lambda_K = 0$.

Suppose $\hat{k}=3$ and $K=11$, then equations (22) - (27) would comprise a system of 25 equations in 25 unknowns the solution of which could be plotted in a bar graph and might look something akin to the equilibrium shown in Figure 1 where X_k and Y_k are optimal steady state stocks and yields respectively.

Several points about steady state equilibrium might be noted:

1. Natural and fishing mortality guarantee that $X_k > X_{k+1}$ for $k = \hat{k}, \ldots, K$. This need not be the case out of equilibrium (for example, along an approach path).

FIGURE I. STEADY STATE EQUILIBRIUM



2. While the steady state stock of older cohorts cannot increase the yield from such cohorts may decrease, remain constant, or increase; i.e., $Y_k \lesseqqgtr Y_{k+1}$, depending on economic and biological factors.

3. While the steady state stock of successively older cohorts cannot increase, their biomass (equal to $B_k X_k$) may decrease, remain constant, or increase.

4. If $\lambda_K = 0$ why would $X_K > 0$? A positive level for the last cohort might be maintained if it was not economic to completely harvest the (K-1) cohort. One would suspect that as $X_{K-1} \rightarrow 0$, $C_{K-1}(\ ) \rightarrow \infty$. If it is not profitable to harvest all of the next to last year class some will survive to become members of the last cohort.

5. As Clark (1973) has noted it is possible that economic factors might lead to the extinction of certain commercial species. This could occur if the market value of the last few individuals exceed their harvest cost. In a multiple cohort model we might observe truncation of some of the older cohorts, with remaining (younger) adults providing for regeneration of the species; i.e., $X_k = 0$ for $\bar{k}, \ldots, K$; and $X_0 = \sum_{k=k^*}^{\bar{k}-1} \lambda_k X_k$.

III. The Hard Clam in Great South Bay

The hard clam (Mercenaria, mercenaria) is a bivalue mollusk found in intertidal and subtidal zones from Cape Cod, Massachusetts to the Gulf of Mexico. Great South Bay, on the southern edge of Long Island, New York, supports a highly productive hard clam fishery. With the exception of a 13,000 acre tract which is privately owned, (dating from a colonial grant), the resource is harvested on an open access basis. The recent decline in the level of clams harvested from the bay is symptomatic of overfishing. Part of the problem undoubtedly stems from the fact that a premium is placed on the youngest legally harvestable cohorts (k=3,4,5); collectively referred to as "littlenecks", and generally eaten raw on the half shell. Table 1 shows the approximate relationship between shell thickness at the hinge, commercial classification, (littleneck, cherrystone, and chowder), and price per bushel. Because each harvestable year class falls into a commercial classification we do not need to explicitly

Table 1: Average Growth and Commercial Class

Year Class (a)	Commercial Class	Price per Bushel (b)
k = 0, 1, 1 shell thickness < 25.4 mm	Sublegal	----
k = 3, 4, 5 shell thickness 25.4-36.4 mm	Littleneck	\$63-65
k = 6, 7 shell thickness 36.4-41.2 mm	Cherrystone	\$22-23
k = 8, 9, 10 shell thickness 41.3 mm +	Chowder	\$11

Source: (a) Smith (1971, p. 27) (b) National Marine Fisheries Service, Fishery Market News Report, October 1, 1980, p. 3

consider the weight of a clam meat for each cohort and we may dispense with B_k as it was defined in the general model.

Current state regulations set a minimum size of one inch or 25.4 mm at the hinge before a clam may be legally harvested. This size is reached at about three years of age. The first year of significant spawning also occurs at age three, so it was assumed that $\hat{k}=k^*=3$. More than 95% of the commercial harvest consists of cohorts $3 \leq k \leq 10$ allowing one to set K=11. Based on the prices in Table 1 values for P_k were adopted according to

$$P_k = \begin{cases} \$64/\text{bushel for } k = 3,4,5 \\ \$23/\text{bushel for } k = 6,7 \\ \$11/\text{bushel for } k = 8,9,10 \end{cases} \tag{25}$$

The biology of the hard clam has been studied extensively dating from the classic work by Belding (1912). The first three columns of Table 2 are reported in Smith (1979) and present estimates of the instanteous mortality rate for half year intervals from age one to five. These instantaneous rates were used to solve for a semi-annual mortality rate. The complement of the semi-annual mortality rate is the semi-annual survival rate, shown in column five. Semi-annual survival rates were multiplied together to obtain the fraction of a cohort surviving to the next oldest year class. For cohort four the annual survival rate of 0.67 was not consistent with the trend in earlier survival rates and what is generally regarded to be increased rates of survival as a hard clam outgrows certain predators. Therefore S_4 was arbitrarily adjusted upward to 0.80. The annual survival rate for cohorts k = 5,6, ..., 10 seems to level off at 0.85 according to Greene (1978, p. 54) and Smith (1979, p. 49).

This author was unable to find any published estimates of mortality or survival from spawning to year one. Given the fecundity of the hard clam, (to be discussed shortly), one must assume that the survival rate is exceedingly low. An estimate of $S_o = 2.0 \times 10^{-9}$ was obtained by the admittedly ad hoc

Table 2: Mortality and Survival of Hard Clams in Great South Bay

Age (years=k)	Length (mm) (a)	Instantaneous Mortality (a) (m(kj)) j=1,2	Semi-Annual Mortality (b) (M(k,j))	Semi-Annual Survival (c) (S(k,j))	Annual Survival (d) S_k
0	< 15	---	---	---	---
					2.0×10^{-9} (e)
1.0	15.0				
		0.42	0.52	0.48	
1.5	24.5				0.35
		0.24	0.27	0.73	
2.0	34.0				
		0.16	0.17	0.83	
2.5	41.0				0.69
		0.16	0.17	0.83	
3.0	48.0				
		0.13	0.14	0.86	
3.5	53.5				0.73
		0.14	0.15	0.85	
4.0	59.0				
		0.16	0.17	0.83	
4.5	62.5				0.67 (f)
		0.17	0.19	0.81	
5.0	66.0				
6,7,8,9,10	> 66.0	---	---	---	0.85 (g)

Source or Method:
(a) Smith (1979, p. 90)
(b) $M(k,j) = e^{m(k,j)} - 1$
(c) $S(k,j) = 1 - M(k,j)$
(d) $S_k = S(k,1)\ S(k,2)$
(e) See Appendix A
(f) Questionable - opt for $S_4 = 0.80$
(g) Greene (1978, p. 54) Smith (1979, p. 49)

procedure described in Appendix A. Based on previous biological studies and the calculations in Appendix A it was assumed that the rate of survival was not dependent on cohort size; that is, $S_k(\cdot) = S_k X_{k,t}$, and specifically that

$$S_k = \begin{cases} 2.0 \times 10^{-9} & \text{for } k=0 \\ 0.35 & \text{for } k=1 \\ 0.69 & \text{for } k=2 \\ 0.73 & \text{for } k=3 \\ 0.80 & \text{for } k=4 \\ 0.85 & \text{for } k=5,6,\ \ldots,\ 10 \end{cases} \qquad (26)$$

Bricelj (1979) has studied the reproductive biology of the hard clam and has derived estimates of the number of eggs spawned per female at various sizes. Because size and age is highly correlated the fecundity for a bushel of standing stock for each cohort could be calculated. These fecundities (γ_k) are given in Table 3 and may be summarized according to

$$\gamma_k = \begin{cases} 7.50 \times 10^8 & \text{for } k = 3,4,5 \\ 8.25 \times 10^8 & \text{for } k = 6,7 \\ 5.25 \times 10^8 & \text{for } k = 8,9,10 \end{cases} \qquad (27)$$

Great South Bay is a shallow body of water attaining a maximum depth of 15-20 feet. Commercial harvest is restricted to hand tongs and rakes. The commerical fisherman or "bayman" will typically work from a small shallow draft vessel, propelled by an outboard motor. Thus there is a limited capital investment to enter the fishery. Estimates of yearly fixed costs range from \$500 to \$1,000. Determination of optimal harvest and age structure will be determined based only on variable cost considerations. When maintained at a steady state optimum it will be possible to make a comparison between net variable revenues, (or fishery rents), and fixed costs payments to determine the ultimate "bottom line."

The stock dependent variable cost function specified for the hard clam model took the form

$$C_k = B_k Y_{k,t} \left(\ln\left(\frac{X_{k,t}}{Y_{k,t}}\right)\right)^{-1} \qquad (28)$$

Table 3: Fecundity of the Hard Clam in Great South Bay

Year class (commercial class)	Eggs per female $\bar{F}_k$ (a)	Clams per bushel U_k (b)	Fecundity per bushel of standing stock γ_k (c)
k = 1, 2 (sublegal)	1.5×10^6	--	--
k = 3, 4, 5 (littleneck)	3.0×10^6	500	7.50×10^8
k = 6, 7 (cherrystone)	6.0×10^6	275	8.25×10^8
k = 8, 9, 10 (chowder)	6.0×10^6	175	5.25×10^8

Source or Method: (a) Bricelj (1979, p. 83)
(b) Smith (1979, p. 27)
(c) $\gamma_k = \frac{1}{2} U_k \bar{F}_k$

where B_k may be interpreted as a unit cost per bushel and the term $(\ln(\frac{X_{k,t}}{Y_{k,t}}))^{-1}$

is a stock dependent term which reduces the cost of a fixed level of harvest for larger initial cohort stocks. The principal components of B_k would be the opportunity cost of the baymans time and the fuel expended in harvesting a bushel of clams. It was assumed that

$$B_k = \$25 \qquad \text{for } k = 3, \ldots, 10 \tag{29}$$

given that under current stock conditions it takes four to five hours to harvest a mixed bushel of littlenecks, cherrystones, and chowders.

Equations (25) - (29) summarize all the parameter values and functional forms sufficient to formulate a well defined multiple cohort model. Maximization of the present value of net revenues subject to spawning and recruitment constraints may be written as

$$\max_{\{Y_{k,t}\}} V = \sum_{t=0}^{\infty} \rho^t \{ \sum_{k=3}^{10} (P_k Y_{k,t} - C_k(X_{k,t}, Y_{k,t}))\}$$

$$\text{subject to: } X_{3,\ t+3} = S_2 S_1 S_0 \sum_{k=3}^{10} \gamma_k X_{k,t} \tag{30}$$

$$X_{k+1,\ t+1} = S_k X_{k,t} - Y_{k,t} \qquad (\text{for } k = 3, \ldots, 10)$$

The Lagrangian and first order necessary conditions are derived in Appendix B. We will immediately proceed to the set of equations describing steady state. These may be written as:

$$P_k = B_k(\ln(\frac{X_k}{Y_k}))^{-1} (1 + (\ln(\frac{X_k}{Y_k}))^{-1}) + \rho\lambda_{k+1} \tag{31}$$

$$\text{for } k = 3, \ldots, 10$$

$$\lambda_{k,t} = B_k(\frac{Y_k}{X_k}) (\ln(\frac{X_k}{Y_k}))^{-2} + \rho S_k \lambda_{k+1} + \rho^3 S_2 S_1 S_0 \gamma_k \lambda_3 \tag{32}$$

$$\text{for } k = 3, \ldots, 10$$

$$X_3 = S_2 S_1 S_o \sum_{k=3}^{10} \gamma_k X_k \tag{33}$$

$$X_{k+1} = S_k X_k - Y_k \tag{34}$$

As noted in the preceeding section, with $\hat{k}=3$ and K=11 steady state is defined by a set of 25 simultaneous equations in 25 unknowns, the solution of which might look something like the equilibrium shown earlier in Figure 1.

It turns out that equations (31) - (34) can be solved iteratively by a technique described in Appendix C. The solution, given the preceeding assumptions on cost, price, mortality, and fecundity, is presented in Table 4. The optimum is characterized by the nearly exclusive harvesting of littleneck cohorts (k = 3,4,5). In the solution sequence an arbitratary lower bound of $Y_k \geq 1$ for k = 3, ..., 10 was set to avoid division by zero. The near zero harvest of cherrystone cohorts, (k = 6,7), and chowder cohorts (k = 8,9) would indicate that the dominant value for these cohorts lies in their ability to provide for future littleneck stocks. This fact is also reflected in cohort shadow prices such that $\lambda_k > 23$ for k = 6,7 and $\lambda_k > 11$ for k = 8,9. In words, these cherrystone and chowder cohorts are more valuable in Great South Bay than on the half-shell or in soup. A slight harvest of chowders (Y_{10} = 15,142 bushels) is optimal before the cohort passes into bioeconomic oblivion.

The annual sustainable revenue is $54,411,814 which is 3.75 times the revenue reported in 1979. Variable costs are $12,210,264. If there are n = 3,000 full time baymen with annual fixed costs of $500 the annual revenue net of variable and fixed costs would be $40,701,550. Gross income per baymen would be $18,137. The total standing stock of 7,523,156 bushels is within the high and low estimates of standing stock given by Smith (1979, pp. 29-34). Thus the managed fishery would appear to provide for considerably greater income per bayman than is currently the case.

Table 4: Steady State Optimum for the Hard Clam Fishery in Great South Bay

Stocks (bushels)	Yield (bushels)	Shadow Price ($)
X_{11} = 250,000	Y_{11} = 0	λ_{11} = 0
X_{10} = 311,932	Y_{10} = 15,142	λ_{10} = 9.50
X_9 = 366,980	Y_9 = 1	λ_9 = 16.72
X_8 = 431,742	Y_8 = 1	λ_8 = 22.30
X_7 = 507,933	Y_7 = 1	λ_7 = 31.99
X_6 = 597,569	Y_6 = 1	λ_6 = 39.48
X_5 = 957,000	Y_5 = 215,881	λ_5 = 46.49
X_4 = 1,510,000	Y_4 = 251,000	λ_4 = 48.54
X_3 = 2,590,000	Y_3 = 380,700	λ_3 = 49.30

$X = \sum_{k=3}^{11} X_k$ 7,523,156 $R = \sum_{k=3}^{10} P_k Y_k = \$54,411,814$ $C = \sum_{k=3}^{10} C_k = \$12,210,264$

$F = nF_n = \$1,500,000$ $\pi = R-C-F = \$40,701,550$

The management policies to achieve these gains are simple and would seem feasible to implement. In addition to the current legal minimum thickness (measured at the hinge) the New York State Department of Environmental Conservation should establish a legal maximum of 36.4 mm (see Table 1). This would eliminate harvesting of cherrystone and chowder cohorts. In the short run they may have to impose catch quotas less than the steady state levels for $k = 3,4,5$ so as to establish the equilibrium levels for cherrystone and chowder stocks. Even in equilibrium quotas on the littleneck landings would have to be imposed to maintain cohort stocks. Such quotas may require a limitation on the number of full time baymen, and possibly a catch quota per man.

IV Summary, Conclusions, and Limitations

The bioeconomic modelling of age structured species has heretofore been limited by a number of unrealistic assumptions necessary for mathematical tractability. This paper develops a reasonably general multiple cohort model which would seem applicable to a wide variety of renewable resources where age structure is important for biological or economic reasons.

One such resource is the hard clam (Mercenaria, mercenaria) in Great South Bay on Long Island. Almost all the commercial harvest is comprised of clams 10 years of age or younger. Minimum legal size is typically reached at about three years of age, which is also the age where significant spawning takes place for the first time. Thus a model containing eight, ($k = 3, \ldots, 10$), commercially exploited cohorts seemed adequate to describe the fishery. Steady state equations were derived from a discrete time control model which maximized the present value of net revenues subject to spawning and recruitment constraints. This system of 25 equations in 25 unknowns (cohort stocks, yields, and shadow prices) was solved in an interative fashion and converged to a solution where harvesting activity was restricted to the littleneck cohorts ($k = 3,4,5$), but permitting sufficient survival to cherrystone and chowder cohorts, (k = 6,7 and k = 8,9,10 respectively), to allow for regeneration of littleneck stocks.

Management prescriptions called for the establishment of a legal maximum which would prohibit the harvest of cherrystone and chowder cohorts. These cohorts are more valuable in Great South Bay as spawning stock than in the market. Short run dynamics may require reduced littleneck landings to establish the necessary cherrystone and chowder cohorts; and in equilibrium it would be necessary to impose baywide quotas so as not to redissipate fishery rents. Limited entry coupled with a catch quota per baymen would seem a feasible way to maintain the optimal harvest and age structure.

Applied bioeconomic modelling requires a considerable amount of information. In the general form described in Section I information on survival, fecundity, growth, market price and harvesting cost was required. In the more valuable commercial fisheries, biologists and economists may have assembled much of the information necessary for applied modelling.

Establishing a managed fishery, operating at or near a bioeconomic optimum, is not a costless undertaking. This study, as with most previous empirical studies, has not assessed the cost of achieving and maintaining optimal fishery conditions. With respect to the hard clam, Great South Bay is a shallow and highly accessable body of water, and enforcement of any kind of program of quotas and limited entry would be expensive. Perhaps the principal value of applied bioeconomic studies lies in their ability to estimate the opportunity cost associated with an open access or a poorly managed fishery. Depending on the magnitude of this opportunity loss a decision can be made as to whether the gains from management can be expected to exceed research, administration, and enforcement costs.

Appendix A: Calculation of the First Year Survival Rate S_o

In steady state equilibrium the number of three year old littlenecks will equal

$$X_3 = S_2 S_1 S_o \sum_{k=3}^{10} \gamma_k X_k \qquad (A.1)$$

solving for S_o we obtain

$$S_o = \frac{X_3}{S_2 S_1 \sum_{k=3}^{10} \gamma_k X_k} \qquad (A.2)$$

Shellfish biologists currently believe that third year littlenecks constitute about one-third of the standing stock; i.e., $X_3 = 0.33X$, where X is the unknown standing stock equal to the sum of all commercial littleneck,cherrystone and chowder stocks, and that collectively littlenecks constitute 60% of the standing stock, (0.60X), while cherrystones and chowders each constitute 20% of the standing stock, (0.20X). Then with the values for S_2, S_1 and γ_k given in Tables 2 and 3 in the text we can calculate a value for S_o such that

$$S_o = \frac{0.33X}{S_2 S_1 [\gamma_l (0.60X) + \gamma_{cr}(0.20X) + \gamma_{co}(0.20X)]}$$

$$S_o = \frac{0.33 \times 10^{-8}}{(.69)(.35)[7.5(0.60) + 8.25(0.20) + 5.25(0.20)]}$$

$$S_o = 1.90 \times 10^{-9} \simeq 2.0 \times 10^{-9}$$

Appendix B: Derivation of First Order Necessary Conditions in the Hard Clam Model

For the hard clam model summarized by the optimization problem in equation (30) in the text, the Lagrangian expression becomes

$$\max_{\{Y_{k,t}\}} L = \sum_{t=0}^{\infty} \rho^t \{\sum_{k=3}^{10} (P_k Y_{k,t} - C_k(X_{k,t}, Y_{k,t})) + \rho^3 \lambda_{3,\ t+3} (S_2 S_1 S_o \sum_{k=3}^{10} \gamma_k X_{k,t} - X_{3,\ t+3}) + \rho \sum_{k=3}^{10} \lambda_{k+1,\ t+1} (S_k X_{k,t} - Y_{k,t} - X_{k+1,\ t+1})\} \tag{B.1}$$

Values of $Y_{k,t}$, $X_{k,t}$, and $\lambda_{k+1,\ t+1}$, (the Lagrange multipliers) which maximize the present value of net revenues must satisfy a first order condition which equates the partial derivatives of L to zero. These derivatives may be written as

$$\frac{\partial L}{\partial Y_{k,t}} = \rho^t \{(P_k - \frac{\partial C_k(\cdot)}{\partial Y_{k,t}}) - \rho \lambda_{k+1,\ t+1}\} = 0 \tag{B.2}$$

$$\frac{\partial L}{\partial X_{k,t}} = \rho^t \{- \frac{\partial C_k(\cdot)}{\partial X_{k,t}} + \rho^3 S_2 S_1 S_o \gamma_k \lambda_{3,\ t+3} + \rho S_k \lambda_{k+1,\ t+1}\} - \rho^t \lambda_{k,t} = 0 \tag{B.3}$$

$$\frac{\partial L}{\partial \lambda_{3,\ t+3}} = \rho^{t+3} \{S_2 S_1 S_o \sum_{k=3}^{10} \gamma_k X_{k,t} - X_{3,\ t+3}\} = 0 \tag{B.4}$$

$$\frac{\partial L}{\partial \lambda_{k+1,\ t+1}} = \rho^{t+1} \{S_k X_{k,t} - Y_{k,t} - X_{k+1,\ t+1}\} = 0 \tag{B.5}$$

With $C_k(\cdot) = B_k Y_k \left(\ln \frac{X_{k,t}}{Y_{k,t}}\right)^{-1}$ the partial derivatives with respect to $Y_{k,t}$ and $X_{k,t}$ simplify to

$$\frac{\partial C_k(\cdot)}{\partial Y_{k,t}} = B_k(\ln(\frac{X_{k,t}}{Y_{k,t}}))^{-1} \; (1+(\ln(\frac{X_{k,t}}{Y_{k,t}}))^{-1}) \tag{B.6}$$

and

$$\frac{\partial C_k(\cdot)}{\partial X_k} = -B_k(\frac{Y_{k,t}}{X_{k,t}}) \; (\ln(\frac{X_{k,t}}{Y_{k,t}}))^{-2} \tag{B.7}$$

In steady state the first order conditions become

$$P_k = B_k(\ln(\frac{X_k}{Y_k}))^{-1} \; (1 +(\ln(\frac{X_k}{Y_k})^{-1}) + \rho\lambda_{k+1} \tag{B.8}$$

for $k = 3, \ldots, 10$

$$\lambda_k = B_k(\frac{Y_k}{X_k}) \; (\ln(\frac{X_k}{Y_k}))^{-2} + \rho S_k\lambda_{k+1} + \rho^3 S_2 S_1 S_o \gamma_k \lambda_3 \tag{B.9}$$

for $k = 3, \ldots, 10$

$$X_3 = S_2 S_1 S_o \sum_{k=3}^{10} \gamma_k X_k \tag{B.10}$$

$$X_{k+1} = S_k X_k - Y_k \qquad \text{for } k = 3, \ldots, 10 \tag{B.11}$$

Equations (B.8) - (B.11) are the same as (31) - (34) in the text.

Appendix C: Solution Sequence for Steady State Optimum

Equations (31) - (34) can be expanded and ordered as follows:

$$P_3 = B_3(\ln(\frac{X_3}{Y_3}))^{-1} \; (1 + (\ln(\frac{X_3}{Y_3}))^{-1}) + \rho\lambda_4 \qquad (C.1)$$

$$P_4 = B_4(\ln(\frac{X_4}{Y_4}))^{-1} \; (1 + (\ln(\frac{X_4}{Y_4}))^{-1}) + \rho\lambda_5 \qquad (C.2)$$

$$\vdots$$

$$P_{10} = B_{10}(\ln(\frac{X_{10}}{Y_{10}}))^{-1} \; (1 + (\ln(\frac{X_{10}}{Y_{10}}))^{-1}) \qquad (C.8)$$

$$\lambda_3 = \frac{B_3(\frac{Y_3}{X_3}) \; (\ln(\frac{X_3}{Y_3}))^{-2} + \rho S_3\lambda_4}{(1 - \rho^3 S_2 S_1 S_o \gamma_3)} \qquad (C.9)$$

$$\lambda_4 = B_4(\frac{Y_4}{X_4}) \; (\ln(\frac{X_4}{Y_4}))^{-2} + \rho S_4\lambda_5 + \rho^3 S_2 S_1 S_o \gamma_4 \lambda_3 \qquad (C.10)$$

$$\vdots$$

$$\lambda_{10} = B_{10}(\frac{Y_{10}}{X_{10}}) \; (\ln(\frac{X_{10}}{Y_{10}}))^{-2} + \rho^3 S_2 S_1 S_o \gamma_{10} \lambda_3 \qquad (C.16)$$

$$X_3 = S_2 S_1 S_o \sum_{k=3}^{10} \gamma_k X_k \qquad (C.17)$$

$$X_4 = S_3 X_3 - Y_3 \qquad (C.18)$$

$$X_5 = S_4 X_4 - Y_4 \qquad (C.19)$$

$$\vdots$$

$$X_{11} = S_{10} X_{10} - Y_{10} \qquad (C.25)$$

The iterative procedure to solve (C.1) - (C.25) employed the following steps:

(1) Assume a value for X_{11}. Then (C.25) and (C.8) $\rightarrow [X_{10}, Y_{10}]$

(2) Assume a value for λ_{10}. Then $[X_{10}, Y_{10}, \lambda_{10}]$ + (C.16) $\rightarrow [\lambda_3]$

(3) $[X_{10}, \lambda_{10}]$ + (C.7) + (C.24) $\rightarrow [X_9, Y_9]$

(4) $[X_9, Y_9, \lambda_{10}, \lambda_3]$ + (C.15) $\rightarrow [\lambda_9]$

(5) $[X_9, \lambda_9]$ + (C.6) + (C.23) $\rightarrow [X_8, Y_8]$

.
.
.

(15) $[X_4, \lambda_4]$ + (C.1) + (C.18) $\rightarrow [X_3, Y_3]$

All of the equations are consistent with our assumptions about X_{11} and λ_{10} <u>except</u> equations (C.9) and (C.17) which will yield values for λ_3 and X_3 that may be different from those obtained in steps (2) and (15). Values for λ_{10} and X_{11} can be respecified to bring the entire system into arbitrarily close consistency.

In the process of working through steps (1) - (15) one will encouncer "corner solutions" where for Y_k near zero one observes

$$P_k - \rho\lambda_{k+1} < B_k\left(\ln\left(\frac{X_k}{Y_k}\right)\right)^{-1} \left(1 + \left(\ln\left(\frac{X_k}{Y_k}\right)\right)^{-1}\right) \qquad \text{(C.26)}$$

in which case marginal revenue net of user cost is less than marginal harvest cost. In this instance Y_k was set equal to one, (essentially insignificant), and $X_k = \frac{(X_{k+1} + 1)}{S_k}$. For the parameter values and functional forms defined in (25) - (29) in the text, this iterative procedure yielded the steady state optimum described in Table 4.

REFERENCES

Belding, D. L. 1912. A Report Upon the Quahog and Oyster Fisheries of Massachusetts. Commission of Fish and Game, Boston, Massachusetts.

Bricelj, V. M. 1979. Fecundity and Related Aspects of Hard Clam (Mercenaria, mercenaria) Reproduction in Great South Bay, New York. M.S. Thesis, Marine Environmental Sciences Program, SUNY at Stony Brook.

Clark, C. W. 1976. Mathematical Bioeconomics: The Optimal Management of Renewable Resources. John Wiley & Sons, New York.

Clark, C. W. 1973. "Profit Maximization and the Extinction of Animal Species." Journal of Political Economy 81: 950-961.

Greene, G. T. 1978. Population Structure, Growth, and Mortality of Hard Clams at Selected Locations in Great South Bay, New York. M.S. Thesis, Marine Environmental Science Program, SUNY at Stony Brook.

Lotka, A. J. 1956. Elements of Mathematical Biology. Dover Publications. New York.

National Marine Fisheries Service, October 1, 1980. Fishery Market News Report, (The "green sheet" for Fulton Market), New York.

Smith, C. F. 1979. Aspects of Hard Clam Management in Great South Bay, New York. M.S. Thesis, Marine Environmental Sciences Program, SUNY at Stony Brook.