THE USEFULNESS OF POLYPERIOD LINEAR PROGRAMMING FOR FARM MANAGEMENT ANALYSIS IN DEVELOPING COUNTRIES

by

Eric W. Crawford
Gregory C. Lassiter
Robert A. Milligan

October 1977 No. 77-35

BACKGROUND

This paper grew out of an independent study course organized by the authors and led by Professor Milligan. Our interest was in studying principles and techniques of farm management analysis that are relevant to the special conditions of agriculture in developing countries. Although the nature of subsistence farming and its response to development interventions are increasingly if still imperfectly understood, there remains a need for more careful selection or adaptation of conventional Western techniques of micro-economic analysis to improve their suitability and power in the development context. The material in this paper is intended as a modest contribution in this direction.

Our acknowledgement and thanks go to Peter Matlon, the fourth member of the study group, whose participation in our early discussions was very valuable.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

The Usefulness of Polyperiod Linear Programming for Farm Management Analysis in Developing Countries

by

Eric W. Crawford
Gregory C. Lassiter
Robert A. Milligan [1]

Introduction

Polyperiod linear programming appears to be a powerful tool for farm planning and farm management in developing countries. Linear programming (LP), of which polyperiod linear programming (PLP) is a special type, has been used extensively in the analysis of agricultural firms in developed countries. More recently, LP has been widely applied by farm management specialists as part of agricultural research and development programs in the Third World. The results have been fruitful even though the physical, economic, and social characteristics of the agricultural environment in developing countries differ substantially from those in Europe and North America. Although commercial agriculture is important in many developing countries, a relatively high proportion of farm firms are small and primarily subsistence oriented.

The special utility of PLP derives from its ability to handle the unique role of the time dimension of subsistence-oriented agriculture. Because the timing of inflows and outflows and the maintenance of reserves between periods is critical in such a system, realistic analysis of subsistence farming requires careful attention to the links between periods and to intertemporal resource allocation. PLP is designed precisely to incorporate such interperiod relationships.

In the U. S., PLP has been used almost exclusively for examining growth paths for farm firms; however, it is also particularly suitable for farm planning (choice of enterprise, optimum allocation of resources, etc.) in developing countries. In other words, PLP can be used advantageously in the context of subsistence-oriented agriculture as LP is generally used in the more commercial agricultural environment of the U. S.

[1] Eric W. Crawford and Gregory C. Lassiter are Ph.D. candidates, and Robert A. Milligan is Assistant Professor in the Department of Agricultural Economics, Cornell University.

The body of this paper reviews the main features of PLP for the reader, who is presumed to have a working knowledge of the standard LP format and technique. Later sections discuss more fully how and why polyperiod LP can be a powerful instrument for subsistence-oriented farm planning, and illustrate in detail practical ways to incorporate some of the typical features of such farm systems in the PLP format.

Polyperiod Linear Programming

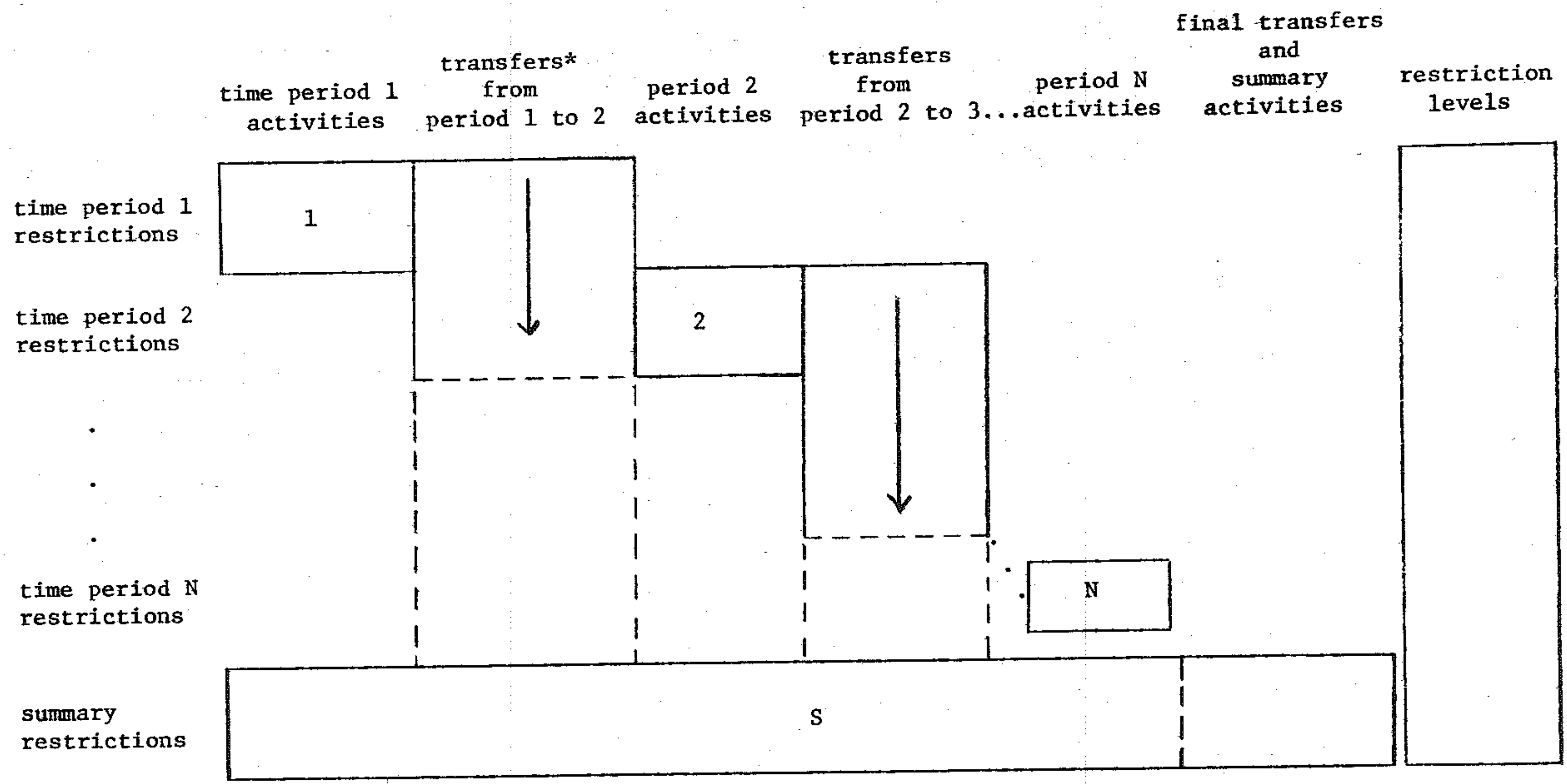
Polyperiod linear programming, also referred to as dynamic linear programming or multi-period linear programming, is a special form of LP in which the tableau or linear programming matrix is structured to incorporate more than one time period, and links between them. PLP is needed only when resources or goods--cash, investments, stores of grain--can be used in a current time period or transferred to a future time period. PLP is not required if the restrictions in various time periods are independent of each other.

Figure 1 shows a generalized illustration of the tableau structure for a PLP problem. Some activities and restrictions are specific to each of the N time periods; others are required to summarize, transfer and/or accumulate incomes, production, savings, etc. across time periods, and to maintain consistency between periods. For example, if an enterprise such as maize is to be undertaken in all time periods, each time period must contain that maize activity. Restrictions or activities affecting a single time period need be entered only once. The coefficients for these activities are located in the appropriate block corresponding to the time period in question, 1, 2, ..., N.

Each time block also contains activities and restrictions for items that can or must be carried over or transferred to the next time period. Examples include crops planted in one period but cultivated and harvested in future time periods, an investment in livestock or physical structures that continue to provide economic services in future time periods, and inventories of assets such as cash, food, or trading goods. Transfer activities are the means by which periods are linked together; they have nonzero coefficient entries in the time block in which the transfer is initiated, and in the later time period(s) where the transfer is received. For example, a period one cash transfer normally adds the residual from the period one cash inventory to the period two cash inventory. Restrictions and activities for periods 1, 2, ..., N will therefore have nonzero entries along the diagonal and to the left (see Figure 1). The area above and to the right of the coefficient blocks will be blank.

Block S in Figure 1 groups together the coefficients for restrictions that pertain to all time periods. RETBAL3 and MAXDEBT are examples of such restrictions, as shown in the detailed PLP model, Figure 3. As noted above, these rows are used for obtaining cumulative totals and maintaining consistency over all time periods.

Figure 1. Diagrammatic Representation of the Tableau Structure for Polyperiod LP.



* Transfers may also be made from period one to any of the periods from 2 to N.

Figure 2 illustrates a very simple PLP model. Maize is produced (grown and harvested) in each of the year's two periods. Cotton can be planted in period 1 and harvested in period 2. It costs $20 to produce maize, with a net return of $40, cotton costs $30 to grow and $10 to harvest, with a gross return of $70. Actual cash returns accrue in the period following harvest. The usual LP restrictions are: a maximum of 8 hectares (ha) of land in each period, and a maximum of 5 ha of maize in period 1. The special PLP features include: (a) activities that transfer cash from period 1 to 2 and period 2 to 3; (b) a restriction that for every hectare of cotton harvested in period 2 a hectare must be planted in period 1; (c) a "net cash" activity that summarizes the cash available for the third period; and (d) an activity and restriction that accumulate the total hectares planted in maize. The cash transfers (a) maintain cash availability during the year. Modelling this important real-life feature requires the use of PLP.

Farm Management in Developing Countries

The nature of farm organization in developing countries poses special problems for the farm management economist. Endemic poverty, agroclimatic factors, and a lack of market integration often result in a rural sector characterized by small scale farming units pursuing goals of self-sufficiency and risk minimization. Enterprise combinations are highly diverse and individualized. In addition, a variety of culturally-derived non-market objectives influence farm management decisions. As a result, conventional farm management analysis must be adjusted to reflect these characteristics.

In particular, the following interrelated aspects of farm organization in developing countries should be recognized in any application of farm management analysis.

1) Subsistence orientation -- Because of poorly developed transportation and communication infrastructure, and a lack of commercial and financial services, market integrating mechanisms are ineffective. Input supply and product marketing do not function smoothly or reliably, especially during seasonal bottleneck periods. This reduces the range of feasible and profitable enterprises open to the farmer, and forces him to become largely self-sufficient. This self-sufficiency requires that farm production provide most family food requirements throughout the agricultural cycle, and also generate the resources needed to meet some non-food needs such as housing, farm tools, and household utensils.

2) Poverty -- Low asset levels, limited land and labor, and restricted access to more productive agricultural technologies also circumscribe the farmer's production and income-earning opportunities, and make it difficult to initiate and sustain a process of capital accumulation and farm expansion or modernization. Without significant food or cash reserves to cushion farm families against output or price variations on a seasonal or year-to-year basis, resource constraints on farm operations often become binding during particular periods within the agricultural cycle.

Figure 2. A Simple Polyperiod LP Tableau.

	$40 Produce Maize (period 1)	-$30 Plant Cotton	0 Transfer Cash (per. 1)	$40 Produce Maize (per. 2)	$60 Harvest Cotton	0 Transfer Cash (per. 2)	0 Total Maize (ha)	0 Net Cash (year 2)	RHS
Land (1) (ha)	1	1							$\leq$ 8
Maize max (ha)	1								$\leq$ 5
Cash (1)	20	30	1						=200
Land (2) (ha)				1	1				$\leq$ 8
Cotton (Plant= harvest)		1			-1				= 0
Cash (2)	-60		-1	20	10	1			= 0
Accumulate maize (ha)	1			1			-1		= 0
Beginning year 2 cash				-60	-70	-1		1	= 0

Note: The solid lines enclose coefficients for periods 1 and 2; the dashed lines outline coefficients for the interperiod transfer and summary activities and restrictions.

3) <u>Risk minimization</u> -- The farmer naturally accords top priority to meeting essential family subsistence needs. For this he depends primarily on his own production. Since the agricultural environment is unstable, and the farmer's own resources are minimal, he is highly vulnerable to risk. This combination of factors leads the farmer to assign more importance to minimizing the risks of his farming operation than to goals such as profit maximization or long-term growth which often conflict with the self-sufficiency objective.

4) <u>Diversification and Complexity</u> -- The combination of self-sufficiency and risk minimization objectives typically results in farm production activities that are numerous, diverse, and tightly interrelated. As a result, the allocation of land, labor, and cash is very complex. On a typical farm, for example, the crop mix is highly variegated, intercropping is the rule and pure stands the exception, planting times are staggered, fields may be fragmented, and the family seasonally engages in non-agricultural activities to complement its agricultural output and earnings.

Although by no means an exhaustive list, these four characteristics of farm organization in developing countries are sufficient to pose a challenge to farm management economists. As demonstrated by several authors (Clayton, Collinson, Heyer, Low), linear programming is one technique that can be useful in such situations. LP permits analysis of the complex configuration of farm planning which accompanies diverse, small-scale, subsistence production by incorporating in the farm model the farm-specific factors of production, and the restrictions implied by the limited availability of productive resources and by marketing constraints or institutional factors. To some extent, production goals other than profit maximization can be incorporated, and culturally specific non-market objectives or restrictions taken into account. For example, risk minimization can be treated by using constraints that either guarantee minimum food stocks and cash balances, force crop diversification, or place greater emphasis on maximizing production in bad years.2/ Minimum production levels can be introduced to reflect domestic tastes and self-sufficiency goals.

The Usefulness of Polyperiod Linear Programming

There is one vitally important factor affecting farm organization in developing countries that cannot be introduced with standard LP--the time dimension of subsistence-oriented agriculture. Several writers (Collinson, Low, Casey) have argued pursuasively that the time pattern of subsistence farming differs distinctively from that of commercial farming, and that the impact of this time pattern dramatically affects farm organization. The relevant planning period is consequently shorter than the annual period typically used in the U. S. In addition, these writers have emphasized the

---

2/ McInerney (1967, 1969) has developed models that embody a maximin objective by incorporating a pay-off matrix directly in the LP constraint set.

existence of critical links between periods which are associated with vital intertemporal flows of cash and labor resources and the carryover of subsistence food stocks.

In subsistence agriculture, the allocation and timing of inputs must be carefully determined in the face of highly variable agroclimatic[3/] conditions in order to maintain an even and reliable output flow consistent with nutritional requirements and family taste. The farmer's aim is not merely to achieve maximum net returns at the end of the season, but to assure domestic consumption needs and seasonal input requirements at key points in time. Because of poverty, the farmer's operating resources are minimal. To survive from period to period, he must have left at the end of one period what he needs to continue in the next. Food stocks, cash supplies, and healthy family labor are the necessities of the farming operation, and the subsistence farmer is much less able than his modern counterpart to purchase or borrow them. Large transportation costs and imperfect or limited markets create regional input shortages and marketing bottlenecks at peak periods which further restrict opportunities to compensate for short-term shortages. Consequently, he must rely on what he produces and carries over from period to period, and the latitude or margin for error in this process is very small.

It is this time dimension of subsistence farming which makes polyperiod linear programming a suitable tool for farm planning. Because a more accurate representation can be made of the time related production conditions that are so characteristic of agriculture in developing countries, the use of PLP can increase the scope and accuracy of farm management analysis beyond that allowed by standard LP. For example, food security and self-sufficiency can be modeled with ordinary LP by requiring a minimum food balance; however, this can more realistically be treated by PLP because the minimum food stock must be maintained not only at the end of the year but at various points throughout the crop cycle, such as the "hungry season" prior to harvest. Similarly, the temporal state of cash balances is often more important than the end of year cash position. In economic terms, PLP provides a mechanism for allocating food stocks and cash reserve by the equimarginal principle rather than by maximums and minimums. For example, the commencement of a farming activity at a time when inputs, such as family labor, are readily available may prove uneconomic if the activity will be completed at a future time when its required inputs will be either scarce or more productively utilized elsewhere. Similarly, assurance of minimum food or cash stocks at future periods affects the allocation of resources at the present time.

The analytical usefulness of PLP as a farm management tool is probably best demonstrated by a brief summary of the types of research questions that it can address. Since PLP permits examination of intertemporal relationships, it can be used to determine how dynamic resource

3/Weather fluctuations plague farmers in temperate zones too, but rainfall variance is generally much higher in the non-equatorial tropics (Lipton). Pest and disease attacks can also be severe, and most farmers in the tropics have little defense against them.

flow constraints, such as temporal input availability, influence decisions on the use of purchased inputs, the intensity and frequency of specific husbandry practices, extent and timing of market sales, and the allocation of family labor. This can be useful for answering the following questions: How great is the effect of seasonal cash or credit shortages on the use of purchased inputs? Do input bottlenecks, particularly of labor, cause planting and harvesting at times which are not agronomically optimal? Is the crop harvested all at once when at optimum maturity, or must it be harvested several times after it reaches the green stage in order to provide a regular family food supply? Do debts that are accumulated over the season require the immediate sale of all crop surpluses, or can some be stored for either family use or for future sales at more favorable prices? What kind of cash operating balance must the farmer keep on hand? Are financial savings possible, and if so, what might be the likely pattern of deposits and withdrawals?

In addition, PLP can be useful for predicting the effects of technological change. For example, the introduction of animal power, as a means of improving agricultural productivity, would imply substantial deviation from previous land, labor, and cash resource allocation patterns. PLP could be useful for analyzing the intertemporal impact of such change. Incorporation of the time related restriction would permit a more accurate calculation of returns to farmers from adoption of innovations, or from produce or input price changes. PLP would provide a more realistic prediction of the adoption of new technology by including cash and food balance restrictions.

Detailed Example

To illustrate some of the points made thus far, and to compare the application of polyperiod LP with that of standard LP, let us construct a simple model of traditional African farming, drawn from the dryland savannah agriculture of northern Nigeria. Figures 3 and 4 show the tableaus for the PLP and LP versions of the model. Figure 4 gives a key for the tableau symbols, and Tables 1 and 2 contain definitions of the variables.

In the PLP model, the agricultural year is divided into three periods corresponding to planting, cultivating, and harvesting. Four hectares (ha.) are available for the food crop (sorghum) and/or the cash crop (groundnuts). In addition, the family may buy and consume sorghum, hire and sell labor, engage in wholesale sorghum trading, borrow money, and build traditional or improved grain storage. Reflecting seasonal price variation, each of the three periods has different objective function coefficients for labor hiring and selling, borrowing, and sorghum purchase and sale. Five transfer activities link the periods together; they permit the transfer of cash, stored grain, and storage capacity from one period to the next. Storage losses and the cost of maintaining storage facilities are embodied in the coefficients. NETCASH accumulates the cash balance available for the next year--the period 3 cash transfer plus the period 3 sorghum selling activities.

Figure 3. Detailed PLP Tableau.

Row	Type	PLANTS.	PLANTG.	USEFAML1	HIREL.1	SELLAB1	CONS.S.1	BUYRET1	SELLRET1	BUYWHL1	SELLWHL1	BORROW$1	BLDTST1	BLDAST1	TRDEBT1	TRCASH1	TRTSTC1	TRASTC1	TRSST1	TRSSA1
NETREV.	N		-B		-U	U	-U	-T	T	-U	T	-U	-B	-B			-A	-A		
S.INV.1	E	S			U		1	-1	1										1	
CASHINV1	E		B		U	-U	U	T		U		-1	B	B		1	A	A		
DEBTINV1	E											-1			1					
TSTCAP1	L						-V	V					-1				1			
ASTCAP1	L									V				-1				1		
WHLBAL1	E									-1	1									1
RETBAL1	L							-1	1											
L.INV.1	L	C	C	-1	-1	1			T		U		B	B			B	A		
MAXHIRE1	L				1															
MINCONS1	G			-U			1													
MINCASH1	G															1				
MINSORG1	G																		1	1
MAXFAML1	L			1		1														
LANDINV	L	1	1																	
S1=S2	L	-1																		
G1=G2	L		-1																	
S.INV.2	E																		-T	
CASHINV2	E								-T		-T					-1				
DEBTINV2	E														-A					
TSTCAP2	E								-V								-1		V	
ASTCAP2	E										-V							-1		V
WHLBAL2	E																			-T
RETBAL2	L							-T	T											
L.INV.2	E																			
MAXHIRE2	L																			
MINCONS2	G																			
MINCASH2	G																			
MINSORG2	G																			
MAXFAML2	L																			
S2=S3	L																			
G2=G3	L																			
S.INV.3	E																			
CASHINV3	E																			
DEBTINV3	E																			
TSTCAP3	E																			
ASTCAP3	E																			
WHLBAL3	E																			
RETBAL3	E							-T	T											
L.INV.3	L																			
MINCONS3	E																			
MINCASH3	G																			
MINSORG3	G																			
MAXFAML3	L																			
MAXHIRE3	L																			
MAXDEBT	L											1								
Y92DEBT	E																			
Y92CASH	E																			

Row	CULTS.2	CULTG.2	USEFAML2	HIREL.2	SELLAB2	CONSS.2	BUYRET2	SELLRET2	BUYWHL2	SELLWHL2	BORROW$2	REPAY$2	BLDTST2	BLDAST2	TRDEBT2	TRCASH2	TRTSTC2	TRASTC2	TRSST2	TRSSA2	SHARVEST	GHARVEST
NETREV.		-A		-T	T	-U	-T	T	-T	T	-U		-B	-B			-A	-A				C
S.INV.1																						
CASHINV1																						
DEBTINV1																						
TSTCAP1																						
ASTCAP1																						
WHLBAL1																						
RETBAL1																						
L.INV.1																						
MAXHIRE1																						
MINCONS1																						
MINCASH1																						
MINSORG1																						
MAXFAML1																						
LANDINV																						
S1=S2	1																					
G1=G2		1																				
S.INV.2				U		1	-1	1											1			
CASHINV2		A		T	-T	U	T		T		-1	1	B	B		1	A	A				
DEBTINV2											-1	1			1							
TSTCAP2						-V	V						-1				1					
ASTCAP2									V					-1				1				
WHLBAL2									-1	1										1		
RETBAL2							-1	1														
L.INV.2	C	C	-1	-1	1			T		U			B	B			B	A				
MAXHIRE2				1																		
MINCONS2			-U			1																
MINCASH2																1						
MINSORG2																			1	1		
MAXFAML2			1		1																	
S2=S3	-1																				1	
G2=G3		-1																				1
S.INV.3																			-T		-C	
CASHINV3								-T		-T						-1						-C
DEBTINV3															-A							
TSTCAP3								-V									-1		V		T	
ASTCAP3										-V								-1		V		
WHLBAL3																				-T		
RETBAL3							-T	T														
L.INV.3																					C	C
MINCONS3																						
MINCASH3																						
MINSORG3																						
MAXFAML3																						
MAXHIRE3																						
MAXDEBT											1											
Y92DEBT																						
Y92CASH																						

Row	USEFAML3	HIREL.3	SELLAB3	CONSS.3	BUYRET3	SELLRET3	SELLSOR3	BUYWHL3	SELLWHL3	BORROW$3	REPAY$3	BLDTST3	BLDAST3	TRDEBT3	TRCASH3	TRTSTC3	TRASTC3	TRSST3	TRSSA3	NETDEBT	NETCASH		RHS
NETREV.		-U	U	-U	-T	T	T	-T	T	-U		-B	-B			-A	-A						---
S.INV.1																						=	660.
CASHINV1																						=	50.
DEBTINV1																						=	0.
TSTCAP1																						<	2.5
ASTCAP1																						<	0.
WHLBAL1																						<	0.
RETBAL1																						<	0.
L.INV.1																						<	0.
MAXHIRE1																						<	16000.
MINCONS1																						>	175.
MINCASH1																						>	40.
MINSORG1																						>	400.
MAXFAML1																						<	1700.
LANDINV																						<	4.0
S1=S2																						<	0.
G1=G2																						<	0.
S.INV.2																						=	0.
CASHINV2																						=	0.
DEBTINV2																						=	0.
TSTCAP2																						=	0.
ASTCAP2																						=	0.
WHLBAL2																						<	0.
RETBAL2																						<	0.
L.INV.2																						=	0.
MAXHIRE2																						<	8000.
MINCONS2																						>	175.
MINCASH2																						>	40.
MINSORG2																						>	400.
MAXFAML2																						<	1700.
S2=S3																						<	0.
G2=G3																						<	0.
S.INV.3		U		1	-1	1	1											1				=	0.
CASHINV3		U	-U	U	T			T		-1	1	B	B		1	A	A					=	0.
DEBTINV3										-1	1			1								=	0.
TSTCAP3				-V	V							-1				1						=	0.
ASTCAP3								V					-1				1					=	0.
WHLBAL3								-1	1										1			=	0.
RETBAL3					-1	1																=	0.
L.INV.3	-1	-1	1			T	U		U			B	B			B	A					<	0.
MINCONS3	-U			1																		=	175.
MINCASH3															1							>	40.
MINSORG3																		1	1			>	400.
MAXFAML3	1		1																			<	1700.
MAXHIRE3		1																				<	16000.
MAXDEBT										1												<	500.
Y92DEBT														-A						1		=	0.
Y92CASH						-T	-T		-T						-1						1	=	-0.

Figure 4. Detailed LP Tableau.

		GROWSORG	SELLSORG	PRODGNUT	CONSORG	FAMLAB1	FAMLAB2	FAMLAB3	HIRELAB1	HIRELAB2	HIRELAB3	SELLAB1	SELLAB2	SELLAB3	BLDTST	BUYSORG	BORROW$	WHLTRADE	SAVECASH	SAVESORG	NETCASH	RHS 1
NETREV	N		T	C	-U				-U	-T	-U	U	T	U	-B	-T	-T	V				---
SORG.INV	E	-C	1		1				U	U	U					-1				1		= 660.
CASHINV	E		-T	B	U				U	T	U	-U	-T	-U	B	T	-1	U	1			= 50.
TSTCAP	L	T	-V		-V										-1	V				V		≤ 2.5
MAXFAML1	L					1						1										≤ 1700.
MAXFAML2	L						1						1									≤ 1700.
MAXFAML3	L							1						1								≤ 1700.
MAXHIRE1	L								1													≤16000.
MAXHIRE2	L									1												≤ 8000.
MAXHIRE3	L										1											≤16000.
LABINV1	L	C		C		-1			-1			1										≤ 0.
LABINV2	L	C		C			-1			-1			1									≤ 0.
LABINV3	L	C		C				-1			-1			1								≤ 0.
TLABINV	L	C	U	C		-1	-1	-1	-1	-1	-1	1	1	1	B			U				≤ 0.
LANDINV	L	1		1																		≤ 4.
MINCONS	G				1	-U	-U	-U														≥ 525.
MINSAV$	G																		1			≥ 40.
MNSAVSOR	G																			1		≥ 800.
MAXDEBT	L																1					≤ 500.
YR2CASH	E																A	-T	-1		1	= 0.

SYMBOLS	RANGE
V	.001 to .0099
U	.01 to .099
T	.10 to .99
A	1.01 to 10.0
B	10.01 to 100.0
C	100.01 to 1000.0
D	1000.01 to 10,000.0
E	10,000.01 to 100,000.0

Table 1. Definition of Variables, PLP Model.

Activities

PLANTS.	Plant sorghum
PLANTG.	Plant groundnuts
USEFAML1	Use family labor, period 1
HIREL.1	Hire labor, period 1
SELLAB1	Sell family labor, period 1
CONS.S.1	Consume sorghum, period 1
BUYRET1	Buy sorghum retail, period 1
SELLRET1	Sell sorghum retail, period 1
BUYWHL1	Buy sorghum wholesale, period 1
SELLWHL1	Sell sorghum wholesale, period 1
BORROW$1	Borrow dollars, period 1
BLDTST1	Build traditional storage capacity, period 1
BLDAST1	Build advanced (improved) storage capacity, period 1
TRDEBT1	Transfer debt (loans), period 1
TRCASH1	Transfer cash, period 1
TRTSTC1	Transfer traditional storage capacity, period 1
TRASTC1	Transfer advanced storage capacity, period 1
TRSST1	Transfer sorghum stored (traditional), period 1
TRSSA1	Transfer sorghum stored (advanced), period 1
CULTS.2	Cultivate sorghum, period 2
CULTG.2	Cultivate groundnuts, period 2
REPAY$2	Repay dollars, period 2
SHARVEST	Harvest sorghum, period 3
GHARVEST	Harvest groundnuts, period 3
SELLSOR3	Sell sorghum (farm gate), period 3
NETCASH	Net cash balance at end of period 3
NETDEBT	Net borrowing outstanding at end of period 3

Restrictions

NETREV.	Net revenue (objective function)
S. INV.1	Sorghum inventory, period 1
CASHINV1	Cash inventory, period 1
DEBTINV1	Debt inventory, period 1
TSTCAP1	Traditional storage capacity, period 1
ASTCAP1	Advanced storage capacity, period 1
WHLBAL1	Wholesale balance, period 1
RETBAL1	Retail balance, period 1
L.INV.1	Labor inventory, period 1
MAXHIRE1	Maximum labor hired, period 1
MINCONS1	Minimum sorghum consumption, period 1
MINCASH1	Minimum cash transfer, period 1
MINSORG1	Minimum sorghum stored, and transferred, period 1
MAXFAML1	Maximum family labor available, period 1
LANDINV	Land inventory, period 1
S1=S2	Sorghum area period 1 equals sorghum area period 2
G1=G2	Same for groundnuts
MAXDEBT	Maximum debt ceiling
YR2CASH	Cash availability for year 2

Table 2. Definition of Variables, LP Model

<u>Activities</u>

GROWSORG	Plant, cultivate, and harvest sorghum
SELLSORG	Sell sorghum (farm gate)
PRODGNUT	Produce and sell groundnuts
CONSORG	Consume sorghum
FAMLAB1	Use family labor, period 1
HIRELAB1	Hire labor, period 1
SELLAB1	Sell family labor, period 1
BLDTST	Build traditional storage capacity
BUYSORG	Buy sorghum
BORROW$	Borrow dollars
WHLTRADE	Wholesale trade in sorghum
SAVECASH	Save cash
SAVESORG	Save sorghum
NETCASH	Net cash available for next year

<u>Restrictions</u>

NETREV	Net revenue (objective function)
SORG.INV	Sorghum inventory
CASHINV	Cash inventory
TSTCAP	Traditional storage capacity
MAXFAML1	Maximum family labor available, period 1
MAXHIRE1	Maximum hired labor available, period 1
LABINV1	Labor inventory, period 1
TLABINV	Overall labor inventory
LANDINV	Land inventory
MINCONS	Minimum consumption of sorghum
MINSAV$	Minimum dollar savings
MNSAVSOR	Minimum sorghum saved (stored)
MAXDEBT	Maximum debt ceiling
YR2CASH	Cash available for year 2

Initial resources include cash and grain reserves, traditional storage capacity, land, and family labor. Minimum levels are specified for consumption and the transfer of cash and stored sorghum. Upper limits are established for family and hired labor available, and for land. Other constraints necessary in the PLP version include wholesale and retail balances (sales cannot exceed current and previous purchases), and the requirement that the same acreage be cultivated and harvested in periods 2 and 3 as was planted in period 1.

Translating the PLP model into regular LP format provides an instructive comparison of the two techniques, in terms of capacity for realistic farm system modeling balanced against size and complexity. The principal changes in moving to ordinary LP arise from treating the whole agricultural year as one period. Labor-related activities and constraints can still be incorporated for each of the three periods, since the labor resource is time-specific and not transferable. Other activities must be consolidated or eliminated since the solution algorithm can no longer distinguish the logical time sequence implicit in the three-period format. Examples of these changes include:

1) crop planting, cultivating, harvesting, and selling activities become one activity for groundnuts (PRODGNUT) and two for sorghum (GROWSORG and SELLSORG), with composite cost and return coefficients;

2) retail selling of sorghum must be eliminated, since its higher price would dominate farm gate sale if both activities were included;4/

3) seasonal price variation cannot be represented with buying and selling activities for each period since the solution would inevitably select the cheapest buying activity and the most profitable selling activity, ignoring any chronological contradictions;

4) the sorghum wholesaling process can now be represented only in drastically simplified form, with price variation and storage costs buried in a single net revenue coefficient that reflects a 10 percent return on cash investment.5/ In order to depict the cash constraint on engaging in wholesaling, the WHLTRADE activity draws on the current cash inventory but yields a cash return available only in year 2. (This activity does contribute to the current year's objective function, however.)

4/ With PLP, both retail and farm gate sale can be included without one dominating the other because farm gate sale can occur only after harvest in period 3. Higher-return retail sales in period 1 or 2 are unavailable as an outlet for period 3's harvest.

5/ The net return to wholesaling in the PLP model is 7-8 percent; it is difficult to assess whether the numerical solutions to the PLP and LP models are significantly influenced by this difference.

5) improved storage construction is eliminated since it is now implicitly included as a cost of wholesaling. In the PLP model, the costs of improved storage make wholesaling profitable only when grain is bought in period 1 and sold in period 2, which is impossible in the LP version;

6) wholesaling is divorced from the sorghum inventory, given the assumption that wholesale purchases equal sales on an annual basis. In the PLP model, purchases had to balance sales on an annual basis for both retail and wholesale activities, but within each period purchases could exceed sales, with the remainder being transferred minus a storage loss;

7) debt transfer and repayment activities are eliminated since the single-period LP model does not incorporate within-year cash flows. However, amounts borrowed are deducted from the cash balance (YR2CASH) available for the following year, with the assumption that year 1 loans are repaid at the outset of year 2.

The major change, then, in moving from PLP to LP, is the elimination of period-specific crop buying and selling activities that reflect seasonal price variations and the costs of transferring grain supplies. Thus standard LP cannot model the possibility that cash or food reserves may become binding constraints during part of the year, with a corresponding effect on the mix of feasible and profitable activities. Another important change is that the LP version cannot represent the wholesale activity in nearly the realistic manner possible with PLP.

Comparing the two techniques for size, cost, and complexity, the PLP version consists of 48 rows and 62 columns, and 297 non-zero elements, and costs about $2.50 for a typical run. The LP version has 20 rows and 20 columns, and 111 non-zero elements, and costs $1.35 on average, a little over half the PLP cost. Modeling the transfer activities and inter-period constraints in the PLP version is particularly tricky, however. This constitutes one of the disadvantages of the technique.

Illustrative Numerical Solution

Tables 3 and 4 show the numerical solution for the sample PLP and LP models. The coefficients for these examples were based on recent field surveys in Zaria Province, Nigeria (Matlon, Hays, Norman). Although the models have some empirical validity, they are obviously too simplified for the numerical results to have much value other than a purely illustrative one.

Solutions were obtained for three resource situations: (1) 4 ha. land and $50 initial cash; (2) 2.5 ha. land and $35 initial cash; and (3) 1.5 ha. land and $25 initial cash. Minimum cash transfers and end-of-year cash balance were set equal to initial cash levels. Other situations modeled were: (4) reducing the allowable debt ceiling from $500 to $25; and (5) eliminating the borrowing activity. These modifications were applied to the version 3 resource situation (1.5 ha., $25 initial cash) in an effort to simulate the poor farmer's limited access to credit.

Table 3. Numerical Solution to the PLP Model for Different Combinations of Land and Cash Availabilities

Activity	Units	Version I*	Version II	Version III	Version IV	Version V
NETREV.	$	375.81	316.66	260.68	252.16	248.37
PLANTS.	ha.	3.0	.9	.9	1.5	1.5
PLANTG.	ha.	1.0	1.6	.6	---	---
USEFAML1	man-hrs	1146.7	1074.3	974.2	954.5	954.4
HIREL.1	man-hrs	---	---	---	---	---
SELLAB1	man-hrs	553.3	625.7	725.8	745.5	745.6
CONS.S.1	kg	197.9	196.5	194.5	194.1	194.1
BUYRET1	kg	239.9	1229.1	1224.1	695.4	468.3
SELLRET1	kg	---	---	---	---	---
BUYWHL1	kg	---	---	---	---	---
SELLWHL1	kg	---	---	---	---	---
BORROW$1	$	8.12	114.22	91.29	25.00	N.A.
BLDTST1	'000 kg	---	---	---	---	---
BLDAST1	'000 kg	---	---	---	---	---
TRDEBT1	$	8.12	114.22	91.29	25.00	N.A.
TRCASH1	$	54.63	36.76	25.00	25.00	25.00
TRTSTC1	'000 kg	2.46	1.47	1.47	1.94	1.93
TRASTC1	'000 kg	---	---	---	---	---
TRSST1	kg	641.1	1674.2	1671.2	1131.3	904.2
TRSSA1	kg	---	---	---	---	---
CULTS.2	ha.	3.0	.9	.9	1.5	1.5
CULTG.2	ha.	1.0	1.6	.6	---	---
USEFAML2	man-hrs	1700.0	1588.2	1288.3	1175.2	1155.2
HIREL.2	man-hrs	---	---	---	---	---
SELLAB2	man-hrs	---	111.8	411.7	524.8	544.8
CONSS.2	kg	209.0	206.8	200.8	198.5	198.1
BUYRET2	kg	---	---	---	---	---
SELLRET2	kg	---	983.7	986.9	476.2	260.9
BUYWHL2	kg	---	---	---	---	---
SELLWHL2	kg	---	---	---	---	---
BORROW$2	$	---	---	---	---	N.A.
REPAY$2	$	---	---	---	26.00	N.A.
BLDTST2	'000 kg	---	---	---	---	---
BLDAST2	'000 kg	---	---	---	---	---
TRDEBT2	$	8.44	118.79	94.95	---	N.A.
TRCASH2	$	40.0	35.00	57.03	43.06	70.93
TRTSTC2	'000 kg	2.03	---	---	1.00	1.22
TRASTC2	'000 kg	---	---	---	---	---
TRSST2	kg	400.0	400.0	400.0	400.0	400.0
TRSSA2	kg	---	---	---	---	---
SHARVEST	ha.	3.0	.9	.9	1.5	1.5
GHARVEST	ha.	1.0	1.6	.6	---	---
USEFAML3	man-hrs	1218.2	1090.5	989.9	979.6	979.6
HIREL.3	man-hrs	---	---	---	---	---
SELLAB3	man-hrs	481.8	609.5	710.1	720.4	720.4

Table 3. cont.

Activity	Units	Version I*	Version II	Version III	Version IV	Version V
CONSS.3	kg	199.4	196.8	194.8	194.6	194.6
BUYRET3	kg	---	---	---	---	---
SELLRET3	kg	216.5	174.7	167.2	175.2	174.8
SELLSOR3	kg	2151.0	389.0	399.7	885.2	885.6
BUYWHL3	kg	---	---	---	---	---
SELLWHL3	kg	---	---	---	---	---
BORROW$3	$	---	---	---	---	N.A.
REPAY$3	$	8.95	125.91	100.64	---	N.A.
BLDTST3	'000 kg	.76	---	---	---	---
BLDAST3	'000 kg	---	---	---	---	---
TRDEBT3	$	---	---	---	---	N.A.
TRCASH3	$	161.0	279.38	214.6	157.36	153.58
TRTSTC3	'000 kg	---	---	---	---	---
TRASTC3	'000 kg	---	---	---	---	---
TRSST3	kg	400.0	400.0	400.0	400.0	400.0
TRSSA3	kg	---	---	---	---	---
NETDEBT	$	---	---	---	---	N.A.
NETCASH	$	425.30	344.53	279.98	277.16	273.37

*Version I: land = 4.0 ha.; initial cash = $50.00
Version II: land = 2.5 ha.; initial cash = $35.00
Version III: land = 1.5 ha.; initial cash = $25.00
Version IV: same as Version III but MAXDEBT reduced to $25.00
Version V: same as Version III with borrowing activities eliminated

N.A. = Not included in Version V.

Table 4. Numerical Solution to the LP Model for Different Combinations of Land and Cash Availabilities

Activity	Units	Version I*	Version II	Version III
NETREV	$	390.25	292.49	226.56
GROWSORG	ha.	2.9	.9	.9
SELLSORG	kg	1674.50	---	---
PRODGNUT	ha.	1.1	1.6	.6
CONSORG	kg	606.3	597.8	588.1
FAMLAB1	man-hrs	1133.3	1064.4	964.9
FAMLAB2	man-hrs	1700.0	1493.3	1194.7
FAMLAB3	man-hrs	1233.2	1081.9	997.4
HIRELAB1	man-hrs	---	---	---
HIRELAB2	man-hrs	---	---	---
HIRELAB3	man-hrs	---	---	---
SELLAB1	man-hrs	566.7	635.6	735.1
SELLAB2	man-hrs	---	206.7	505.3
SELLAB3	man-hrs	466.8	618.1	702.6
BLDTST	'000 kg	---	---	---
BUYSORG	kg	---	---	---
BORROW$	$	---	---	---
WHLTRADE	kg	2318.9	696.4	1301.0
SAVECASH	$	40.0	35.0	25.0
SAVESORG	kg	800.0	800.0	800.0
NETCASH	$	295.08	111.60	168.1

*Version I: Land = 4.0 ha.; initial cash = $50.00
Version II: Land = 2.5 ha.; initial cash = $35.00
Version III: Land = 1.5 ha.; initial cash = $25.00

Several observations can be made about the results:

1) the PLP and LP solutions to versions 1, 2, and 3 do not differ greatly in terms of net revenue, mix of sorghum and groundnuts, consumption, and labor use. Net revenue for the PLP model varies less than in the LP model, and in general the PLP results appear more stable with respect to changes in resource endowment.

2) as the land and cash resource endowment is reduced, both models show a similar pattern of change in crop mix, reduction in family labor use, and increase in off-farm wage employment. Selling of family labor rises to supplement total family income in the face of reduced opportunities for on-farm production and income.

3) the greatest difference in the two models concerns the role of credit. Borrowing occurs in the PLP model, but not the LP model. While solutions to the LP model are therefore unaffected by the borrowing limitations introduced in versions 4 and 5, the PLP solution is significantly altered: all land is planted in sorghum, with none in groundnuts, labor selling increases, and retail buying and selling of sorghum decline. These results no doubt reflect the role of borrowing in overcoming the within-year cash constraints that this paper has argued are important in small-scale subsistence farming. They also provide an example of the added realism possible when using PLP.

4) other results that suggest the value of the multi-period format include sorghum buying for later resale, and the construction of grain storage to enable consumption reserves and trading inventories to be carried from period to period.6/

5) neither model shows any hiring of labor. There are evidently too few activities to generate a realistic total demand for labor.

6) as the initial resource base is reduced, NETCASH available for year 2 declines steadily in the PLP model, but not in the LP example. NETCASH in the LP version 3 exceeds that of version 2 probably because the lower SAVECASH requirement, and the smaller groundnut acreage, free cash for investment in wholesale trade, which is less valuable in the current year than groundnuts but adds to cash receipts available for year 2.

In summary, although it would be unwise to push the results of these simple models too far, nonetheless they appear to confirm the superiority of the PLP technique in facilitating more realistic modeling of within-year price and cost variation, and inter-period linkages, which are a salient

---

6/ The fact that wholesale trade occurs in the LP but not the PLP solutions probably has little significance. It proved difficult to model this activity accurately in the limited format of standard LP.

feature of subsistence agriculture. These advantages of PLP may be weighed against its added complexity when considering the application of linear programming techniques as a tool of farm management analysis in developing countries.

REFERENCES

Casey, H., "Dynamic Programming in Agricultural Production," Chapter 5, pp. 101-115 in Planning Agriculture in Low Income Countries, University of Reading, Department of Agricultural Economics and Management, Development Study No. 14, 1974.

Clayton, Eric S., Economic Planning in Peasant Agriculture, Ashford, Kent: Economics Department, Wye College, 1963.

Collinson, Michael P., Farm Management in Peasant Agriculture, New York: Praeger, 1972.

Hays, Jr., H. M., "The Marketing and Storage of Food Grains in Northern Nigeria," Samaru Miscellaneous Paper 50, Institute for Agricultural Research, Ahmadu Bello University, Zaria, Nigeria, 1975.

Heyer, Judith, "An Analysis of Peasant Farm Production Under Conditions of Uncertainty," Journal of Agricultural Economics, Vol. XXIII No. 2, May 1972, pp. 135-145.

Lipton, M., "The Theory of the Optimizing Peasant," Journal of Development Studies, Vol. 4, 1968, pp. 327-351.

Low, A. R. C., "Linear Programming as a Tool In Innovation Selection for Farms in South East Ghana," Chapter 3, pp. 64-86 in Planning Agriculture in Low Income Countries, University of Reading, Department of Agricultural Economics and Management, Development Study No. 14, 1974.

McInerney, J. P., "Maximin Programming--An Approach to Farm Planning Under Uncertainty," Journal of Agricultural Economics, Vol. XVIII No. 2, May 1967, pp. 279-289.

______________, "Linear Programming and Game Theory Models--Some Extensions," Journal of Agricultural Economics, Vol. XX, No. 2, May 1969, pp. 269-278.

Matlon, Peter J., "The Size Distribution, Structure, and Determinants of Personal Income Among Farmers in the North of Nigeria," Unpublished Ph.D. Dissertation, Department of Agricultural Economics, Cornell University, May 1977.

Norman, David W., "An Economic Survey of Three Villages in Zaria Province," Samaru Misc. Paper 38, Institute for Agricultural Research, Ahmadu Bello University, Zaria, Nigeria, 1972.