RAIL LINE ABANDONMENTS:
NUMBER, LOCATION, AND TIME PHASING

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The U. S. Railway Association has recommended the closure of "potentially excess" rail lines in the Northeast and Midwest. Consequently, many state governments and local communities are in the process of evaluating which lines are worth subsidizing to preserve rail service. The abandonment of excess rail lines may benefit the rail industry by reducing some network costs such as rail maintenance and freight scheduling. Some of the cost savings from rail abandonment may be passed on to users in terms of lower freight rates and/or improved rail service. On the other hand, the abandonment of some lines is certain to impose additional economic and social costs on many rural areas.

As alternative rural transportation systems are evaluated the questions arise: Which of the existing rail lines should be abandoned; and, when should the lines be closed?

This paper presents a method of analysis to determine the number, location and time phasing of rail abandonments to minimize the discounted transportation costs of a community over a planning horizon of T years. A small scale version and numerical example are presented of a Stollsteimer-type location model used to determine the optimal locational pattern of rail lines and grain elevators in a region surrounding Fort Dodge, Iowa [2]. In contrast to the Iowa study, this paper focuses primarily on the questions of rail line abandonment and extends the Iowa study to determine the optimal location and time phasing of rail closures.

Number and Location of Rail Abandonments

Suppose there are G origins within a region. Each origin ships freight to one of M markets. Freight may be shipped directly from origin to market by truck; or, freight may be transshipped to market through a rail reload station. Freight that is transshipped to market is assembled to a rail station by truck and shipped from rail stations to market by rail. The supply of freight shipped from each origin is known; the supply received at each market is variable.

At the beginning of the planning horizon, R rail lines exist in the region. Some rail lines should, perhaps, be abandoned and others should be maintained.

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Transportation costs include a fixed annual maintenance cost required to ensure continuation of local rail services plus a variable cost of shipping by the least costly mode of transportation. The annual maintenance cost of rail lines varies by the number and location of rail lines under consideration and is borne by the community in the form of a subsidy.

Define the symbols

ε = element of

S_g = origin or source g; g = 1, 2, ..., G

R_i = location of rail station i; i = 1, 2, ..., I

L_r = location of rail line r; r = 1, 2, ..., R

M_j = market j; j = 1, 2, ..., M

λ_{km} = k^{th} set of m rail lines, m = 0, 1, 2, ..., R; where m = 0 denotes no rail lines. k = 1, 2, ..., K_m · $K_m = R!/m!(R-m)!$

Note: λ is used as a shorthand notation for λ_{km}

X(g..) = quantity of freight shipped from S_g

X(gij) = quantity of freight transshipped from S_g to M_j through R_i

X(g.j) = quantity of freight shipped directly from S_g to M_j

X(..j) = quantity of freight shipped to M_j

$$= \sum_{gi}\sum X(gij) + \sum_{g} X(g.j)$$

X(gi.) = quantity of freight shipped from S_g to R_i

C(gi.) = marginal cost of transporting freight from S_g to R_i

C(.ij) = marginal cost of transporting freight from R_i to M_j

C(gij) = C(gi.) + C(.ij)

$\alpha(r)$ = annual equivalent cost of maintaining rail line L_r

$TTC|\lambda$ = total truck plus rail transportation cost given rail line locational pattern λ_{mn}

$$= \sum_{r}\alpha(r) + \sum_{gij}\sum\sum C(gij)X(gij) + \sum_{gj}\sum C(g.j)X(g.j); \; r,\ i\varepsilon\lambda$$

$\beta(i)$ = per unit transfer (reload) cost of using rail station R_i

X(g..), X, C(gi.), C(.ij), $\alpha(i)$ and $\beta(i)$ are known constants. X(g.j), X(gij), and X(..j) are variables.

The problem to be solved is: Determine (a) m, the number of rail lines; (b) λ_{km}, the locational pattern of rail lines; and (c) X(g.j) and X(gij) the spatial flow of freight from origins to market to minimize total costs (TC) where

$$(1)\ TC = \sum_{r \varepsilon \lambda} \alpha(r) + \sum\sum_{gi \varepsilon \lambda} \sum_{j} [C(gij) + \beta(i)]\ X(gij) + \sum\sum_{gj} C(g.j)\ X(g.j)$$

This problem may be described as a transshipment rail location problem. The method of solution uses combinatorial search procedures initially developed by Stollsteimer [3] and later expanded by Ladd and Lifferth [2].

The problem is solved in two parts. The first part determines the spatial flow (allocation) of freight from origins to market for each λ_{km}. That is, part one is carried out once for each locational pattern of rail lines. Finding the optimal path of freight from each origin for a given network of rail lines is basically a problem of finding the shortest route in an acyclic network and, as such, may be determined using a recursive (sequential) search algorithm [4, p. 235]. Network λ_{km} is acyclic because freight is shipped from an origin to a rail station or to a market but not from origin to origin; and freight is shipped from a rail station to a market, but not from a rail station to an origin or to another rail station to be transshipped again.

Once the shortest route for each origin is found, the total variable transportation and reload costs for each locational pattern of rail lines, denoted as $TVC|\lambda_{km}$, is determined.

The second part determines the one locational pattern of rail lines for which total variable transportation and reload costs <u>and</u> fixed costs of rail line maintenance are minimized. The optimal number and location of rail lines are determined by systematically comparing total cost for each λ_{km} and selecting that constellation of rail lines for which joint total cost is minimum.

<u>Part One</u> determines $TVC|\lambda_{km}$ as follows: Let $AVC(gi_g j_i)$ denote the average variable transportation and reload cost of shipping one unit of freight over the shortest route from origin g to market if freight is transshipped through a rail station. For $R_{i \varepsilon \lambda}$ compute

(2) $AVC(gi_g j_i) = \min_i \{\min_j [C(.ij) + \beta(i)] + C(gi.)\}$ for all g. Expression (2) provides a G x 1 matrix of $[AVC(gi_g j_i)]$

Denote $AVC(g.j_g)$ as the lowest attainable average variable cost of shipping freight from origin g to market if freight is transported directly from origin to market. Compute $AVC(g.j_g)$ for all g as follows:

$$(3)\ AVC(g.j_g) = \min_j C(g.j)$$

Let $AVC(g...)|\lambda$ denote the shortest route from origin g through network λ_{km} and compute as follows:

$$(4)\quad AVC(g..)|\lambda_{km} = \min\,[AVC(gi_g j_i),\ AVC(g.j_g)]$$

Total variable cost given λ_{km} is

$$(5)\quad TVC|\lambda_{km} = \sum_{g} AVC(g..)\ X(g..)|\lambda_{km}$$

<u>Part Two</u> determines the locational pattern of rail lines and spatial flow of freight that minimizes (1). Let $\overline{TC}|\lambda_{km}$ denote the minimum total cost given λ_{km} and compute as follows:

$$(6)\quad \overline{TC}|\lambda_{km} = TVC|\lambda_{km} + \sum_{r\varepsilon\lambda_{km}} \alpha(r)$$

Compute (6) for all k and for all m. The minimum total cost from m rail lines is

$$(7)\quad \overline{TC}|m = \min_{k}\ \overline{TC}|\lambda_{km}$$

The number and locational pattern of rail lines that minimizes total cost is

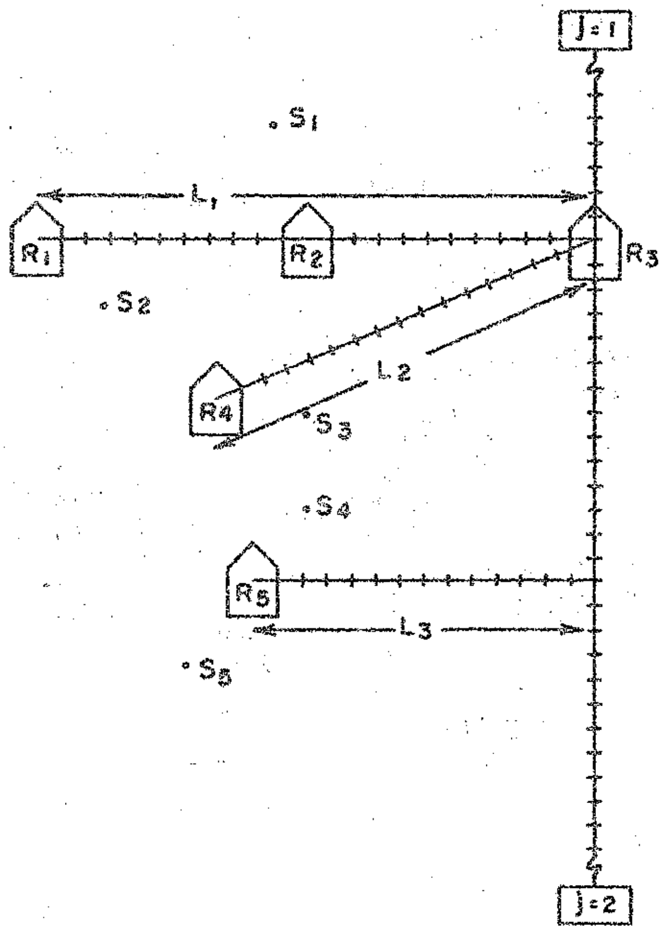
$$(8)\quad \overline{TC} = \min_{m}\ \overline{TC}|m$$

<u>Example 1</u>

The method of solution used to solve the rail location problem can be illustrated as follows: Suppose there are five origins, two markets, five rail stations, and three rail lines classified as "potentially excess" that connect with a major trunk line. The supply of freight at origins is known. Freight may be shipped from origins to market directly, or it may be trans-shipped to market through rail stations.

Figure 1 shows the location of origins, rail stations, final markets, and potential rail linessites. Final destinations are denoted by □ ; rail stations are denoted by ⌂ ; dots, • , represent origins; and ++++ represents a rail line.

FIGURE 1. LOCATIONAL PATTERN OF ORIGINS, RAIL STATIONS, MARKETS, AND RAIL LINES FOR λ_{13}



Data used for this example are presented in the following order: (a) alternative rail locational patterns; (b) supply at origins; (c) transportation costs; and (d) reload costs at rail stations.

Rail locational options: alternative rail line locational patterns used in this example are defined as follows:

λ_{10} = no feeder lines

λ_{11} = rail line L_1

λ_{21} = rail line L_2

λ_{31} = rail line L_3

λ_{12} = rail lines L_1 and L_2

λ_{22} = rail lines L_1 and L_3

λ_{32} = rail lines L_2 and L_3

λ_{13} = rail lines L_1, L_2 and L_3

The rail line network defined by λ_{13} is presented in Figure 1.

Supply at origins: The volume of freight shipped from each origin is predetermined.

X(1..) = 100,000; X(2..) = 100,000; X(3..) = 100,000; X(4..) = 100,000; X(5..) = 100,000

Transportation costs: the per unit transportation cost of shipping from origin S_g to rail station R_i, and from S_g to M_j are presented in Table 1. Table 2 presents the per unit transportation costs of shipping freight from rail station R_i to market M_j. The annual cost of maintaining rail line L_1 is $\alpha(1)$ = \$275,000; L_2 is $\alpha(2)$ = \$325,000; and L_3 is $\alpha(3)$ = \$125,000.

Table 1. TRANSPORTATION COSTS FROM ORIGINS TO RAIL STATIONS, AND FROM ORIGINS TO MARKETS IN DOLLARS PER UNIT: C(gi.) and C(g.j)

Source	Rail station					Market	
	R_1	R_2	R_3	R_4	R_5	M_1	M_2
S_1	1.0	.4	1.2	1.0	1.6	5.8	7.4
S_2	.3	.8	1.8	.6	1.2	4.4	7.0
S_3	1.2	.6	1.2	.3	.6	6.8	6.8
S_4	1.4	1.0	1.5	.5	.3	7.2	6.6
S_5	1.5	1.4	2.0	.8	.2	7.6	6.0

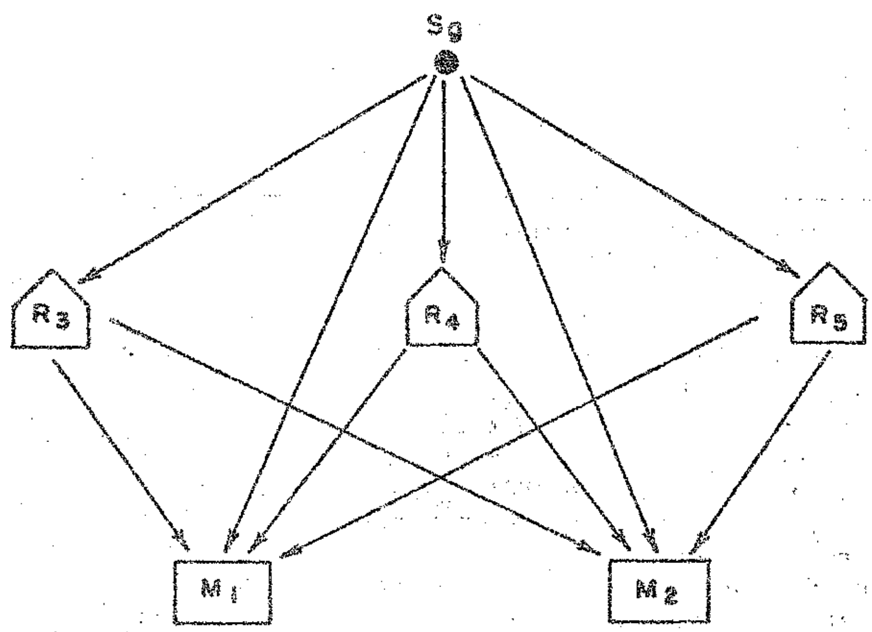
Table 2. AVERAGE VARIABLE TRANSPORTATION COSTS FROM RAIL STATIONS TO MARKET IN DOLLARS PER UNIT, C(.ij)

Rail station	Market	
	M_1	M_2
R_1	2.0	2.3
R_2	1.8	2.1
R_3	1.5	1.8
R_4	1.8	2.1
R_5	2.1	1.8

Reload costs: The reload cost at rail stations represents the per unit cost of transferring freight from one mode to rail. For this example $\beta(i)$ = \$.50 for all i.

Part One determines the optimal routing of freight from origin g and the total variable cost for each constellation of rail lines. The location of rail stations is also determined in part one and depends upon the number and location of rail lines. The routing options, for example, of S_g given λ_{32} are illustrated schematically in Figure 2. For λ_{32} freight may be transshipped from S_g through R_3, R_4, or R_5. For λ_{11} freight may be transshipped from S_g through R_1, R_2, or R_3.

FIGURE 2. SHIPPING OPTIONS AVAILABLE FROM ORIGIN S_g GIVEN λ_{32}



The optimal routing pattern for each λ_{km} is determined using a sequential routing algorithm. The first step is to determine the best route from rail station R_i to market for all i. Table 3 presents an I x J matrix of AVC(.ij) where AVC(.ij) = C(.ij) + β(i). The best market for R_i is found by min [C(.ij) + β(i)] = AVC(.ij$_i$). The symbol * is used to denote AVC(.ij$_i$) in the matrix AVC(.ij). For example, since [AVC(.1,1) = 2.5] is less than [AVC(.1,2) = 2.8], M_1 is the best market for R_1 and all freight received at R_1 should be shipped to M_1 to minimize costs.

Table 3. AVERAGE VARIABLE COST OF RELOADING FREIGHT AT R_i SHIPPING FREIGHT FROM RAIL STATION R_i TO MARKET j WHERE $AVC(.ij) = C(.ij) + \beta(i)$

Rail station	Market M_1	M_2
R_1	2.5*	2.8
R_2	2.3*	2.6
R_3	2.0*	2.3
R_4	2.3*	2.6
R_5	2.6	2.3*

* Denotes $AVC(.ij_i) = \min_j AVC(.ij)$

The next step computes a matrix of $[AVC(gij_i) = AVC(.ij_i) + C(gi.)]$ for each rail locational pattern λ_{km}. Table 4 presents $AVC(gij_i)$ for the three possible combinations of one rail line; i.e. $\lambda_{1,1}$, $\lambda_{2,1}$, and $\lambda_{3,1}$. The best route for S_g when freight is transshipped through a rail station to market is denoted by * where $AVC(gi_gj_i) = \min_i AVC(gij_i)$. For example, $AVC(1,1,j_1)|\lambda_{1,1}$ represents the average variable cost of shipping freight from origin S_1 to rail station R_1 plus the transfer costs at R_1 plus the average variable cost of shipping from R_1 to the best market. From Table 3 it was determined that M_1 is the best market for R_1. Thus, $AVC(1,1j_1)|\lambda_{1,1} = AVC(.1,1_1)|\lambda_{1,1} + C(1,1.) = 2.5 + 1.0 = 3.5$. Repeating this step for each λ_{km} completes the operations of expression (2).

Table 4. AVERAGE VARIABLE COST FROM TRANSSHIPPING FREIGHT FROM ORIGIN S_g THROUGH RELOAD STATION R_i TO MARKET M_j FOR ALL COMBINATIONS OF ONE ($M = 1$) RAIL LINE; WHERE $AVC(gij_i)|\lambda_{km} = AVC(.ij_i) + C(gi.)$

Source	λ_{11} R_1	λ_{11} R_2	λ_{11} R_3	λ_{21} R_3	λ_{21} R_4	λ_{31} R_3	λ_{31} R_5
S_1	3.5	2.7*	3.2	3.2*	3.3	3.2*	3.9
S_2	2.8*	3.1	3.8	3.8	2.9*	3.8*	3.9
S_3	3.7	3.1*	3.2	3.2	2.6*	3.2*	3.2
S_4	3.9	3.3*	3.5	3.5	2.8*	3.5	2.9*
S_5	4.0	3.7*	4.0	4.0	3.1*	4.0	2.8*

* Denotes $AVC(gi_gj_i)|\lambda_{km} = \min_i AVC(gij_i)|\lambda_{km}$

The values for expression (3) are found in Table 5. Table 5 is a G X J matrix of [AVC(g.j) = C(g.j)]. The best route for S_g when freight is shipped directly to market is denoted by * where $AVC(g.j_g) = \min_j C(g.j)$.

Table 5. AVERAGE VARIABLE COST OF SHIPPING FREIGHT DIRECTLY FROM SOURCE S_g TO MARKET M_j, WHERE $AVC(g.j) = C(g.j)$

Source	Market M_1	Market M_2
S_1	5.8*	7.4
S_2	4.4*	7.0
S_3	6.8*	6.8
S_4	7.2	6.6*
S_5	7.6	6.0*

*Denotes $AVC(g.j_g) = \min_j C(g.j)$

Expression (4) is found by selecting, for each λ_{km}, the best route for origin S_g given $AVC(gi_gj_i)$ and $AVC(g.j_g)$. Table 6 presents the minimum attainable average variable cost for each S_g given λ_{km}.

Table 6. THE LOWEST ATTAINABLE AVERAGE VARIABLE COST OF SHIPPING FREIGHT FROM ORIGIN S_g FOR SELECTED RAIL LINE LOCATIONAL PATTERNS, $AVC(g..)|\lambda_{km}$[a]

Source	Rail line locational options							
	λ_{10}	λ_{11}	λ_{21}	λ_{31}	λ_{12}	λ_{22}	λ_{32}	λ_{13}
S_1	3.2	2.7	3.2	3.2	2.7	2.7	3.2	2.7
S_2	3.8	2.8	2.9	3.8	2.8	2.8	2.9	2.8
S_3	3.2	3.1	2.6	3.2	2.6	3.1	2.6	2.6
S_4	3.5	3.3	2.8	2.9	2.8	2.9	2.8	2.8
S_5	4.0	3.7	3.1	2.8	3.1	2.8	2.8	2.8

[a] See expression (4).

Table 7 presents total variable cost, fixed cost of rail maintenance, and the minimum attainable total cost for each rail line locational pattern. Total variable cost, $TVC|\lambda_{km}$, is found by computing $\sum_g AVC(g..)\ X(g..)|\lambda_{km}$.

Table 6 presents AVC(g..) for each λ_{km}; and, the volume of freight at S_g, X(g..), is predetermined.

Table 7. TOTAL VARIABLE COST, FIXED COST OF RAIL LINE MAINTENANCE, AND MINIMUM ATTAINABLE TOTAL COST FOR SELECTED RAIL LOCATIONAL PATTERNS, IN THOUSANDS OF DOLLARS[a]

	Rail line locational options							
	λ_{10}	λ_{11}	λ_{21}	λ_{31}	λ_{12}	λ_{22}	λ_{32}	λ_{13}
$TVC \mid \lambda_{km}$	3,540	3,120	2,920	3,180	2,800	2,860	2,860	2,740
$\sum_{r \varepsilon \lambda_{km}} \alpha(r)$	0	275	325	125	600	400	450	725
$\overline{TC} \mid \lambda_{km}$	3,540	3,395	3,245	3,305	3,400	3,260	3,310	3,465

[a] See expressions (5) and (6).

Part Two determines the optimal number and location of rail lines that minimizes total variable costs plus total fixed costs of rail maintenance. From Table 7 the minimum total cost from M rail lines can be identified. The number and location of rail lines that provide the lowest attainable cost for expression (1) is also identified from Table 7. Locational pattern λ_{21} with a total systems cost of $3,245,000 represents the optimal number and location of rail lines.

It is not necessary to compare every possible combination of rail lines to find a global optimum. If the best location of 4 rail lines is better than the best location of both 3 rail lines and 5 rail lines, then the best location of 4 rail lines is better than any other number and location of rail lines. The change in total variable costs between the best location of M rail lines and the best location of M + 1 rail lines will never be positive. The best location of M rail lines is included in the different locational combinations of M + 1 rail lines. An increase in total variable costs from adding an additional rail line can always be avoided by selecting the best location of M rail lines and refusing to use the M + 1st rail line. Thus, once a local optimum is found, additional comparisons are unnecessary since any local optimum is also global.

In this simple illustration the trade-off between rail maintenance costs and shipping costs is demonstrated. As rail lines are abandoned there are some savings of rail maintenance costs. As rail lines are abandoned, however, freight must be shipped from origins by truck to more distant rail stations for transshipment to market; or, freight must be trucked directly from origin to market. The additional cost of shipping freight resulting from rail abandonment must be balanced against the cost savings of maintaining fewer lines.

Time Phasing of Rail Abandonments

Expressions (1) through (8) describe a method of solving for the optimal number and location of rail lines for a given volume of freight and for a given level of costs. Changes in the parameters of the model, however, may influence both the pattern of shipments that minimizes total cost as well as the locational pattern of rail lines. Rail abandonment decisions, therefore, that are based only on current economic conditions may not be in the long run interest of the public if changes in the volume of freight and/or level of transportation costs are anticipated. Abandoning a rail line, in most cases, is an irreversible decision. On the other hand, rail abandonment decisions should not be based on future transportation needs disregarding current conditions. Maintaining excess rail lines, for future use, for example, may impose costs that are greater than the long run benefits.

Suppose, for example, that the data [X(g..), C(gi.), C(.ij), $\alpha(r)$, and $\beta(i)$], used in example 1 reflect the economic conditions prevailing during period 1; and further, suppose that the volume of freight increases from the period 1 level of 200,000 units per origin to 500,000 units per origin in period 2. The best locational pattern for period 1, disregarding period 2, is λ_{21}. The best locational pattern for period 2, disregarding period 1, is λ_{22}. The problem is clear: If rail abandonment decisions are irreversible and if rail lines L_1 and L_3 are abandoned in period 1 to minimize total system cost for the users in period 1, then λ_{22} is not a feasible alternative for period 2. As an alternative, λ_{22} may be implemented during period 1 and maintained for use in period 2. In this case, the additional cost of maintaining λ_{22} during period 1 ($\overline{TC}|\lambda_{22} - \overline{TC}|\lambda_{21} = 5{,}000$) must be balanced against the discounted value of λ_{22} over λ_{21} in period 2.

The procedures outlined below extend the rail location model presented in expressions (1) through (8) to account for the temporal as well as the spatial interdependencies that influence the number and location of rail lines. The problem is to determine (a) the number, location and time phasing of rail abandonments, and (b) the flow of freight from origins to market for each period to minimize discounted total costs over a finite time horizon of T years. The method of solution involves the following four steps:

Step 1: Compute the minimum total system cost in time t for each rail locational network λ_{km} given the predetermined level of costs and volume at origins during time t; t = 1, 2, ..., T. Let $TC(t)|\lambda_{km}$ denote the minimum total cost in time t given λ_{km}. $TC(t)|\lambda_{km}$ is determined by solving expressions (1) through (6) for each period t. Predetermined data [X(g..), C(gi.), C(.ij), $\alpha(r)$, and $\beta(i)$] are changed for each t to reflect the economic conditions prevailing during time t.

Step 2: Construct a decision tree representing the feasible rail network options over a time horizon of T periods. At the start of time t = 0 there is 1 decision node; at the end of time t = 1 there are $\sum_m K_m$ possible decision nodes. Each node represents one rail line network as defined by λ_{km} where k = 1, 2, ..., K_m and m = 0, 1, ..., R. At the end of time t = 2 there is another set of decision nodes (or rail network options) which branch from the nodes of the previous period. The number of decision nodes in time t = 2 depends upon the feasible rail network options available in period 2 given the combinations of rail networks in period 1. This branching process continues until all feasible adjustment paths over time are identified.

Step 3: Label each decision node. Let N_{st} denote decision node s in time t; s = 1, 2, ..., S_t. During time t = 0, S_t = 1. During time t = 1, $S_t = \sum_m K_m$. During other periods, $t \geq 2$ S_t depends upon the number of network decision nodes feasible during the period.

Step 4: Compute the discounted total cost of alternative rail abandonment options and select that adjustment path which minimizes discounted total cost.

Let

$\overline{TC}|N_{st} = \overline{TC}(t)|\lambda_{km}$ where

λ_{km} corresponds to N_{st}, by definition as specified in Step 3.

= minimum total cost in time t given the rail locational network identified by decision node N_{st}.

D = single period discount rate.

$\overline{DTC}|N_{st}$ = minimum discounted total cost at N_{st} from t to T.

$\overline{DTC}|N_{10}$ represents the minimum discounted total cost over a finite time horizon of T periods. $\overline{DTC}|N_{10}$ is found by solving expression (9) for each time period beginning with the last period (t = T) and working backwards (sequentially) through time.

$$(9)\quad \overline{DTC}|N_{st} = D \min_{\acute{s}\in N_{st}} [\overline{DTC}|N_{\acute{s},\, t+1}] + \overline{TC}|N_{st}$$

for s = 1, 2, ..., S_t and t = T - 1, T - 2, ..., 1, 0.

where

$\overline{DTC}|N_{st} = \overline{TC}|N_{st}$

and $\overline{TC}|N_{10} = 0$

Example 2

Steps 1 through 4 are illustrated as follows. Let the data of example one represent the conditions prevailing during period one. Assume that in period two the volume at each origin increases from the period 1 level to: X(1..) = 500,000; X(2..) = 500,000; X(3..) = 500,000; X(4..) = 500,000; and X(5..) = 500,000. Table 8 presents $TC(t)|\lambda_{km}$ for t = 1 and t = 2.

Table 8. MINIMUM ATTAINABLE TOTAL COST FOR TIME 1 AND TIME 2 GIVEN SELECTED RAIL LOCATIONAL PATTERNS, IN THOUSANDS OF DOLLARS, $TC(t)|\lambda_{km}$

	λ_{10}	λ_{11}	λ_{21}	λ_{31}	λ_{12}	λ_{22}	λ_{32}	λ_{13}
Time t = 1	3,540	3,395	3,245	3,305	3,400	3,260	3,310	3,465
Time t = 2	8,850	8,075	7,625	8,075	7,600	7,550	7,600	7,575

Step 2 identifies the various rail line abandonment options that are feasible over time. In Step 3 the various nodes of the decision tree identified in Step 2 are labeled. Figure 3 presents the decision tree of the feasible rail line abandonment options over two time periods for m = 0, 1, and 2. Decision nodes are identified by both λ_{km} and N_{st}.

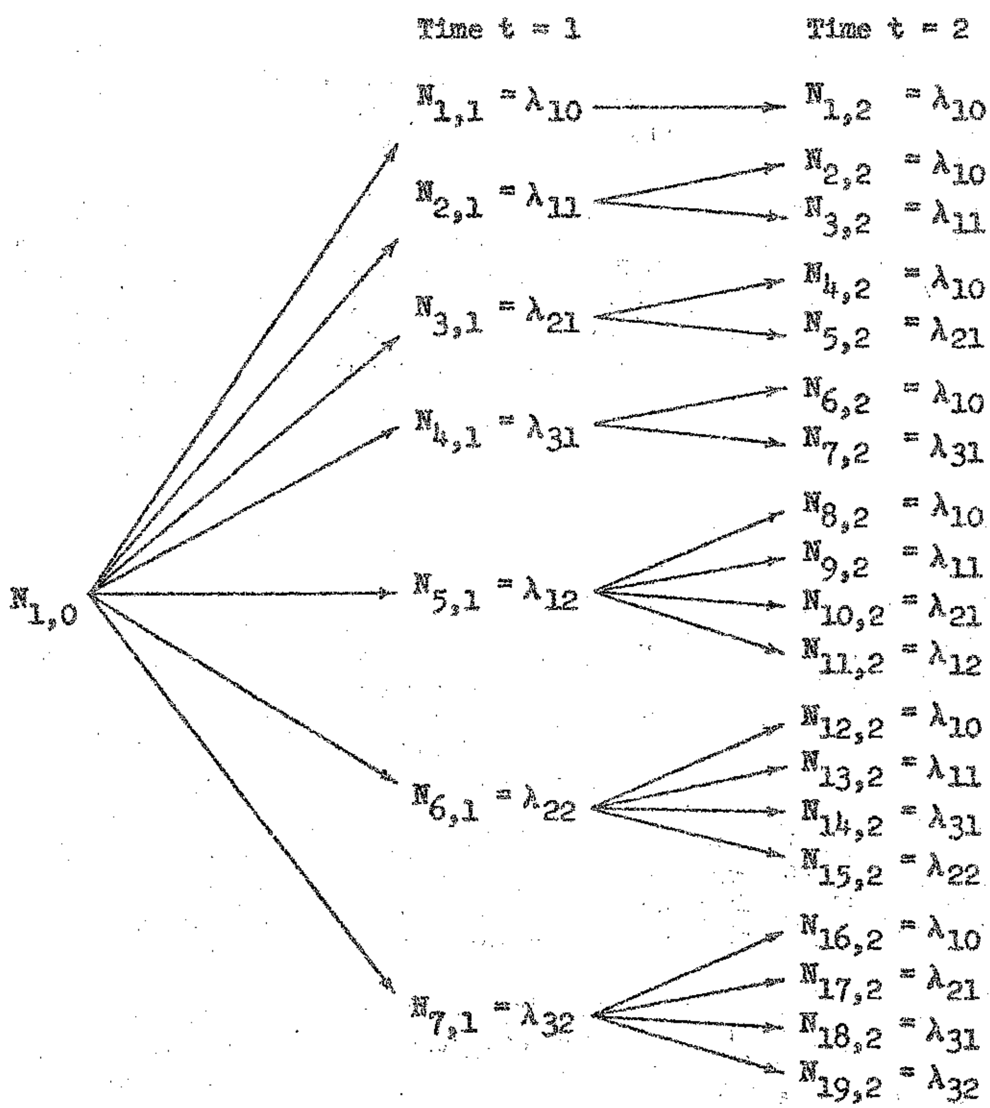


Figure 3. RAIL LINE ABANDONMENT OPTIONS OVER A TIME HORIZON OF 2 PERIODS

Step 4 identifies the optimal adjustment path of rail abandonments over time. The optimal time phasing of abandonment is determined by selecting that adjustment path that minimizes discounted total costs. $DTC|N_{10}$ is found by sequentially computing $DTC|N_{st}$ for each time period beginning with t = T and working backwards through time. For t = 2, = T, $DTC|N_{s,2}$ can be read directly from Table 8; e.g., $DTC|N_{1,2} = 8{,}850$; $DTC|N_{2,2} = 8{,}850$; $DTC|N_{3,2} = 8{,}075$; ..., $DTC|N_{19,2} = 7{,}600$. For t = 1, $DTC|N_{s1}$ is computed as described by expression (9). For example, $DTC|N_{2,1} = \{D \min [DTC|N_{2,2}; DTC|N_{3,2}] + \overline{TC}|N_{2,1}\} = \{.9091 \min [8{,}850; 8{,}075] + 3{,}395\} = 10{,}736$ where $D = 1/(1 + .10)$.

This step is repeated for each N_{s1}; s = 1, 2, ... S_1. Step 4 is completed when $DTC|N_{1,0}$ is determined.

$$DTC|N_{1,0} = \{D \min [DTC|N_{1,1}; DTC|N_{2,1}; \ldots; DTC|N_{6,1}; \ldots; DTC|N_{8,1}] + 0\}$$

$$= \{.9091 \min [11{,}586; 10{,}736; \ldots; 10{,}124; \ldots; 10{,}329] + 0\}$$

$$= .9091\ (10{,}124) = \$9{,}204$$

In this example, discounted total cost was minimized by selecting $DTC|N_{6,1}$ which represents implementing λ_{22} during period 1 and λ_{22} during period 2. Any other number, location, and time phasing of rail abandonments would be suboptimal.

In summary, the number and location of rail lines to minimize total transportation costs during period one, disregarding period two, is defined by option λ_{21} with one rail line located at L_2. When the future conditions of period two are considered along with period one in making the rail abandonment decisions, option λ_{22} is best. Locational pattern λ_{22} includes rail lines L_1 and L_3. The additional cost of maintaining too many lines during period one is less than the additional cost of having too few lines in period two.

Application and Conclusion

A variation of the model presented in this paper was used to determine the optimal locational pattern of rail lines and grain elevators in a region surrounding Fort Dodge, Iowa. Joint net revenue of grain producers was maximized by abandoning all feeder (or branch) rail lines in the region and hauling freight by truck from origins to subterminal elevators located on major trunk lines. The cost savings of maintaining fewer rail lines plus the quantity discounts available for shipping larger volumes of grain from subterminal elevators were balanced against the additional trucking costs plus the investments required to establish subterminals on trunk rail lines. Additional energy requirements, road use costs, and pollution emission resulting from rail line abandonment were also estimated.

The planning horizon extended from 1971 to 1980. The location and capacity of grain handling and transportation facilities existing at the beginning of the planning horizon were taken into account in designing the transportation network. In the Fort Dodge study the problem of estimating the optimal phasing of rail abandonments over the planning horizon was a moot question. The best locational pattern of rail lines in 1980, disregarding other time periods, was identical to the best locational pattern of rail lines in 1971, given the level of costs and volume of freight existing in 1971. Discounted joint net revenue of producers was maximized, therefore, by abandoning all feeder (branch) rail lines at the beginning of the planning horizon. Detailed results of the study are reported in Baumel, et al. [1].

Many communities or geographical regions in the Northeast and Midwest are presently sustaining rail lines that have been classified by the U.S. Railway Association as "potentially excess." The classification of such lines, however, has been based primarily on single line analysis without regard to intramodal interdependencies that exist over space as well as over time. The procedures for multiple-line analysis presented in this paper provide one method of including intra- and inter-modal interdependencies in the evaluation and design of alternative rural transportation systems.

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