A GENERALIZED RESOURCE LEASING POLICY EVALUATION MODEL

BY

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September 1975 No. 75-16

The work upon which this publication is based was supported in part by funds provided by the National Science Foundation under Grant No. SIA74-21846.

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Introduction

The effects of government policy on domestic energy supplies are nowhere more apparent than in the disposal of energy resources held in the public domain. With the continuing depletion of proven energy reserves located on private lands, it is now clear that these areas will become a major energy supply source over the next 25 years. Consequently, the strategy and schedule for leasing the millions of acres involved becomes increasingly important.

Regardless of the energy resource being considered, private sector response to public policy actions in this area will normally follow a similar logic. Assuming competitive lease sales and a profit maximization objective function for the private sector, discounted cash flow techniques (appropriately constrainted for public rules and market rigidities) can be used to simulate these responses. This paper provides the detailed specification and description of a model, incorporating these techniques, which can be used to evaluate a number of the policy questions pertaining to the various energy resources located on the public domain. This generalized leasing model incorporates a number of factors important for public policy decisions into a framework of private market behavior. Economic, geological and engineering considerations relevant to private producer decision making are included so that the model may be useful for quantitatively testing the effects of a wide range of public policy alternatives. For example, the model is designed to determine the impacts of a number of alternative federal policies aimed at reducing risk for private sector resource development. A wide range of leasing policy alternatives are also incorporated into the model so that it may be used to analyze the effects of alternative leasing strategies.

This paper is designed to provide readers with an in-depth understanding of how the model works. Both the theoretical and mechanical aspects are covered in great detail, in order that the reader will understand not only the theoretical rationale behind the relationships modeled but will also comprehend the means used to translate the theoretical structure into actual equations and solution procedures.

Basic Concepts

The model is designed to simulate the actions of the winning bidder in competitive leasing situations. In general, it utilizes exogenously supplied estimates of energy reserves on an individual or group of leaseholds, along with estimates of the associated production costs (investment and operating) and market prices to determine the actions of a potential leasehold developer which would maximize his after tax net present value. In so doing, the model determines the production capacity to be installed on the leasehold and the length of time that capacity is used for production. Uncertainty with respect to the key variables supplied exogenously (reserves, production costs and market prices) is incorporated via use of Monte Carlo simulation techniques which are described subsequently. Net present value calculations are carried out using discount cash flow techniques with exogenously supplied rates of return as discount rates.

Given this basic model logic, several approaches to model solution can be used. The solution algorithm can be designed to handle installed capacity (q_o) as either a continuous value (one which can take on any value in arriving at an overall optimum solution) or as a lumpy value (one in which only pre-specified capacities are permitted in model solution due to the type of production equipment which must be installed). This distinction, in large part, leads to the different model algorithms. In the former situation, equations are specified which solve for and optmize installed capacity simultaneously with other model outputs. In the latter, the discrete installed capacities which are allowable are exogneously entered into the model and the optimal capacity is determined. One advantage of this approach is that it permits economies of scale with respect to installed capacity to be considered in model solutions since unique cost relationships can be entered with each capacity examined.[1]

Figures 1 and 2 are flow diagrams for the two alternative solution algorithms. Both approaches have been programmed for model execution. The model description will follow these two flow diagrams and will separately describe the solution algorithm with continuous q_o and with q_o input exogenously. It may be helpful for the reader to refer back and forth between these two flow diagrams and the text. To make the description easier to follow, a list of all model input variables with the associated computer code, the symbol used in this description, and a brief definition is provided in Table 1. All symbols in the text and future references to variable names will refer to the variable definitions in Table 1. A more comprehensive description of some of the variables is provided in Appendix A together with a complete list of input variables (Table A-1).

After the variables are read in and stored if necessary, the first step in the model solution is to run completely through the model once using mean values for all input variables. This step determines the after-tax net present value (ATNPV) if all mean values are used and converts that value into a bonus bid payment to be used in subsequent calculations.[2] This conversion is assumed

1 However, economies of scale with respect to reserve size can be used under both algorithm approaches.

2 The amount of the bonus bid is necessary for use in the tax calculations. The use of mean input values to calculate the bonus serves to approximate the actual value. This can then be used in subsequent calculations where uncertainty is considered. Optionally, the bonus may also be recalculated after any number of Monte Carlo iterations for use in subsequent iterations.

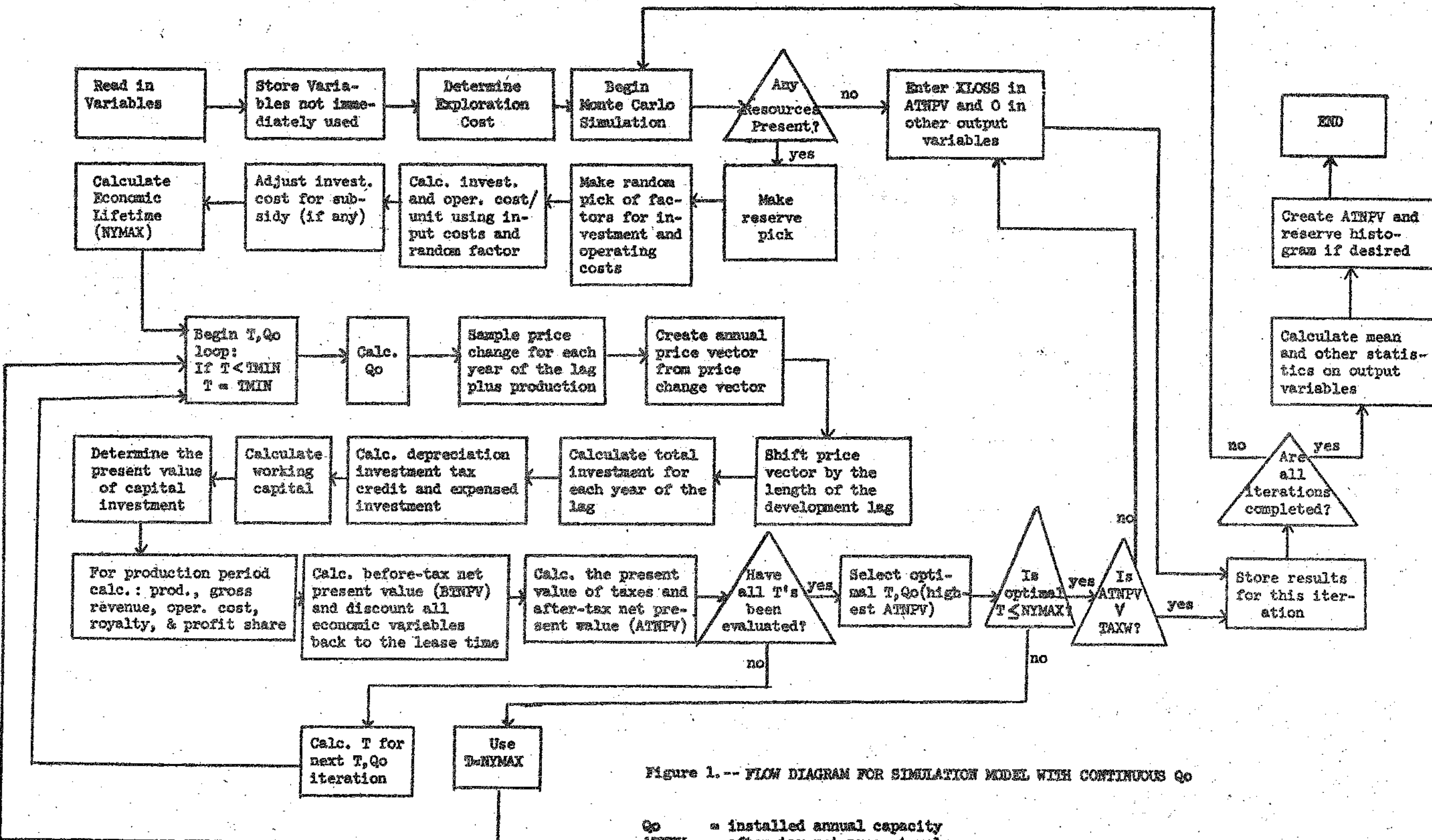

Figure 1.-- *FLOW DIAGRAM FOR SIMULATION MODEL WITH CONTINUOUS Qo*

Qo	= installed annual capacity
ATNPV	= after-tax net present value
TAXW	= tax write-off available if lease is not developed after exploration
XLOSS	= loss incurred from exploration of lease is not developed (exclusive of bonus)
TMIN	= minimum production time period

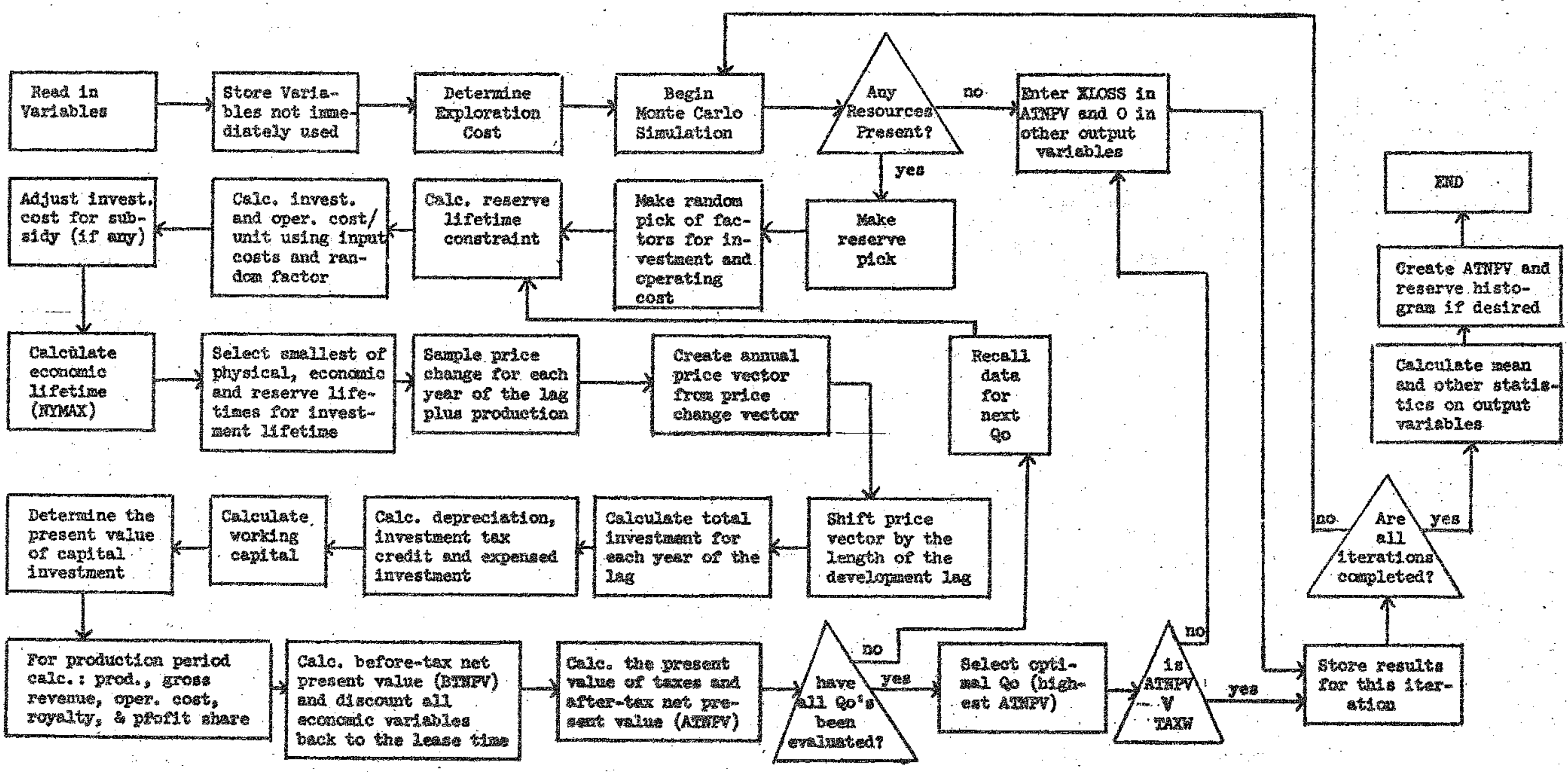

Figure 2.--FLOW DIAGRAM FOR SIMULATION MODEL WITH INPUT Qo

Qo = installed annual capacity
ATNPV = after-tax net present value
TAXW = tax write-off available if lease is not developed after exploration
XLOSS = loss incurred from exploration if lease is not developed (exclusive of bonus)

Table 1.--Some Input Variables for the Generalized Leasing Model.

Symbol	Computer Code	Definition
P_o	RPO	Initial Price for the Resource
λ	RLAMB	Royalty Rate (%)
N	RN	Depreciation Period (years)
α	RALPHA	Investment Salvageable (%)
Ω	OMEGA	Investment Tax Credit Rate (%)
r	RR	Discount Rate (%)
a	X(I)	Production Decline Rate (%)
R	RCAP	Reserves (if Monte Carlo not used
θ	RTHETA	Annual Change in Operating Cost (%)
z	RZ	Depletion Rate (%)
ϕ	RPHI	Tax Rate (%)
i	RI	Interest Rate for Capital Recovery (%)
T_p	LT	Maximum Physical Lifetime for Investment (Years)
β	RBETA	Geologic Parameter (oil)
γ	GAMMA	Geologic Parameter (oil)
P_1	RP1MN	Mean of Normal Distribution of Annual Change in Price
B_1	BFAC	Factor Used to Adjust ATNPV to determine Bonus
B_o	BCØN	Constant Used to Adjust ATNPV to determine Bonus
s	ST	Rate for State Severance Tax (on Gross Value)
B	IBP	Length of Production Build-Up Period
AGFAC	AGFAC	Factor for Determining the Amount of Associated Gas (or any second resource)
GP_o	GPØ	Initial Price for Gas (or any second resource)
R	RMEAN	Mean of Reserve Distribution
F	IFLATP	Length of Time the Initial Production Level is Used
Q_o	QO	Installed Capacity (Annual)
b^o	RBQ	Cost per Unit of Installed Capacity
K_o	RKO	Operating Cost per Unit
L^o	LAG	Investment Lag--Construction Period (Years)
f	F	Proportion of Investment Expended in each Lag Year (vector of L dimension)
y	YZ	Proportion of Yearly Investment which is tangible (oil)
RENT	RENT	Annual Rent per Acre
h_i	BPP	Factor Applied to Capacity to Determine Prod. During IBP

to be linear according to equation 1:

(1) $\text{BONUS} = B_o + B_1 \cdot \text{ANTPV}$

where B_o and B_1 are the input values BCON and BFAC, respectively.[3]

The Exploration Phase of Resource Development

The next step in the model solution is to determine the exploration cost for the lease tract or area in question. For example, gross oil exploration costs (EC) are a function of the number of wells to be drilled per acre, the number of acres in the tract, and the cost per well.

(2) EC = Wells/acre x acres x dollars/well

This amount is adjusted by deducting tax savings from expensed investment, depreciation, investment tax credit, and other tax deductions; and adding rental payments during the exploratory period.

In addition to calculating the net expenses of exploration, the potential tax write-off available to the company if the lease is not developed is also calculated. This potential tax write-off is the bonus payment plus the book value of depreciable exploration expenses multiplied by the tax rate. The value is used later in the program to compare with the potential present value of the lease if developed to decide whether or not it is advantageous to develop the lease.

For resources such as coal or oil shale, the same principles are involved in determining exploration expenses and potential tax write-offs, but the functional relationships used in determining exploration costs would differ.

Uncertainty and the Monte Carlo Analysis

For policy analysis, it is important to determine the potential effects on private decisions of uncertainty with respect to future prices, production costs, and reserves. Using the mean (average) values of probability distributions is inadequate for this analysis because only outputs resulting from these mean values are produced. No measure of the spread (variance) of potential outcomes is obtained. In other words, in the absence of some type of simulation, no measure of the potential riskiness of the final outcome is derived (and of course, the probability of the expected mean actually occurring is zero). For policy purposes, it is desirable to learn not only how the mean output values are affected by various policy options but also how the variance or range of the outcomes is changed.

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[3] If B_o and B_1 are set equal to 0 and 1, respectively, the bonus will equal ATNPV. The values of B_o and B_1 depend on the bidder's risk preference function.

For example, suppose two policy options have identical effects on the means of relevant policy objectives (model outputs), have identical costs (in whatever terms cost is measured), but have differential effects on the expected outcome variances. Naturally, the policy maker would choose that option which offers the greatest reduction in variance under these circumstances. In other cases, there will be trade-offs between changes in means, differences in relative cost, and variances which must be weighed by the decision makers. In every case, however, the variance or range of possible outcomes is a piece of information which is valuable to the decision maker attempting to influence private market behavior.

Monte Carlo simulation is one technique for handling the problem of uncertainty in input values and to estimate the variance in potential outcomes. Rather then using point estimates of uncertain input variables, an assumed probability distribution is provided from which samples are taken to be used as inputs for the analysis. The process of sampling each variable from its unique probability distribution and performing the model calculation is repeated many times to produce a range of model output values. A frequency distribution of these output values can be derived and the mean and variance of the expected outcomes determined. In performing this type of simulation we replace the unknown actual population of future prices, costs and reserves by an assumed probability distribution from which samples are drawn. By sampling many times it is possible to generate many possible combinations of prices, costs and reserves that together produce outcomes, each in the appropriate proportion (King, p. 303).

Any type of probability distribution may theoretically be specified for the uncertain variables. Table 2 lists the uncertain variables used in this model and the type of distribution which is used for each. Figures 3, 4, 5, and 6 depict the normal, triangular, uniform, and log-normal distributions respectively. The rationale for using these distributions is provided below.

Table 2--Distributions Used for Uncertain Variables

Variable	Distribution
Annual price change	Normal
Investment cost contingency factor	Triangular
Operating cost contingency factor	Triangular
Presence or absence of resources (Bernoulli)	Uniform
Amount of reserves	Log-normal or normal

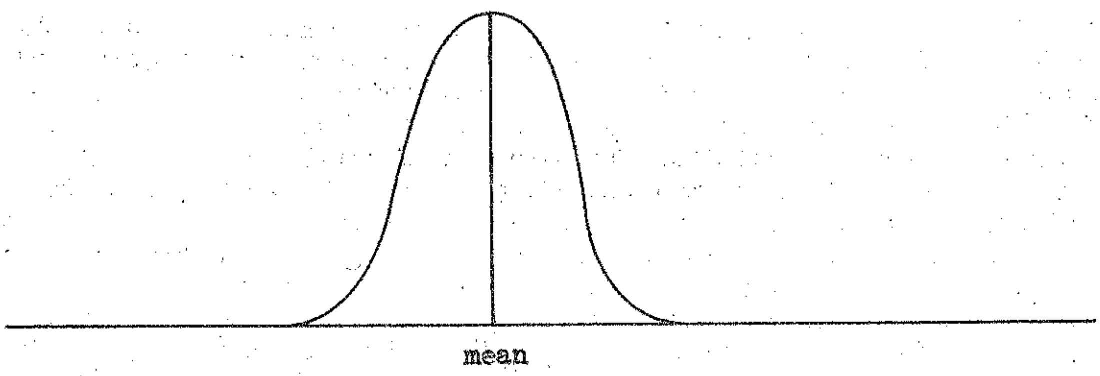

Figure 3.-- Normal Distribution Used for Annual Price Change and Reserves

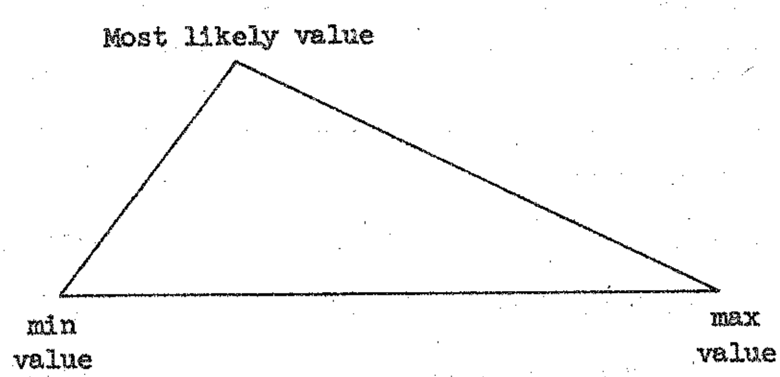

Figure 4.--Triangular Distribution

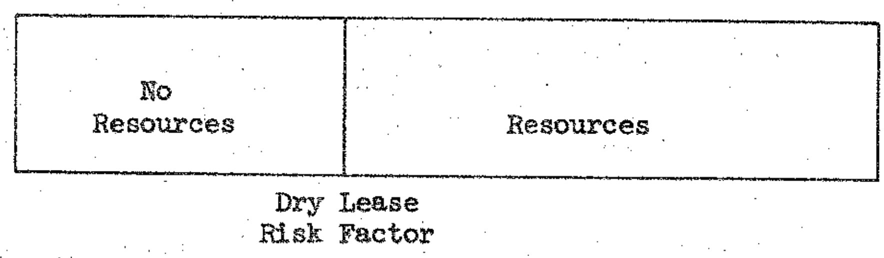

Figure 5.--Uniform Distribution

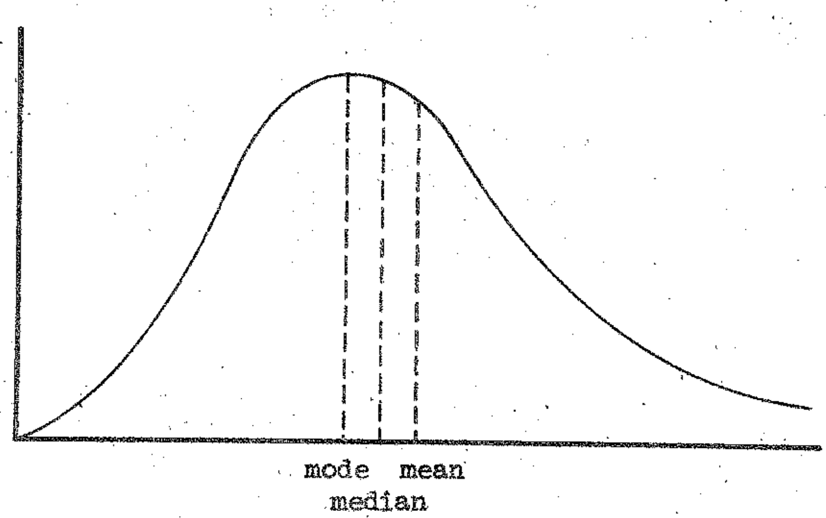

Figure 6.--Log Normal Distribution

Future Resource Prices: Uncertainty in future resource prices is handled by randomly selecting the annual change in price each year from a normal distribution with a specified mean and variance. This vector of annual price changes is converted to a vector of initial annual price. Equation 3 illustrates this process.

$$P_o(n) \cdot e^{P_1(n)} = P_o(n+1) \quad (3)$$

Since this procedure is repeated independently for each Monte Carlo iteration, a separate price distribution emerges for each year of the production period. Because the annual price change has a compound effect upon the initial price, the mean and variance of these annual price distributions would also change through time.[4]

The price change used in this analysis is the expected price change in excess of general inflation. It is not the total expected change in price of the resources; rather, it is the difference between the expected change in price of the resource and the expected general rate of inflation. This same principle applies to investment and operating cost factors. Thus, the relative inflation rate between revenues expected from the resource and cost to obtain the resource is a derivative of the inputs to the model. Because both cost and revenue inflation factors are keyed to general inflation, relative inflation between costs and revenues for a particular investment can be automatically accounted for using this procedure.

Investment Cost Contingency Factor: Investment costs are uncertain for at least three reasons, and a cost contingency factor is used to incorporate this uncertainty into the model. The contingency factor is a percentage of the estimated investment cost and is selected from a triangular distribution with an input minimum, maximum and most likely value.

One of the most important reasons for a contingency factor in investment cost is that inflation in construction costs in recent years has taken place at a rate higher than the rate of general inflation. Although this experience will not necessarily continue, it is uncertain what the rate will be over the next decade. Since the construction and start-up period for an energy extraction facility may be five years or more, the rate of inflation can have a

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[4] The resulting annual price distributions can be truncated and possibly skewed. For example, if policy options involving price support levels are simulated or minimum prices are used, the support levels may be high enough to affect prices, truncate the lower end of the price distribution and implicitly the price change distribution.

significant effect on total construction costs. Second, investment costs may be uncertain because technology for extracting and refining some resources is relatively new. For example, no large oil shale processing plants have yet been constructed. As another example, sub-sea completions required in some offshore areas represent a new technology. Unforeseen engineering and technical problems could raise such investment costs substantially. A third reason for an investment cost contingency factor is that the length of the development and construction period required for facilities of the type and scale required may not be known with certainty. Changes in the assumed period will have a significant impact on the present value of investment costs.

As is evident from the discussion of these factors, the distribution of investment cost uncertainty tends to be one-sided. In other words, the risk is mainly on the high side, so the contingency factor distribution would be expected to be skewed in that direction.

Operating Cost Contingency Factor: The two factors affecting annual operating costs in the model are θ, the annual increase in cost per unit, and K_o, the initial operating cost per unit. For purposes of analysis, θ is assumed to be known and constant throughout the production period, and a triangular distribution of K_o values is utilized. Uncertainty in initial operating cost arises from the same sources as for investment cost (future relative inflation and unforeseen technological difficulties) plus uncertainty in the future cost of environmental protection. Since future government regulations are unknown or are subject to modification, it is difficult to forecast the environmental control costs which must be borne by the private sector. However, once production has begun with technological problems solved and environmental control equipment in place, future changes in operating cost should be subject to less uncertainty. Therefore, the initial operating cost, K_o, was assumed to be uncertain with risk mainly on the high side.

Presence or Absence of Resources: This variable is particularly relevant for oil and natural gas production. When some quantity of resource is known with certainty to be present, the variable may be set to zero, and the model then assumes resources are always present on the lease area. When the variable is operative, a random number generator is used to generate a random number between zero and one from a uniform distribution. This random number is then compared with the dry lease risk factor to determine if resources are present for this iteration. If the random number is greater than or equal to the dry lease risk factor, then resources are assumed to be present and the model computations continue. For example, if the random number generated were .13 and the dry lease risk factor, .10, then resources would be present for this iteration. Clearly, if the dry lease risk factor is set at zero, then all random numbers between zero and one will be greater than or equal to the dry lease risk factor and resources will always be present.

Amount of Reserves: For some resources such as oil and natural gas, the greatest source of uncertainty is the amount of reserves present on a leasehold. For almost all resources some degree of uncertainty about the total quantity of resources in place exists.

Relating to petroleum exploration, a number of researchers have found that the log normal distribution provides a good fit for experimental data on the size of petroleum deposits (Uhler and Bradley; U.S.G.S., 1975; Kaufman, 1963).

Therefore, the log normal distribution is used for the size distribution of petroleum resources and in other situations where deemed appropriate.

For resources which are not distributed log normally, the normal distribution may be used in the simulation program. In either case, the mean and standard deviation and distribution desired are model inputs.

The Model Description with Monte Carlo Simulation

Once the Monte Carlo simulation begins, each of the procedures is repeated for each iteration of the simulation. In other words, if 200 Monte Carlo iterations are specified, all of the steps from this point on are repeated 200 times. The results of each iteration are stored and used to calculate the mean and other statistics on output variables.

The first step in the Monte Carlo simulation is to determine if there are any resources present on the lease. The chance of the lease having no resources is an input variabe, DTRSK. A random number is selected from a uniform distribution and compared with this factor to determine if resources are present for each iteration, as explained above. If no resources are present, the loss incurred from exploration is entered into the after-tax net present value factor (ATNPV) and used in calculating the expected present value of the lease over all iterations. The iteration is terminated and a new iteration is begun.

If resources are found on the tract, the next step in the process is to make a random selection of factors to be used in determining total investment and operating costs. A choice of three methods is allowed in making this selection of factors. First, the investment and operating cost input values may be used without any random component added. In this case, the random selection process is bypassed. Alternatively, a cost adjustment factor may be selected from the triangular cost distributions supplied for both investment and operating costs. For both investment and operating cost the minimum adjustment factor, the most likely adjustment factor and the maximum adjustment factor are inputs determining the shape of the triangular density function. For example, the cost factor could range from 0 to .2 with a most likely value .1. In this case an equilateral triangular density function would be employed. Either the mean of the triangular distribution or a random selection from that distribution may be used to determine the actual adjustment factor. The adjustment factor is then multiplied by the base cost with the result being added to the base cost. In essence, the random cost component which results from the adjustment factor is a contingency. The actual amount of the contingency may be zero (if the base value is used), equal to the mean of the triangular distribution, or randomly selected from the distribution. Normally, the random selection method would be used because contingency is considered a random component of total cost. Hence, the random selection method is considered to better reflect actual operating conditions.

The next two steps in the model simulation vary depending upon whether installed capacity is an input vector or determined within the model. If installed capacity is internally determined, the random factors for investment and operating costs are immediately used to determine the investment and operating cost values which will be used for each installed capacity. If installed capacity is an input, associated investment and operating cost values are also input along with each installed capacity. The same random factor is applied to each of the investment and operating cost values for each installed capacity to determine a unique set of cost values. In other words, there is a fundamental difference between the two versions of the model in that economies of scale with respect to installed capacity are permitted if installed capacity is input to the model, but are not permitted if installed capacity is solved for within the model. However, economies of scale with respect to reserve size are permitted under both approaches. Once investment and operating costs are calculated, an investment subsidy may be subtracted if one is used for purposes of policy analysis.

If installed capacity is an input vector to the model, each capacity together with reserves and other input variables is used to determine the maximum production time horizon which can be used given the installed capacity and the amount of reserves. On the other hand, if installed capacity is solved within the model, a time horizon and the corresponding (maximum) installed capacity is determined internally. Since each of these procedures represent a different solution to the same basic structural relationship, we will develop that relationship carefully and explain the correlation between the two procedures.

<u>Economic, Engineering and Geologic Relationships</u>: We begin with the simple depiction of the relationship between reserves and production. Reserve estimates enter the calculus of profitability both as a basis for the investment and as a constraint on the production from an investment. The production constraint is represented in equation (4):

$$xR \geq \sum_{t=1}^{T} q(t) \qquad (4)$$

where R represents the amount of the resources in place, x the recoverable fraction with a given technology, q(t) the amount of annual production, and T, the production time horizon. This equation merely states that the sum of production through time can be no greater than the recoverable portion of the reserves in place (with a given technology). Given this constraint, the producer attempts to select an initial plant capacity which will maximize his return through time. In other words, the producer attempts to select the investment which maximizes his after-tax net present value of revenue subject to the production constraint.

Assume for the moment that production declines exponentially through time. Annual production may then be expressed as a function of initial installed capacity as in equation (5):

$$q(t)_i = \int_{t-1}^{t} q(o)_i e^{-at} \qquad (5)$$

where $q(o)_i$ represents initial installed capacity of the ith plant which is one of a group of possible initial capacities.[5] While this simple relationship between installed capacity and annual production may be adequate for oil resources after a period of time, it is not adequate for other resources or for oil resources during the early production phase. A typical resource production pattern includes a production build-up period during which production is increasing each year as installed capacity is coming on stream followed by a flat production period which continues indefinitely or is followed by a declining production period as shown in Figure 7. Under this scenario, total production during the lease life is given by equation (6):

$$(6) \qquad PROD = q(o)_i \cdot \sum_{j=1}^{B} h_j + q(o)_i \cdot (F-B) + \int_{o}^{T-F} q(o)_i e^{-at}$$

where the build-up period is the period between year one and year B, the flat production period is between year B and year F, and the declining production period (perhaps at a zero rate) is the period from F to T; T being the production life of the lease as determined below.[6] Equation 6 gives the sum of production during each of the three phases of production. Production during the build-up period is equal to the sum over the build-up period of the annual factors h_j times installed capacity; production during the flat period is simply the number of years in which production is constant times installed capacity; and production during the decline period is equal to the integral over the number of years production is declining.

Recalling from equation (4) that total production must be less than or equal to recoverable reserves we may now combine equations (4) and (6) to yield the relationship between recoverable resources and installed capacity:

$$(7) \qquad xR - \beta q(o)_i e^{-a} - \gamma q(o)_i \geq q(o)_i \cdot \sum_{j=1}^{B} h_j + q(o)_i \cdot (F-B) + \int_{o}^{T-F} q(o)_i e^{-at}$$

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5 For some resources, the value of the production decline rate, a, may be set equal to zero. In that case annual production, $q(t)_i$, becomes equal to initial installed capacity $q(o)_i$ throughout the production period.

6 The integral for the decline period goes from zero to T-F rather than F to T because this integral properly measures the sum of production over the decline period.

The β and γ parameters are geologic variables applicable to oil which relate total recovery to the rate of recovery. (The faster the oil is produced, the lower is total recovery.) For resources such as coal and oil shale or any resource other than petroleum, the geologic factors β and γ may be set equal to zero. In that case, recoverable reserves, xR, is greater than or equal to production as defined in equation (6).

By assuming that recoverable reserves are exhausted, we may change equation (7) from an inequality to an equality and solve for either q_o or T.[7] Equation (8) represents the solution of equation (7) for T which is used in the case of input q_o:

$$(8)\quad T = \left\{ \ln \left[1 + a(-xR/q_o + \beta e^{-a} + \gamma + \Sigma h_i + F - B)\right] \right\} / -a + F$$

Equation (9) represents the solution to equation (7) when installed capacity, q_o, is solved within the model:

$$(9)\quad q_o = \frac{axR}{\left[1 + a(\beta e^{-a} + \gamma + \Sigma h_i + F - B) - e^{-a(T-F)}\right]}$$

Equations (8) and (9) are derived by integrating equation (7) and solving algebraically.

Given that q_o and T have been determined either by input or within the model, the production time horizon, T, must be subjected to two constraints before it can be employed. These constraints are the physical and economic lifetimes of the proposed investment. The production time horizon for a given investment can be no greater than the actual physical lifetime of the initial plant.[8] Nor can the production time horizon exceed the time at which variable unit cost of producing the product exceeds the revenues per unit obtained from marketing it. In other words, when the steadily increasing unit costs of production (assuming a rising MC curve) exceed the revenues per unit of production, production would cease.

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7 The notation, $q(o)_i$, is here changed to q_o representing one potential investment, but the reader should be aware that the optimization process to be used covers all available investment opportunities.

8 This does not necessarily mean that the energy resource on the leasehold has been exhausted. As a result, the winning bidder may want to reinvest in order to continue production until the point where resource exhaustion takes place. Whether such investment will, in fact, occur depends upon economic considerations present at that point in time. The extent of the remaining resource will play a substantial role in this decision. The model can be modified to incorporate this later investment decision if it is assumed important (in a present value sense) for initial bidding behavior.

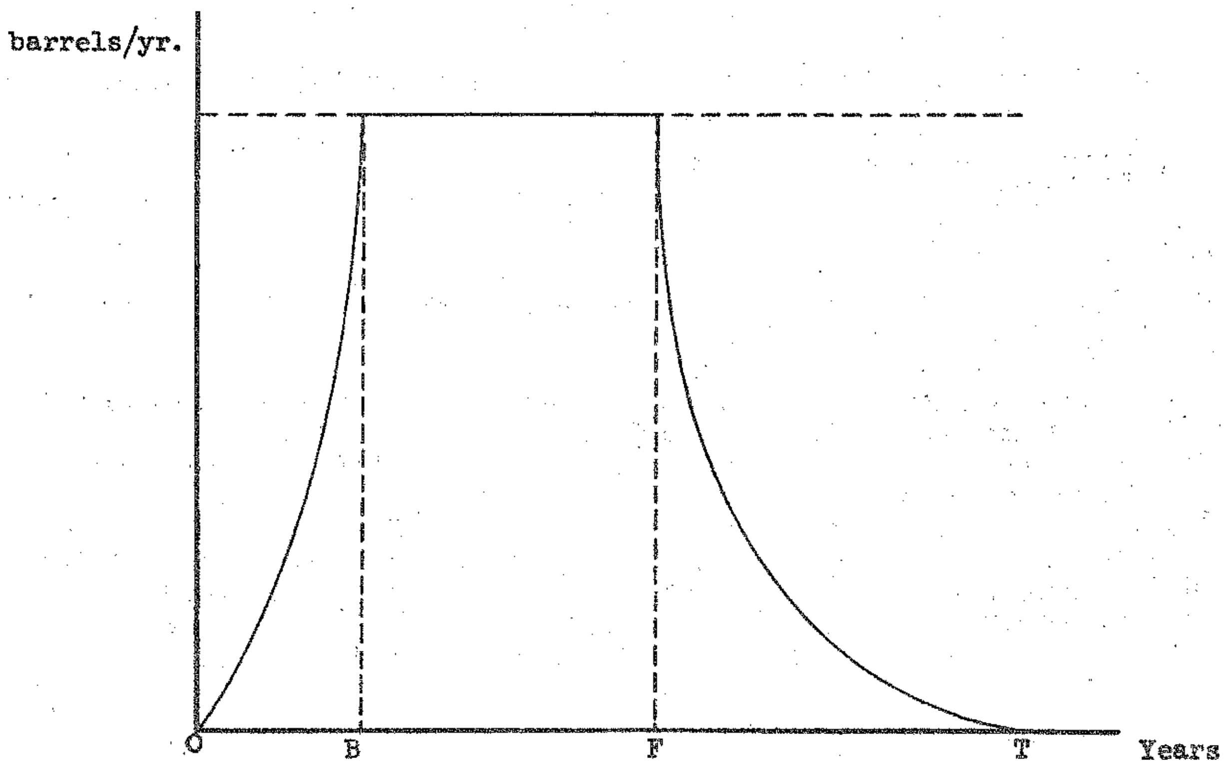

Figure 7.--Production Profile Through Time

The first constraint is simply expressed as an exogenously determined constant:

$$(10) \qquad T \leqq T_p$$

where T_p equals the maximum physical lifetime of the investment. The second constraint is the limit obtained when marginal cost equals marginal revenue. Equation (11) states that the economic limit occurs when operating costs plus taxes exceed or equal revenue minus royalties and severance taxes:[9]

$$(11) \qquad (1 - \lambda - s)P_o e^{P_1(t+L)} \leqq K_o e^{[(\theta+a)t-aF]} + \phi[(1-\lambda-s)P_o e^{P_1(t+L)} - z(1-\lambda-s)P_o e^{P_1(t+L)} - K_o e^{[\theta+a)t-aF]}$$

Solving equation (11) for the time constraint yields:

$$(12) \qquad T \leqq \left\{ (\ln[\frac{(1-\phi)K_o}{(1-\phi+\phi z)(1-\lambda-s)P_o}]) - aF - P_1 L \right\} / (P_1 - \theta - a)$$

Note that this equation may be negative or undefined when the rate of change in price is greater than or equal to the decline rate plus the rate of change in unit operating cost ($P_1 \geq \theta + a$). The negative sign occurs because the marginal revenue-marginal cost curve intersection is in the negative quadrant to the left of the origin as shown in Figure 8. The correct interpretation for this negative sign is that the economic time constraint is infinite.

We now have each of the equations and relationships necessary to determine the production time horizon. The production time horizon is that T

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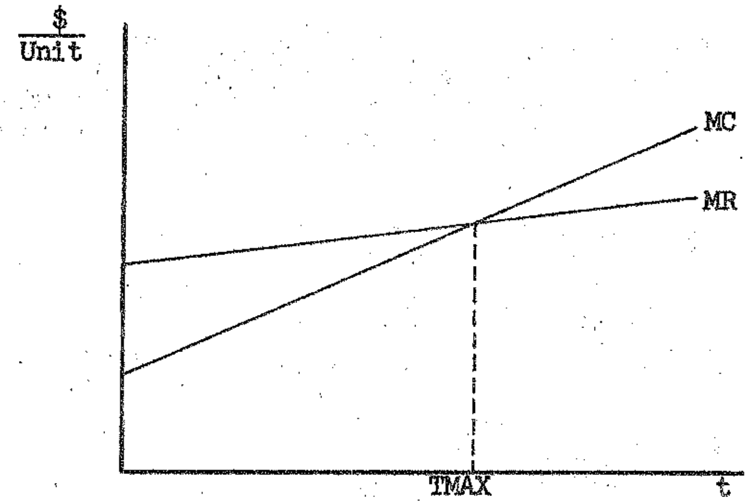
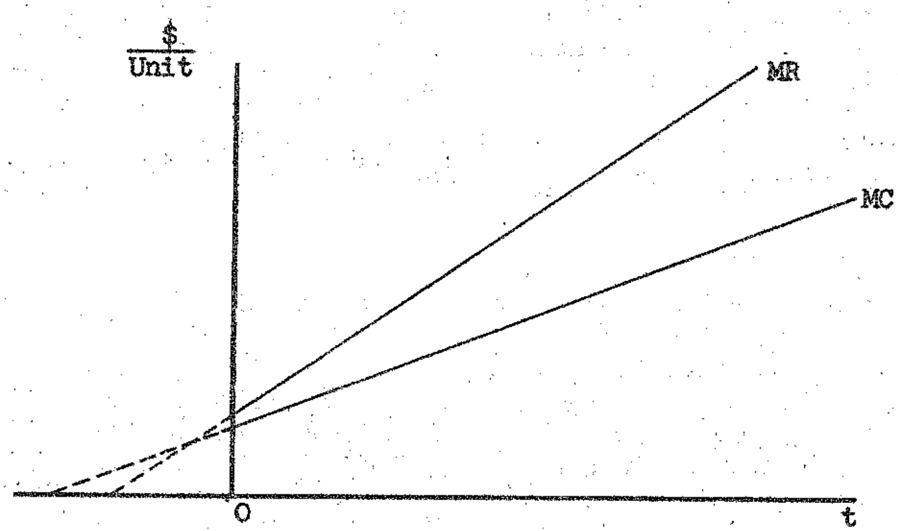
[9] Since total operating costs increase by the value θ through time, but remain constant in any time period regardless of the decline rate, unit costs increase at an exponential rate through time. This phenomena can be due to equipment obsolescence, logistical problems with production and/or increased maintenance costs and relative inflation. (Arps; Davidson; U.S. Department of the Interior Officials). In notation form, total operating costs in any time, t, are expressed as $q_o K_o e^{\theta t}$. Thus, units costs become:

$$q_o K_o e^{\theta t} / q_o e^{-a(t-F)} = K_o e^{[(\theta+a)t-aF]}$$

The denominator of this fraction is derived from the last term of equation (7).



Finite Solution



Infinite Time Horizon

Figure 8.--Solution to the Economic Time Horizon

determined in the model either by equation (8) or through the q_o, T optimization procedure, subject to the physical and economic lifetime constraints given by equations (10) and (12). Hence, the production time horizon is the minimum of the resource exhaustion time period, the time period for the physical life of the plant, or the economic production time constraint. Mechanically, these equations differ slightly depending on whether installed capacity is input or determined by the model as explained above and as outlined in Figures 1 and 2.

For the first q_o-T set to be evaluated in each Monte Carlo iteration, the next step is to create a vector of prices covering each year in the production period. The first step in this process is to create a vector of annual price change covering at least the period from the time of the lease sale to the end of production. This vector may be created by randomly sampling from a normal distribution of price change with an input mean and standard deviation as explained above. Alternatively the mean annual price change may be used for each year in the vector.

If desired, more than one price change distribution may be used in generating the price change vector. The model allows for as many as four unique price change distributions to be input for up to four specified time periods. For example, price could be expected to rise at an annual rate of eight percent for three years, fall at a rate of three percent for six years, remain relatively constant for eight years, and then rise at a rate of four percent through the end of production. Each of the expected price change values would have a unique variance, so that the variance as well as the expected value of annual price change can differ through time. The price change vector is created by utilizing the appropriate distribution for each year in the vector.

The next step is to create a vector of prices from the lease time until the end of production using the initial input price P_o along with the vector of price changes. The vector is created by multiplying the price at the beginning of each year by the exponential price change during that year to get the price at the beginning of the next year (see equation 3). This process is repeated until prices have been generated for each year until the end of production.

For computational purposes, only prices during the production period are relevant; prices during the construction and development period are not needed for the analysis. The price vector must, therefore, be truncated by the length of the development period (lag) and reindexed. In other words, a new price vector which begins with the initiation of production must be created from the original price vector which begins at the point of the lease sale. Correspondingly the vector of annual change in price must also be reindexed by this same amount. Once this is accomplished, the vectors of price and price change correspond to the years of production.

The next step is to calculate the investment for each year of the construction and development lag and the discounted value of total capital investment. Total capital investment is determined by multiplying installed capacity, q_o, by the investment cost per unit of installed capacity, b, as determined in the cost subroutine for each resource.

To determine the discounted value of total investment, the total investment figure must be multiplied by the percentage of total investment occurring in each year of the development period and the resulting investment value for each year discounted back to the beginning of the lease. Both development costs and exploration costs for each year are summed together and discounted back to the beginning of the lease. In functional form this relationship is expressed in equation (13):

$$(13) \qquad PVI = \sum_{i=1}^{L} (q_o \cdot b \cdot f_i + EX_i)/(1+r)^i$$

where PVI represents the present value of investment, f_i the factor used to determine the proportion of investment in each year of the lag, and EX_i the exploration expense during each year of the lag. The values for total annual investment are then used to calculate depreciation streams for both the lag and production periods; and to calculate expensed investment and the investment tax credit.

The sum of years digits method is used to calculate depreciation. However, other IRS approved methods can be incorporated if desired. Separate depreciation streams are maintained for each year of investment. Equation (14) illustrates the calculation for one depreciation stream:

$$(14) \qquad DP_j = \sum_{i=1}^{N} \left(\frac{N-i+1}{(N^2+N)/2}\right)(y_j \cdot b \cdot f_j \cdot q_o(1-\alpha))/(1+r)^{(i)}$$

In the actual computations, each of the streams calculated as in equation (14) is split into two streams: one for the development period and one for the production period. If the construction and development period (lag) is four years, eight depreciation streams are calculated. For both the development and production periods, the present value of depreciation is summed to provide one depreciation value for use in calculating tax savings during development and another to use in the profitability calculations during production. In addition, the undiscounted annual depreciation during production is saved to be used later in any profit share calculations.

The present value of tax savings during the development period is calculated from the present value of the sum of depreciation (DEPLG) plus expensed investment (EXINV) multiplied by the tax rate, (ϕ), plus the present value of the investment tax credit (IVTC) as shown in equation (15):

$$(15) \qquad TXS = \phi(DEPLG + EXINV) + IVTC$$

Working capital is then calculated as a function of the first year's operating cost. Once this calculation is complete, the model then enters the production loop. In this loop annual and total production, gross revenue, operating cost, royalty, severance tax, depletion, and profit share are calculated. Because many of the equations are in integral form, yet many of the values are needed on an annual basis, integral solutions are obtained over each year of production and then summed over the production period. For example, production is obtained from point zero to the end of year 1 and then from the beginning of year two to the end of year two and so on through the beginning of the last year of production to the end of the last year of production. These values are then summed to determine total production. In this way both annual and total values can be obtained for variables such as production, profit share, and royalty; and continuous discounting is maintained for variables such gross revenue and operating cost.

The methods used to determine annual production in each year of the production period are described in detail above. In addition to calculating production for the basic resource, production is also calculated for any associated resource such as associated gas with petroleum production. The ratio of production between the major resource and the secondary resource is assumed to to be a constant factor. In other words, to determine the production of associated natural gas in each period, the production of oil is multiplied by the factor (AGFAC) to determine the production of natural gas. In the equations that follow the annual production of the major resource will be denoted by q_t and production of the secondary resource will be denoted by g_t.

A number of equations are used in calculating the economic variables for each year of the production period. So that this process may be clearly understood, the development of the equation for gross revenue is presented below in three sequential forms:

1. The generalized integral form.

2. The integral form divided into annual periods.

3. The computational form actually used in the model.

Equations (16), (17), and (18) represent these three forms of the gross revenue equation respectively:[10]

$$(16)\qquad GR = q_t P_t \int_0^T e^{(P_1-r)t} + g_t GP_t \int_0^T e^{(GP_1-r)t}$$

$$(17)\qquad GR = \sum_{t=1}^{T} \left(q_t P_t \int_{t-1}^{t} e^{(P_1-r)t} + g_t GP_t \int_{t-1}^{t} e^{(GP_1-r)t} \right)$$

$$(18)\qquad GR = \sum_{t=1}^{T} \left[\frac{Q_t P_t (e^{(P_1-r)t} - e^{(P_1-r)(t-1)})}{(P_1-r)} + \frac{g_t GP_t (e^{(GP_1-r)t} - e^{(GP_1-r)(t-1)})}{(GP_1-r)} \right]$$

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[10] Actually P_1 and GP_1 are also time indexed variables as explained above, but they are written here in unindexed form for clairty of exposition.

Note that the annual values calculated from equation (18) are discounted (continuously) to the beginning of the production period. Calculation of annual operating cost (OC) proceeds in the same manner. The generalized integral form used in calculating operating cost is given in equation (19):

$$(19) \qquad OC = q_o K_o \int_0^T e^{(\theta - r)t} + RENT$$

The marginal cost of extracting the secondary resource is assumed to be zero, <u>or</u> included in the cost of extracting the primary resource.

According to IRS regulations, the bonus payment may be depleted (depreciated) in proportion to the depletion of reserves held. Accordingly, the proportion of total production produced in each year is multiplied by the original bonus and discounted to calculate the present value of bonus depletion. The annual values of gross revenue and cost, depreciation (DP_t), rent, and bonus depletion (BDP_t) are used to calculate the annual profit share base (PSB) as shown in equation (20):

$$(20) \qquad PSB = (1-\lambda-s)[P_t \cdot q_t + GP_t \cdot g_t - OC_t - DP_t - RENT - BDP_t]$$

To determine before-tax net present value (BTNPV), the difference between gross revenue and operating cost is discounted to the beginning of the lease and the discounted values of royalty, capital investment, profit share, and severance tax are subtracted. For resources for which depletion is still allowed, depletion is calculated as the present value of gross revenue minus the present value of bonus depletion (BDF) multiplied by one minus the royalty rate (λ); that quantity multiplied by the depletion rate (z) as illustrated in equation (21):[11]

$$(21) \qquad DPL \doteq z * (1-\lambda)(GR-BDP)/(1+r)^L$$

Taxable income is the present value of investment plus before-tax net present value minus the present value of depreciation during production minus the present value of bonus depletion as shown in equation (22):

$$(22) \qquad TXINC = BTNPV + PVI - DP - BDP - DPL$$

The present value of taxes paid is simply the taxable income multiplied by the tax rate minus the tax savings during the development period. A check is included in the model to eliminate the possibility of negative taxes. The implication of this constraint is that companies are not allowed to calculate investment profitability for any particular investment based on excess tax write-offs to be obtained from that investment. Excess tax write-offs

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[11] A check is provided in the program to make sure that depletion is no greater than one-half of the net income before depletion as stipulated in IRS regulations.

are allowed in the simulation program when development does not occur, but excess write-offs are not allowed ex ante as a basis for calculating investment profitability when development does occur.

After-tax net present value (ATNPV) is simply the difference between before-tax net present value and present value of taxes paid plus the present value of the original investment at the end of production as shown in equation (23):

$$\text{(23)} \qquad ATNPV = BTNPV - TAX + (Y\alpha q_o b + w)/(1+r)^T$$

where $Y\alpha q_o b$ represents salvage and w, working capital. The after-tax net present value calculated as described above represents the net worth of the lease. It also represents the residual economic rent to the resource. The relevance of this variable to better decisions and government policy is discussed in more detail below.

Once the after-tax net present value is determined for a particular q_o, other output variables associated with that ATNPV are stored. The model then checks to determine if all q_o's or T values have been evaluated. If not the model returns to the beginning of the q_o-T loop and repeats the procedure outlined above. If all possible T values or all input q_o values have been evaluated, the model then proceeds to select the optimal q_o-T combination for this Monte Carlo iteration. The optimal set is the one with the highest ATNPV. This optimal ATNPV is then compared with the potential tax write-off calculated earlier during the exploration phase. If the ATNPV is greater than the potential tax write-off the optimal ATNPV value is stored as the result for this iteration. If the potential tax write-off from not developing the lease is greater than the potential gain from developing the lease (ATNPV), the decision is made not to develop the lease and the exploration loss is entered into the after-tax net present value register. A zero is entered into the register for other output variables such as production, production time horizon, profit share, royalty, and tax. This result corresponds to the real world situation in which some quantity of resource is discovered during the exploration phase but the economics dictate that the quantity is so small that it is not commercial and the lease is not developed.

Monte Carlo Results and Model Outputs: With the final values of all output variables determined for this Monte Carlo iteration the model then checks to see if all Monte Carlo iterations specified have been completed. If not, the model returns to the beginning of the Monte Carlo simulation and repeats the entire process. If all the Monte Carlo iterations have been completed, then the mean, standard deviation, and other statistics on each output variable are calculated. If desired, histograms can be constructed for after-tax net present value (ATNPV) and reserves. The histograms illustrate the distribution of output for these two variables. The distribution of after-tax net present value provides the range of potential outcomes and the frequency with which each outcome occurs.

In the above described model, economic rent is composed of royalty and profit share payments, tax payments, and the after-tax net present value (ATNPV). These rent components can be manipulated in the model to determine expected bidding behavior and associated impacts for various leasing policy alternatives. For example, in a bonus bidding system with a fixed royalty rate, the expected bonus bid is a function of after-tax net present value.[12] The sum of the bonus bid, royalty income, and taxes is equal to total economic rent.

Under a royalty bid system, the winning bid would be expected to be the one which eliminates after-tax net present value. In other words, when after-tax net present value is constrained to equal zero, royalty payments and taxes alone would compose economic rent, and the royalty bid rate can be determined. Hence, the discounted value of cumulative royalty payments and taxes equals the anticipated economic rent.

One of the policy options programmed into the model is the ability to determine what the royalty bid rate would be under the above assumptions. In addition to the fixed royalty and royalty bid options, a sliding scale royalty system is also incorporated into the model. Under this system, the royalty rate in each period is a function of the production in that period. This system attempts to capture economies of scale and prevent early termination of production by increasing the royalty rate when production is high and decreasing the rate when production is low. Similarly a variable profit share system is incorporated into the model which allows the profit share rate to vary in each production period with the amount of profit in that period.

A number of other profit share systems are also included in the model. A capital recovery system, which provides for recovery of capital at a specified rate of interest over a predetermined time period before the government takes its profit share, is one of the profit share variations. Also, a profit share system based on the British profit share plan is included.

The model is also programmed to handle any of three variations of advanced royalty payments. Specific advanced royalty systems with the advanced royalty based on either a certain value per ton or a certain percentage of the gross value at the point of the lease are two of the advanced royalty options. The third advanced royalty option (ad valorem) provides for collecting advanced royalties at a predetermined rate based on the actual price prevalant throughout the production period. In conjunction with any of the advanced royalty systems an exogenous delay in production may be input to the model and the effects of any of the advance royalty systems with alternative input values determined. Alternatively, changes in the expected production delay caused by different advanced royalty parameters or price expectations may be evaluated.

Summary

Clearly a wide range of leasing policy options including bonus bidding systems, royalty systems, profit share systems, and a number of combinations of these systems and their many variants may be analyzed with the generalized leasing model. In addition to the wide range of leasing policy

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[12] Actual bonus bids are a result of bidding strategies formulated from game theoretic approaches combined with bidders estimates of lease value.

options, a number of tax policy options are also included in the model. In addition, a number of general policy options such as price subsidies, purchase guarantees, price supports, investment subsidies and other policy options designed to increase certainty for private investors are included. Furthermore, other tax policy, general policy, or leasing policy options can easily be incorporated in the model framework.[13] Hence, the model is ideally suited for analysis of a wide range of government policy options dealing with the disposition of federally owned natural resources.

Outputs of the basic model include statistics on the following variables: production time horizon, installed capacity, present value of royalty payments, present value of depletion, present value of taxes, present value of profit share payments, production, reserves, total resource cost, and after-tax net present value. Additional outputs are provided for specialized leasing or other policy options such as the royalty bidding system.

The use of Monte Carlo simulation with uncertain variables provides an additional dimension to government policy analysis. Not only can the change in expected value of model outputs be determined when a policy variable is changed, but also the change in variance of the model outputs can be determined. This information may be quite useful for government policy makers attempting to influence private sector decisions. In addition, the simulation process more closely approximates the decision making procedure used in the private sector when evaluating potential resource investments.

This model description has been both detailed and comprehensive. The aim has been to give the reader a thorough understanding of not only the rationale behind the model algorithm, but also an understanding of the actual equations and decision functions utilized in the programmed version of the model. All too often, the links between theory and computational forms used in models are not clearly established and readers and model users must tediously grope through the description to provide these links on their own. It is our hope that through providing a complete description of the model mechanism that readers and users will be better able to utilize the model results and to properly establish the links between these model results and informed policy analysis.

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[13] For example, model outputs from individual evaluations may be combined to simulate lease sales through time. This approach is used to determine the impacts through time of alternative leasing strategies.

APPENDIX A

Table 1 in the text provided a partial listing of input variables used in this model; namely those variables which were specifically used in the main text. Table A-1 provides a complete list and brief definition of all input variables used in the model in the order in which they are input. This appendix also provides an expanded definition for a number of variables about which the economic or geologic interpretation might be uncertain or about which the usage in the model might be unclear. Variable definitions not expanded in this section are thought to be clearly explained in the text or by the definition given in Table A-1.

The control variables, (input on the first card) determine the options which are used in analyzing the data and calculating and printing the results. NCASE determines the number of cases, situations, leases or fields which are to be evaluated. Each case requires a complete set of input cards. The variable NCHKPT is a print option. When the variable is set equal to one, the detailed output statistics for each Monte Carlo iteration are printed. Model users are cautioned to set this variable to zero if a large number of Monte Carlo iterations are specified to avoid excess printing. Variable NSTAT determines the number of output variables for which statistics are calculated and printed. When the variable is set equal to zero only the ATNPV statistics are calculated and printed. When the variable is set equal to one, statistics for all variables are printed. Variables NPLP and NLMB are the control variables for the price loop option and the royalty bid loop options, respectively. When either of the variables are set to one, the appropriate loop option is utilized, and the model iterates over all possible resource prices or royalty rates to determine that price or royalty rate which sets the after-tax net present value approximately equal to zero. Variable LGLP is the control variable which is used in conjunction with an advanced royalty system to determine the optional production initiation point (from the producers point of view). When this variable is set equal to one the program loops through a specified range of production delays to determine the lag which maximizes ATNPV given the producers price and cost expectations and the advanced royalty parameters. Because these options create another loop outside the main model which must itself be iterated several times, they tend to be very expensive to run. Of course, more than one option cannot be run at the same time and users are cautioned in this regard. The NHIST variable is specified as 1 if a histogram for ATNPV is desired, at 2 if a reserve histogram is also desired, and at 0 if no histogram is desired. Variable HSTNN determines the number of statistics to be calculated for each output variable. If this variable is equal to zero the mean for each output variable is the only statistic calculated and printed. If the variable is set equal to 1, the mean, standard deviation, coefficient of skewness, and coefficient of kurtosis are calculated for each output variable.
The variable NMRES is the reserve code. The following codes are used to identify resources analyzed by the model:

1 = oil

2 = gas

3 = oil shale

4 = coal

Table A-1.--Complete List of Generalized Leasing Model Input Variables

Symbol	Computer Code	Definition
Control Card:	NCASE	No. of cases
	NCHKPT	Print option for statistics on each MC iteration (1=PT)
	NSTAT	Use 1 to get all statistics, use 0 for ATNPV only
	NPLP	1 for price loop option, 0 otherwise
	NLMB	1 for royalty bid loop option, 0 otherwise
	NHIST	1 if an ATNPV histogram is desired, 2 for reserve histogram also
	HSTMN	0 if mean is the only statistic needed, 1 for all stats
	NMRES	Reserves code (1=oil, 2=gas, 3=oil shale, 4=coal)
	LGLP	1 if lag loop is desired (used with advance royalty)
Card 1:		
	SUB	Price Subsidy (Dollars)
P_o	RPØ	Initial Price for the Resource
λ	RLAMB	Royalty rate (%)
N	RN	Depreciation period (years)
α	RALPHA	Investment salvageable (%)
Ω	ØMEGA	Investment Tax Credit Rate (%)
r	RR	Discount rate (%)
a	X(I)	Production Decline Rate (%)
Card 2:		
R	RCAP	Reserves (if Monte Carlo not used) (Barrels)
θ	RTHETA	Annual Change in Operating Cost (%)
z	RZ	Depletion Rate (%)
ø	RPHI	Tax Rate (%)
i	RI	Interest Rate for Capital Recovery (%)
T_p	LT	Maximum physical lifetime for investment (Years)
β	RBETA	Geologic Parameter (oil)
γ	GAMMA	Geologic Parameter (oil)
Card 3:		
	NQO	Number of capacities to evaluate; if 0, continuous form is used
	RLAMF	Royalty adjustment factor for sliding royalty
	RLAMX	Maximum royalty rate
	PRØDF	Production max for base royalty
	PFSHRM	Profit share rate (minimum rate if sliding scale used)
	PFSRTX	Maximum profit share rate
	PFSRF	Profit share factor for sliding profit share
	PFBASM	Profit maximum for base profit share rate
	NCAP	Captial recovery period (years)
	BPF	British plan factor

Table A-1.--Continued

Symbol	Computer Code	Definition
Card 4:		
	PMIN	Price support level (or min price to be used)
	NPMETH	Price generation method; 0=means, 1=random
	NSEED	Price generation seed
	MPT	Length of price support period (years)
P_1	RP1MN	Mean of normal distribution of annual change in price
	RP1STD	Standard deviation of dist. of annual price change
	NPC	No. of price change distributions used
	NP1	End of period for first P_1 distribution
	NP2	End of period for second P_1 distribution
	NP3	End of period for third P_1 distribution
	NP4	End of period for fourth P_1 distribution
AGFAC	AGFAC	Factor for determining the amount of associated gas
	GP1MN	Mean of normal distribution of annual price change(gas)
	GP1STD	Std.Dev. of dist. of annual price change for gas
GP_o	GPØ	Initial price for gas (or any second resource)
	GPMIN	Minimum allowable price for gas (or price support)
	NGPSD	Seed for generation of annual price change for gas
	MGPT	Length of time for which minimum gas price is valid
	NGC	No. of gas price change distributions used
	NG1	End of period for first gas price change dist.
	NG2	End of period for second gas price change dist.
	NG3	End of period for third gas price change dist.
	NG4	End of period for fourth gas price change dist.
Card 4a:		
	P1M2	Mean of P_1 distribution for period 2
	SD2	Std. Dev. of P_1 dist. for period 2
	P1M3	Mean of P_1 dist. for period 3
	SD3	Std. Dev. of P_1 dist. for period 3
	P1M4	Mean of P_1 dist. for period 4
	SD4	Std. Dev. of P_1 dist. for period 4
	G1M2	Mean of GP1 dist. for period 2
	GSD2	Std. Dev. of GP1 dist. for period 2
	G1M3	Mean of GP1 dist. for period 3
	GSD3	Std. Dev. of GP1 dist. for period 3
	G1M4	Mean of GP1 dist. for period 4
	GSD4	Std. Dev. of GP1 dist. for period 4
Card 5:		
	NLØØP	Number of Monte Carlo Iterations
Card 6:		
	NBM	Investment cost selection method: 0=mean, 1=random, 2=base
	NKM	Operating cost selection method: 0=mean, 1=random, 2=base

Table A-1.--Continued

Symbol	Computer Code	Definition
Card 6: (Cont'd)	NBSD	Investment cost seed
	NKSD	Operating cost seed
	SUBI	Investment subsidy (%)
	BMIN	Minimum Value for random component of investment cost RBQ
	BMAX	Maximum Value for random component of investment cost RBQ
	BMØDE	Most likely value for random component of investment cost
	KMIN	Minimum value for random component of operating cost RKO
	KMAX	Maximum value for random component of operating cost RKO
K_o	KMØDE	Most likely value for random component of operating cost RKO
	BYPRCD	Byproduct credit (percent of price added to price)
	LDLM	Climatic variable for OCS investment cost
	ITMIN	Minimum allowable production time horizon
	LAG1	Exploration period (years)
	F1	Proportion of exploration expense for each year
	YZ1	Proportion of yearly exploration cost which is tangible
RENT	RENT	Annual rent per acre
CARD 7:		
	WELLS	No. of wells per 1000 acres
	ACRES	No. of acres in lease area
	RBEXP	Cost per exploratory well drilled
	DTRSK	Chance of no resource find at all
B_1	BFAC	Factor used to adjust ATNPV to determine bonus
B_O	BCØN	Constant used to adjust ATNPV to determine bonus
	NDTSD	Seed for random determination of dry tracts
	WCF	Proportion of first year's operating cost = working capital
	FR	Fraction of reserves used to set advance royalty prod.
	CNT	Rate for specific advance royalty (cents/ton)
S	ST	Rate for state severance tax (on gross value)
	LLIFE	Lease life used for advance royalty calculations
	LAGD	(Delayed) lag used in conjunction with advance royalty

Table A-1.--Continued

Symbol	Computer Code	Definition
Card 7:		
(Cont'd)	MADRØY	Method of calculating advance royalty
	LAGR	Length of period before advance royalties are applied
	MCR	No. of M.C. iterations for second bonus approximation
	ALAMB	Advance royalty rate
CArd 8:		
R	RMEAN	Mean of reserve distribution (log normal)
	RSTD	Standard deviation of reserve distribution
	NRSD	Seed for reserve value generation
F	IFLATP	Length of time the initial production level is used
	KRS	Reserve distribution: 0=log normal, 1=normal
Card 9:		
Qo	Qo	Installed capacity (annual)
b	RBQ	Cost per unit of installed capacity
Ko	RKO	Operating cost per unit
L	LAG	Investment Lag--Construction period (years)
f	F	Proportion of investment expended in each lag year
y	YZ	Proportion of yearly investment which is tangible
Card 10:		
B	IBP	Length of production build-up period
h_i	BPP	Factor applied to capacity to determine prod. during IBP

These nine are the only control variables used by the model.

The remainder of the input variables are either geologic, economic, or policy input variables. The first data input card contains eight of these variables. The first variable, SUB, represents a price subsidy for the resource. This is the policy variable designed to determine the impact of a price subsidy provided by the government. The initial price for the resource, RPO, is the price for the resource at the time of the lease. As explained in the main text, a vector of initial annual prices is created in the program starting with RPO.

The discount rate (r), computer coded RR, may be viewed in whatever fashion is considered appropriate by the model user. The authors normally consider an appropriate discount rate to be the opportunity cost for capital in low risk portions of the private sector (such as the prime interest rate). Because this opportunity cost is related to the rate of return on capital in less risky industries, the discount rate is essentially a risk free rate. Riskiness of the investments in energy resources is handled through the Monte Carlo simulation process. Alternative discount rate conceptualizations may be utilized by model users.

The production decline rate, a, X(I) in the computer code, is the continuous annual rate of decline in production after the peak production period has ended. For resources with no anticipated decline in production the peak production may be extended throughout the production life of the resource and the decline rate becomes inoperative.

On the second card, the annual change in operating cost (θ), RTHETA in the computer code, may be thought of as composed on two components. First, θ is composed of the expected difference in the inflation rate for operating cost and general inflation rate in the economy. Second, annual operating costs might be expected to increase because of equipment obsolescence and increased maintenance cost.

Although depletion is no longer allowed for oil and natural gas, depletion is still permissable for other resources. The depletion rate (z), RZ in the computer code, varies from resource to resource. Revenue depletion (as opposed to cost depletion) is always utilized in the model calculations. This form of depletion is almost always used in industry as well.

The variable coded RI is the interest rate used in the capital recovery profit share system. Beta and gamma, RBETA and GAMMA in the computer code, are geologic parameters relating only to petroleum resources. The functional form in which they are utilized relates the penalty in terms of total resource recovery to the rate of extraction. In other words, the faster the petroleum is extracted, the less the total recovery which is achieved.[1]

Card three contains the variables used in setting flexible royalty and profit share rates. The minimum (base) royalty rate is input on card one. The annual royalty rate is determined each period by subtracting the maximum production allowed for the base royalty, PRODF, from the production in that period and then using the royalty adjustment factor, RLAMF to determine the actual rate. The maximum royalty rate, RLAMX, cannot be exceeded in any time period. Similarly, the profit share rate which applies in any given year is determined by subtracting the profit share base from the profits in a given year and applying the profit share factor to that difference to determine the actual rate, which must be less than or equal to the maximum profit share rate. If a capital recovery profit share plan is used, the length of time over which the capital is recovered, NCAP, is also input on this card. If the British profit share plan is used in which some multiple of the capital investment is recovered before a profit share is taken by the government, the amount of the multiple or factor (BPF) is input on this card.

On card four the price support level, PMIN, enables the user to determine the effects of a government instituted price support for any resource through the model optimization routine. The length of the price support may be varied using the variable MPT. Each of the random variables such as the annual change

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[1] In the authors analysis, beta is usually set between .5 and 1 and gamma is set equal to 0 for petroleum. Both parameters are set equal to 0 for other resources.

in price uses its own unique seed for the random number generation. Our analysis has shown that the choice of seed is crucial to obtaining a set of random numbers without bias for any given generator. After resting more than 25 possible seeds, we have selected a seed for each variable which produced Monte Carlo output distributions very similar to the input distribution.

On card six, the variable LCIM, is used to index the climatic condition for offshore oil development if the power function contained in the cost subroutine for oil is used. This variable may also be used in other cost subroutines as desired. If the variable is set equal to zero, it becomes inoperative.

The remainder of the variables have been explained in the main text or an adequate explanation is provided in Table A-1. Of course, if there are further clarifications needed or questions pertaining to the use of the variables, the authors would be more than happy to assist readers and users in this regard.

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