AN ANALYSIS OF PRODUCER BEHAVIOR INCORPORATING TECHNICAL CHANGE

By

Timothy D. Mount

30 December 1970 No. 22

This paper was presented at the Econometric Society Meeting in Detroit on December 30, 1970.

AN ANALYSIS OF PRODUCER BEHAVIOR INCORPORATING TECHNICAL CHANGE

T. D. Mount

1: Introduction

Two factors contribute to observed rates of technical change within an industry. One is the increased technical efficiency of new production methods relative to established methods. The second is the rate at which firms adopt these new methods. This analysis is directed to the latter process, and in particular, to determining conditions under which it is profitable for producers to change an existing production method. It is assumed that any change involves introducing specific types of capital equipment that embody the new method and possibly scrapping some obsolete equipment. Hence, technical changes within a firm are directly related to investment behavior.

A suitable analytical framework has been developed by Salter (11) to provide an economic rationale for the continued use of technically inferior production methods by firms. Salter describes the choice of production method in terms of a competitive model. New firms, using the best available method, determine the product price. Established firms, using other methods, produce as long as current costs are covered by revenue. However, no explicit conditions are developed to determine whether a rational producer should change an existing production method when net returns are positive.

In another study, Smith (12) develops a model that can be used to represent a producer employing a technically inferior production method. Smith assumes that once a production plant is built, the quantity of capital in that plant is fixed. Subsequent investments may be made in a new plant, but not in addition to existing facilities. The investment criterion is to minimize the total cost of producing a specified target output. Consequently, this criterion does not allow for a change of the level of output when a new production method is introduced. If output is not restricted to a target quantity, it pays producers to use existing production methods whenever net returns are positive. This is equivalent to the Salter framework, and once again conditions for introducing a new production method when net returns are positive are not identified.

In the analysis that follows, certain components of a given stock of capital are assumed to be transferable to new production methods. In agricultural production, land provides one example of this type of capital. The possibility of receiving higher net returns to transferable capital acts as an incentive to change existing production methods even though net returns are positive. Any change of production method involves some investment in equipment embodying the new method. The basic economic decision facing producers is whether to buy this equipment and introduce the new method, or whether to invest the same amount of money in expanding facilities using the existing production method. Under these conditions, no specifications exist restricting output to the same level when a new method is introduced.

A theoretical framework for analyzing investment decisions by producers is developed in Section 2 under the assumption that each production method is identified by a different Cobb-Douglas production function. The decision criterion is specified to be maximization of the present value of net returns to capital less discounted replacement costs. A constraint is placed on investment expenditure, and the optimum decision depends both on the level of this constraint and on the initial quantities of capital inputs. The decision criterion implies that producers maximize net returns to capital.

Hence, any empirical application of the theoretical model involves the prediction of net returns, given a stock of capital inputs and a set of current input prices.[1/] In contrast, the standard estimation problem in production function theory is to predict output for given levels of both current and capital inputs. Although it is possible to predict net returns using parameter estimates derived from the standard prediction problem, these estimates may be very inefficient. A more satisfactory estimation procedure, based on a model originally proposed by Hoch (6) and Mundlak (10), is developed in Section 3. In the final section, an attempt is made to apply the estimation procedure to data for the California dairy industry.

2: Theoretical Model

Each production method within an industry is specified by a different Cobb-Douglas production function. The expression for current net returns to capital for a single production method may be written as follows.[2/]

2:1

$$R_i(t) = P_Y(t)\ Y_i - \sum_{n=1}^{N} P_{Xni}(t)\ X_{ni}(t)$$

$$= P_Y(t)\ A_i \prod_{n=1}^{N} X_{ni}(t)^{\beta_{ni}} \prod_{m=1}^{M} K_{mi}(t)^{\gamma_{mi}}$$

$$- \sum_{n=1}^{N} P_{Xni}(t)\ X_{ni}(t)$$

1/ Since Cobb-Douglas production functions are specified, it follows that maximum net returns are proportional to the optimum output level.

2/ In expression 2:1, each production method is specified with the same set of (N+M) inputs, but differences between methods are assumed to be embodied in at least one of the M capital inputs, implying that not all inputs are identical. To account for this, it may be assumed that a specified subset of the input coefficients are zero for each production method. In addition, prices for corresponding current inputs are allowed to vary between production methods.

where $i = 1,2,\ldots,I$ specifies production method i.

$t = 1,2,\ldots,T$ specifies time period t.

$Y_i(t)$ is the quantity of output.

$X_{ni}(t)$ is the quantity of current input n; $n = 1,2,\ldots,N$.

$K_{mi}(t)$ is the quantity of capital input m; $m = 1,2,\ldots,M$.

$P_Y(t)$ is the price of output.

$P_{Xni}(t)$ is the price of current input n; $n = 1,2,\ldots,N$.

A_i, β_{ni}; $n = 1,2,\ldots,N$, and γ_{mi}; $m = 1,2,\ldots,M$ are production function parameters.

The first step in the analysis is to derive optimum conditions for investment in a given production method.[1] Any stock of capital provides a stream of income over several time periods. The present value of net returns to capital may be used as a single criterion for the complete income stream. However, any stock of capital requires additional costs after the initial purchase to cover repairs and maintenance. Two alternative procedures for including these additional costs are generally adopted. The first is to assume that costs occur in every time period in proportion to the level of each capital input. The second is to assume that each item is used for a specified length of time and is then replaced. In reality, both situations occur, although current maintenance costs need not be proportional to capital input levels. In the following analysis, replacement at regular intervals is assumed, and future replacement costs

---

1/ For convenience, the i subscript is dropped until the problem of changing a production method is considered.

are discounted to the present. Hence, each investment decision may be viewed as one in a sequence of decisions occurring at discrete intervals through time. The actual investment criterion is to maximize the present value of a stream of net returns to capital less discounted replacement costs. For an infinite time horizon, the criterion is:[1]

2:2

$$W = \int_0^\infty R(t) \exp(-rt)\, dt - \sum_{m=1}^{M} \left[[K_m^* + \bar{K}_m \exp(-rL_m)] \sum_{s=0}^{\infty} P_{Km}(sQ_m) \exp(-srQ_m) \right]$$

where $R(t)$ is the current net returns to capital defined in 2:1.

$K_m^* \geq 0$ is the increment to capital input m; $m = 1,2,\ldots,M$.

$\bar{K}_m$ is the initial stock of capital input m.

Q_m is the useful life of capital input m.

L_m is the present age of $\bar{K}_m$.[2]

$P_{Km}(t)$ is the price of capital input m.

r is the discount rate.

In 2:2, each type of capital is composed of two parts, the existing stock $\bar{K}_m$ and an increment to this stock K_m^*. Consequently, the level of each capital input is the sum of the two components, and $K_m = \bar{K}_m + K_m^*$; $m = 1,2,\ldots,M$. The non-negativity of capital increments K_m^* implies that

---

1/ This criterion is consistent with a finite investment horizon if it is assumed that the value of the final stock of capital equals the discounted value of potential earnings in the future.

2/ In reality, the existing stock of a single capital input may contain items purchased at different times in the past. However, this complication does not influence the optimum conditions, and L_m may be viewed as some weighted average of the different ages.

capital inputs can only be expanded. This is equivalent to making the scrap value of existing capital zero, since the marginal physical productivity of any capital input is always positive given a Cobb-Douglas production function. However, it is also possible to consider an investment decision when certain items in an existing stock of capital are due for scrapping, implying that the initial stock may be defined without including these items. This type of investment decision provides an opportunity to reappraise past decisions, as it is not necessarily optimum to replace the scrapped equipment.

Generally, the time paths of prices are unknown, and it is convenient to assume that prices are constant over time so that 2:2 may be simplified. A constraint on current investment expenditure is also introduced using a Lagrangian multiplier. Hence, the investment criterion is to maximize the following expression with respect to the (N+M) inputs.

2:3

$$\mathcal{L} = r^{-1} R - \sum_{m=1}^{M} \left[P_{Km} \, [1-\exp(-rQ_m)]^{-1} \, [K_m^* + \overline{K}_m \exp(-rL_m)] \right]$$

$$+ \lambda \, [C - \sum_{m=1}^{M} P_{Km} K_m^*]$$

subject to

$X_n \geq 0; \; n = 1,2,\ldots,N$

$K_m^* \geq 0; \; m = 1,2,\ldots,M$

where $R = P_Y \left[A \prod_{n=1}^{N} X_n^{\beta_n} \prod_{m=1}^{M} (K_m^* + \overline{K}_m)^{\gamma_m} \right] - \sum_{n=1}^{N} P_{Xn} X_n$

C is the constrained level of investment expenditure.

λ is a Lagrangian multiplier.

Necessary conditions for a maximum of 2:3 may be derived using the Kuhn-Tucker theorem to be:[1]

2:4

$$X_n \left[\frac{\beta_n P_Y Y}{r X_n} - \frac{P_{Xn}}{r} \right] = 0 \; ; \; n = 1,2,\ldots,N$$

$$\frac{\beta_n P_Y Y}{r X_n} - \frac{P_{Xn}}{r} \leq 0 \; ; \; n = 1,2,\ldots,N$$

$$X_n \geq 0 \; ; \; n = 1,2,\ldots,N$$

$$K_m^* \left[\frac{\gamma_m P_Y Y}{r(K_m^* + \overline{K}_m)} - \frac{P_{Km}}{[1-\exp(-rQ_m)]} - \lambda P_{Km} \right] = 0; \; m = 1,2,\ldots,M$$

$$\frac{\gamma_m P_Y Y}{r(K_m^* + \overline{K}_m)} - \frac{P_{Km}}{[1-\exp(-rQ_m)]} - \lambda P_{Km} \leq 0; \; m = 1,2,\ldots,M$$

$$K_m^* \geq 0; \; m = 1,2,\ldots,M$$

$$\lambda \left[C - \sum_{m=1}^{M} P_{Km} K_m^* \right] = 0$$

$$C - \sum_{m=1}^{M} P_{Km} K_m^* \geq 0$$

$$\lambda \geq 0$$

As there is no expenditure constraint on current inputs, net returns to capital are maximized, and $X_n > 0$ for all n.[2] The optimum level of

---

1/ Sufficient conditions for a maximum are that the investment criterion 2:2 is quasi-concave [See Arrow and Enthoven (1)].

2/ To conform with the sufficiency conditions for unconstained maximization, returns to scale for any fixed stock of capital must be decreasing, implying $\Sigma_{n=1}^{N} \beta_n < 1$.

each current input is proportional to the optimum level of output Y^*, and may be written:

2:5

$$X_n^* = \frac{\beta_n P_Y Y^*}{P_{Xn}} \quad ; \; n = 1,2,\ldots,N.$$

Substituting 2:5 into 2:1, the expression for maximum net returns to capital is:[1]

2:6

$$R^* = [1-\beta_s] \left[A\, P_Y \prod_{n=1}^{N} \left[\frac{\beta_n}{P_{Xn}}\right]^{\beta_n} \prod_{m=1}^{M} (K_m^* + \bar{K}_m)^{\gamma_m} \right]^{(1-\beta_s)^{-1}}$$

$$= [1-\beta_s] \left[(A\, P_Y)^{(1-\beta_s)^{-1}} \prod_{n=1}^{N} \left[\frac{\beta_n}{P_{Xn}}\right]^{\tilde{\beta}_n} \prod_{m=1}^{M} (K_m^* + \bar{K}_m)^{\tilde{\gamma}_m} \right]$$

where $\beta_s = \sum_{n=1}^{N} \beta_n$

$\tilde{\beta}_n = \beta_n (1-\beta_s)^{-1}; \; n = 1,2,\ldots,N.$

$\tilde{\gamma}_m = \gamma_m (1-\beta_s)^{-1}; \; m = 1,2,\ldots,M.$

In contrast to the N current inputs, optimum levels of the M capital inputs depend on the level of the Lagranian multiplier λ unless the increment to existing capital stock is zero ($K_m^* = 0$). For any positive increment, the optimum level of capital input is

2:7

$$(K_m^* + \bar{K}_m) = \frac{\gamma_m P_Y Y^*}{r\, P_{Km} \left[(1-\exp(-rQ_m))^{-1} + \lambda^* \right]} \qquad \text{for } K_m^* > 0,$$

where λ^* is the optimum value of the Lagrangian multiplier.

1/ In addition, the optimum level of output Y^* is directly proportional to the maximum of current net returns to capital ($Y^* = [P_Y (1-\beta_s)]^{-1} R^*$).

The optimum capital input level 2:7 is the same as the level for unconstrained profit maximization if $\lambda^* = 0$, implying the investment constraint is not binding.[1] For situations in which $\lambda^* > 0$, the optimum ratio between two capital inputs is:

2:8

$$\frac{K_u^* + \bar{K}_u}{K_v^* + \bar{K}_v} = \frac{\gamma_u \, P_{Kv} \, [(1-\exp(-rQ_v))^{-1} + \lambda^*]}{\gamma_v \, P_{Ku} \, [(1-\exp(-rQ_u))^{-1} + \lambda^*]} \quad \text{for } K_u^*,\ K_v^* > 0$$

It follows that the optimum ratio is independent of $\lambda^* > 0$ only if the life spans of both types of capital are the same ($Q_u = Q_v$). In general, the relationship between the optimum ratio and λ^* is monotonic, and the first derivative with respect to λ^* is:

2:9

$$\frac{d\left[\dfrac{K_u^* + \bar{K}_u}{K_v^* + \bar{K}_v}\right]}{d\lambda^*} = \frac{\gamma_u \, P_{Kv} \, [(1-\exp(-rQ_u))^{-1} - (1-\exp(-rQ_v))^{-1}]}{\gamma_v \, P_{Ku} \, [(1-\exp(-rQ_u))^{-1} + \lambda^*]^2}$$

The sign of the numerator of 2:9 determines the sign of the whole expression since the denominator is always positive. The numerator sign is positive (negative) if the life span of capital u is shorter (longer) than the life span of capital v, so that $Q_u < Q_v$ ($Q_u > Q_v$). All optimum values of the Lagrangian multiplier λ^* are non-negative, and consequently, any solution with $\lambda^* > 0$ relative to a solution with $\lambda^* = 0$ implies that short life capital is substituted for long life capital. This discrimination in favor of short-life capital increases as λ^* increases. No discrimination is present if investment expenditure is unconstrained or if cost minimization is used as the investment criterion.

1/ The optimum capital input levels for minimizing the cost of producing a target output are also identical to 2:7 with $\lambda^* = 0$.

For any optimum solution of 2:4, the value of λ^* may be interpreted as a marginal cost of the investment constraint measured by discounted profits foregone. Relaxing the constraint implies a corresponding incremental expansion in the scale of production, and λ^* increases (decreases) if the production function exhibits economies (diseconomies) of scale. It follows that the optimum ratio of short-life to long-life increases (decreases) as the investment constraint is relaxed whenever there are economies (diseconomies) to scale.

In general, no explicit solution for the Lagrangian multiplier λ^* exists, but the optimum value may be determined by an iterative procedure for any given level of the investment constraint. It is necessary, however, to allow for the possibility that some of the increments to capital stock are zero $(K_m^* = 0)$. When the optimum value λ^* is known, values of the optimum capital increments may be determined $(K_m^*;\ m = 1,2,\ldots,M)$, and also the maximum value of the investment criterion 2:3. This gives the optimum investment program for expanding output using the existing production method.

The analysis up to this point makes no allowance for alternative production methods. However, an optimum solution may be derived for any production method given the initial stock of capital inputs and a specified level of the investment constraint. In fact, the existence of alternative production methods implies only that additional feasible solutions exist in the maximization problem. A simple procedure to follow is to determine the maximum values of the investment criterion for all production methods, and to specify the optimum investment program is to adopt the method that yields the highest value. It is still necessary to consider how to define the initial stock of capital inputs for an alternative production method, as generally equipment embodying the existing method is scrapped when a new method is introduced. Hence, initial stocks are not identical for different production methods. It is assumed that the initial capital stocks for method j given that method i is the existing method may be written as follows:

2:10

$$\overline{K}_{mj} = (1-a_{mij})\ \overline{K}_{mi}\ ;\ m = 1,2,\ldots,M \text{ and } i,j = 1,2,\ldots,I,$$

where $0 \leq a_{mij} \leq 1$ is the proportion of the capital input that is scrapped when changing from method i to method j. In the simplest case, the a_{mij} are zero for all but the single capital input in which the production method is embodied.

If the price of scrapped equipment is zero, the constraint on investment expenditure is identical for all production methods. However, if the price is positive, this extra revenue may be used to increase investment expenditures.[1] The effective level of the constraint for production method j given that method i is currently used is:

2:11

$$C_{ij} = C_i + \sum_{m=1}^{M} a_{mij}\ \tilde{P}_{Km}\ (L_{mi})\ ,$$

where C_i is the initial level of the investment constraint

$\tilde{P}_{Km}\ (L_{mi})$ is the scrap price of capital in m after L_{mi} years using method i.

The optimum investment program using production method j may be determined from the necessary conditions 2:4 by redefining the initial

---

1/ It is important to know the age of equipment that is scrapped even though the ages of the initial levels of capital inputs have no bearing on the optimum conditions. The reason is that a comparison of the discounted replacement costs for capital stock using alternative production methods is directly influenced by these differences in initial stock levels.

levels of capital inputs using 2:10 and the investment constraint using 2:11. It is generally true that if the investment constraint is relaxed sufficiently, the production method exhibiting the highest returns to scale in 2:6 ($\Sigma_{m=1}^{M} \tilde{\gamma}_{mi}$ is largest for all i) will be optimum regardless of which method is currently used. However, it may be necessary to expand the size of the existing firm many times for this change to be optimum, and limits on the availability of credit would prevent such changes occurring in practice.

3: Estimation Procedures

In Section 2, necessary conditions for maximizing the investment criterion 2:4 imply that producers maximize net returns to capital stock. As Cobb-Douglas production functions are specified, optimum levels of expenditure on each current input are proportional to the optimum level of revenue. A model that incorporates these proportional relationships has been developed in the literature by Hoch (6) and Mundlak (10). The logarithmic transformation of this model may be written for any specified production method as follows:

3:1

$$y_t = \alpha + \sum_{n=1}^{N} \beta_n x_{nt} + \sum_{m=1}^{M} \gamma_m k_{mt} + \omega_t; \; t = 1,2,\ldots,T$$

$$y_t - x_{1t} = \delta_1 + p_{1t} + \epsilon_{1t}$$

$$y_t - x_{2t} = \delta_2 + p_{2t} + \epsilon_{2t}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$y_t - x_{Nt} = \delta_N + p_{Nt} + \epsilon_{Nt}$$

$$E[\omega_t] = E[\epsilon_{nt}] = 0 \text{ for all } t \text{ and } n.$$

$$E[\omega_t \omega_{t'}] = \sigma_\omega^2 \text{ if } t = t'$$

$$= 0 \text{ otherwise.}$$

$$E[\epsilon_{nt}\epsilon_{n't'}] = \sigma_n^2 \text{ if } n = n' \text{ and } t = t'.$$

$$= \sigma_{nn'} \text{ if } n \neq n' \text{ and } t = t'.$$

$$= 0 \text{ otherwise}$$

$$E[\omega_t \ \epsilon_{nt'}] = 0 \text{ for all } t, t' \text{ and } n.$$

where

$y_t = \log Y(t)$

$x_{nt} = \log X_n(t)$; $n = 1,2,\ldots,N$

$k_{mt} = \log K_m(t)$; $m = 1,2,\ldots,M$

$p_{nt} = \log [P_{Xn}(t)\ P_Y(t)^{-1}]$; $n = 1,2,\ldots,N$

$\alpha = \log A$, β_n; $n = 1,2,\ldots,N$, γ_m; $m = 1,2,\ldots,M$ and δ_n; $n = 1,2,\ldots$, N are unknown parameters

ω_t and ϵ_{nt}; $n = 1,2,\ldots,N$ are unexplained residuals

$Y(t)$, $X_n(t)$, $K_m(t)$, $P_Y(t)$, $P_{Xn}(t)$, A, β_n, and γ_m are all defined in 2:1.

Each of the $(N+1)$ equations in 3:1 is specified with an unexplained residual term. The first equation is the linearized production function, and the other N equations show the relationship between output and each current input. If producers actually maximize net returns to capital stock, the following conditions hold in 3:1: $\delta_n = -\operatorname{Log} \beta_n$ and $\epsilon_{nt} = 0$ for all n and t (see 2:5). Hence, the specified model allows for some unexplained variation between the optimum conditions and observed behavior. In addition, no constraint is placed on δ_n $(n = 1,2,\ldots,N)$ so it is possible for producers to follow a policy that is consistently different from the optimum conditions. From an economic viewpoint, it may be interesting to test whether the difference between observed and optimum behavior is significant.

The (N+1) equations in 3:1 form a simultaneous system with output and N current inputs endogenous, and with M capital inputs, N price ratios, and (N+1) constant terms exogenous. Ordinary least squares estimates of the input parameters derived from the first equation in 3:1 are biased since the N current inputs x_{nt} are related to the residual ω_t. Both Hoch (6) and Mundlak (10) have developed procedures for obtaining unbiased estimates of these parameters. However, if the N current input equations in 3:1 are defined in terms of the expected level of output $E[y_t]$ instead of actual output y_t, then the N current inputs x_{nt} are no longer related to the residual ω_t. Hence, ordinary least squares estimates of the input parameters using the production function only are best linear unbiased estimates. This fact was established by Hoch (6), and has been subsequently used to justify single equation estimation procedures.[1/] In addition, Zellner, Kmenta, and Drèze (13) have shown that single equation estimates are unbiased maximum likelihood estimates, if maximizing expected net returns to capital is specified as the decision criterion, and if all residuals are multivariate normal.

1/ Another source of estimation bias discussed in relation to 3:1 is caused by omitting firm and time effects from the specification. However, incorporating these effects in 3:1 is quite straightforward, but no attempt is made to do so explicitly in our analysis.

The use of expected net returns to capital as the decision criterion seems reasonable, and this implies that least squares estimates of the input parameters are satisfactory. However, if the primary objective of the analysis is to predict net returns to capital stock, as it is in our case, estimates derived from the complete system of $(N+1)$ equations are preferable. The reason is that the parameters of interest are not α, β_n and γ_m but the transformed parameters $\tilde{\alpha} = \alpha(1-\beta_s)^{-1}$, $\tilde{\beta}_n = \beta_n(1-\beta_s)^{-1}$ and $\tilde{\gamma}_m = \gamma_m(1-\beta_s)^{-1}$, where $\beta_s = \sum_{n=1}^{N} \beta_n$. This conclusion follows immediately from an inspection of the expression for maximum net returns to capital 2:6. In statistical terms, the objective for the single equation procedure may be viewed as predicting output given levels of both current and capital inputs. In contrast, the objective using the simultaneous system is to predict output and current inputs given current input prices and capital input levels.[1] The discussion that follows consists of two parts. The first part concerns the derivation from 3:1 of direct estimates of the desired parameters $\tilde{\alpha}$, $\tilde{\beta}_n$ and $\tilde{\gamma}_m$. The second part shows that alternative estimates of these parameters, derived using single equation procedures, are relatively inefficient.

An obvious way to obtain parameter estimates in 3:1, since this equation is just identified, is to derive the reduced form and to estimate the resulting coefficients directly.[2] In fact, the reduced form coefficients are equal to $\tilde{\alpha}$, $\tilde{\beta}_n$ and $\tilde{\gamma}_m$, the parameters required for predicting

---

1/ The optimum level of output is proportional to the maximum level of net returns to capital.

2/ The first equation in 3:1 contains $(N+1)$ endogenous variables $(y_t, x_{nt};$ $n=1,2,\ldots,N)$ and N exogenous price variables are omitted $(p_{nt};$ $n=1,2,\ldots,N)$. Consequently, estimates of the reduced form coefficients can be used to give unique estimates of the original parameters α, β_n and γ_m

net returns. If current input prices are not accurately recorded for each observation and this is a problem in some empirical applications, then, the estimated reduced form coefficients may be very unreliable. To overcome this potential problem, an estimation procedure is developed for the reduced form that does not require the direct use of price data. Hence, it is sufficient for prediction purposes to know the average prices for a group of observations rather than the exact prices for each observation.

The (N+1) equations in 3:1 may be written in matrix notation as follows:

3:2

$$[Y \quad X]\begin{bmatrix} 1 & 1_N' \\ -\beta & -I \end{bmatrix} = [1_T \quad J]\begin{bmatrix} \alpha & 0 \\ 0 & \delta \end{bmatrix} + [K \quad P]\begin{bmatrix} \gamma & 0 \\ 0 & I \end{bmatrix} + [\omega \quad \epsilon]$$

$$E[\omega \quad \epsilon] = [0 \quad 0]$$

$$E[\omega \quad \epsilon]\begin{bmatrix} \omega' \\ \epsilon' \end{bmatrix} = \begin{bmatrix} \sigma_\omega^2 & 0 \\ 0 & \Sigma \end{bmatrix} \otimes I$$

where

$$\underset{T\times 1}{[Y} \quad \underset{T\times N}{X]} = \begin{bmatrix} y_1 & x_{11} & x_{21} \cdots x_{N1} \\ y_2 & x_{12} & x_{22} \cdots x_{N2} \\ \vdots & \vdots & \vdots \quad\quad \vdots \\ y_T & x_{1T} & x_{2T} \cdots x_{NT} \end{bmatrix}$$

$$\underset{T\times 1}{[1_T} \quad \underset{T\times N}{J]} = \begin{bmatrix} 1 & 1 & 1 \ldots\ldots 1 \\ 1 & 1 & 1 \ldots\ldots 1 \\ \vdots & \vdots & \vdots \quad\quad \vdots \\ 1 & 1 & 1 \ldots\ldots 1 \end{bmatrix}$$

$$\Big[\underset{T\times M}{K} \quad \underset{T\times N}{P}\Big] = \begin{bmatrix} k_{11} & k_{12} \cdots k_{M1} & p_{11} & p_{21} \cdots p_{N1} \\ k_{12} & k_{22} \cdots k_{M2} & p_{12} & p_{22} \cdots p_{N2} \\ \vdots & \vdots \quad\;\; \vdots & \vdots & \vdots \quad\;\; \vdots \\ k_{1T} & k_{2T} \cdots k_{MT} & p_{1T} & p_{2T} \cdots p_{NT} \end{bmatrix}$$

$$\Big[\underset{T\times 1}{\omega} \quad \underset{T\times N}{\epsilon}\Big] = \begin{bmatrix} \omega_1 & \epsilon_{11} & \epsilon_{21} \cdots \epsilon_{N1} \\ \omega_2 & \epsilon_{12} & \epsilon_{22} \cdots \epsilon_{N2} \\ \vdots & \vdots & \vdots \quad\;\; \vdots \\ \omega_T & \epsilon_{1T} & \epsilon_{2T} \cdots \epsilon_{NT} \end{bmatrix}$$

$$\begin{matrix} 1\times 1 & 1\times N \\ \end{matrix}$$
$$\begin{bmatrix} \alpha & 0 \\ 0 & \delta \end{bmatrix}_{\begin{smallmatrix} N\times 1 & N\times N \end{smallmatrix}} = \begin{bmatrix} \alpha & 0 & 0 & \cdots & 0 \\ 0 & \delta_1 & 0 & \cdots & 0 \\ 0 & 0 & \delta_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & \delta_N \end{bmatrix}$$

$$\begin{matrix} 1\times 1 & 1\times N \\ \end{matrix}$$
$$\begin{bmatrix} 1 & 1'_N \\ -\beta & -I \end{bmatrix}_{\begin{smallmatrix} N\times 1 & N\times N \end{smallmatrix}} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ -\beta_1 & -1 & 0 & \cdots & 0 \\ -\beta_2 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -\beta_N & 0 & 0 & \cdots & -1 \end{bmatrix}$$

$$\begin{matrix} M\times 1 & M\times N \\ \end{matrix}$$
$$\begin{bmatrix} \gamma & 0 \\ 0 & I \end{bmatrix}_{\begin{smallmatrix} N\times 1 & N\times N \end{smallmatrix}} = \begin{bmatrix} \gamma_1 & 0 & 0 & \cdots & 0 \\ \gamma_2 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \gamma_M & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\begin{bmatrix} \underset{1x1}{\sigma_\omega^2} & \underset{1xN}{0} \\ \underset{Nx1}{0} & \underset{NxN}{\Sigma} \end{bmatrix} \otimes I_{NxN} = \begin{bmatrix} \sigma_\omega^2 & 0 & 0 & \dots & 0 \\ 0 & \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1N} \\ 0 & \sigma_{12} & \sigma_2^2 & \dots & \sigma_{2N} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & \sigma_{1N} & \sigma_{2N} & \dots & \sigma_N^2 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

The reduced form of 3:2 is [1]

3:3

$$[Y \quad X] = \left[[1_T \quad J] \begin{bmatrix} \alpha & 0 \\ 0 & \delta \end{bmatrix} + [K \quad P] \begin{bmatrix} \gamma & 0 \\ 0 & I \end{bmatrix} + [\omega \quad \epsilon] \right] \begin{bmatrix} 1 & 1_N' \\ -\beta & -(\beta 1_N' + I\lambda) \end{bmatrix} \lambda^{-1}$$

$$= [1_T \quad J] \begin{bmatrix} \alpha & \alpha 1_N' \\ -\delta\beta & -\delta(\beta 1_N' + I\lambda) \end{bmatrix} \lambda^{-1} +$$

$$[K \quad P] \begin{bmatrix} \gamma & \gamma 1_N' \\ -\beta & -(\beta 1_N' + I\lambda) \end{bmatrix} \lambda^{-1} + [(\omega - \epsilon\beta) \quad [\omega 1_N' - \epsilon(\beta 1_N' + I\lambda)]] \lambda^{-1}$$

where $\lambda = (1 - 1_N' \beta)$

This may be rewritten as follows

3:4

$$[Y \quad (X+P)] = 1_T [\tilde{\alpha} - 1_N' \delta\tilde{\beta}) \quad (\tilde{\alpha} 1_N' - 1_N' \delta \tilde{\beta} 1_N' - 1_N' \delta)]$$

$$+ K[\tilde{\gamma} \quad \tilde{\gamma} 1_N'] + P[\tilde{\beta} \quad \tilde{\beta} 1_N']$$

$$+ [\omega \quad \omega 1_N'] \lambda^{-1} - [\epsilon\tilde{\beta} \quad \epsilon(\tilde{\beta} 1_N' + I)]$$

where [2] $\tilde{\alpha} = \alpha\lambda^{-1}$, $\tilde{\beta} = \beta\lambda^{-1}$, and $\tilde{\gamma} = \gamma\lambda^{-1}$.

1/ The inverse of the (N+1)x(N+1) matrix of endogenous variable parameters is given by Mundlak (10; p.141).

2/ These vectors contain the parameters that are required for predicting net returns to capital.

The final version of the reduced form expresses the logarithm of output Y and the N vectors of current input expenditures $(X+P)$ as linear functions of a constant term, the $(M+N)$ exogenous variables in K and P, and a residual. All $(N+1)$ equations for these endogenous variables contain the same unknown input parameters $\tilde{\gamma}$ and $\tilde{\beta}$. However, the constant terms and residuals are different.

The reduced form of the linearized production function in 3:4 may be written:[1]

3:5

$$Y = 1_T (\tilde{\alpha} - 1_N' \delta \tilde{\beta}) + K\tilde{\gamma} - P\tilde{\beta} + \omega\lambda^{-1} - \epsilon\tilde{\beta}$$

If the price matrix P is unknown a substitution of $(Y1_N' - X)$ for $(1_T 1_N' \delta + P + \epsilon)$ may be made. The equivalence of these two expressions follows directly from the N current input equations in 3:2. With this substitution, 3:5, may be rewritten:

3:6

$$Y = 1_T\tilde{\alpha} + (X - Y1_N')\,\tilde{\beta} + K\tilde{\gamma} + \omega\lambda^{-1}$$

The residual ω is specified in 3:2 to be homoscedastic and λ is a scalar. Consequently, ordinary least squares estimates of $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are consistent unbiased estimates.[2] These estimates are:

3:7

$$\begin{bmatrix} \tilde{a} \\ \tilde{b} \\ \tilde{c} \end{bmatrix} = \begin{bmatrix} T & 1_T'Z & 1_T'K \\ Z'1_T & Z'Z & Z'K \\ K'1_T & K'Z & K'K \end{bmatrix}^{-1} \begin{bmatrix} 1_T'Y \\ Z'Y \\ K'Y \end{bmatrix}$$

---

[1] It is possible to use all $(N+1)$ endogenous variable vectors in 3:4 to obtain a single set of parameter estimates. However, this procedure can be applied only if the price matrix P is known.

[2] The independent variables $(X-Y1_N')$ in 3:6 contain a stochastic element ϵ. Hence, estimates 3:7 are BLUE if conditioned on ϵ, since $E[\epsilon'\omega] = 0$ in 3:2.

where $Z = (X - Y1_T')$

The estimated variance of estimates in 3:7 is:

3:8

$$E\left[\begin{bmatrix}\tilde{a}-\tilde{\alpha}\\ \tilde{b}-\tilde{\beta}\\ \tilde{c}-\tilde{\gamma}\end{bmatrix}\begin{bmatrix}\tilde{a}-\tilde{\alpha}\\ \tilde{b}-\tilde{\beta}\\ \tilde{c}-\tilde{\gamma}\end{bmatrix}'\right] = \tilde{s}_\omega^2\begin{bmatrix}T & 1_T'Z & 1'K\\ Z'1_T & Z'Z & Z'K\\ K'1_T & K'Z & K'K\end{bmatrix}^{-1}$$

where $\tilde{s}_\omega^2 = (T-N-M-1)^{-1}\ [(Y-\tilde{a}-Z\tilde{b}-K\tilde{c})'\ (Y-\tilde{a}-Z\tilde{b}-K\tilde{c})]$

is an unbiased estimate of $\sigma_\omega^2 \lambda^{-1}$

Ordinary least squares estimates 3:7 are, in fact, equivalent to using $(J\delta + P + \epsilon)$ as an instrumental variable for X in the first equation of the original specification 3:1.[1/] Once again $(Y1_T' - X) = -Z$ may be substituted for $(J\delta + P + \epsilon)$. In matrix notation, the first equation in 3:1 may be written:

3:9

$$Y = \alpha + X\beta + K\gamma + \omega$$

Estimates of α, β, and γ using instrumental variables are:

3:10

$$\begin{bmatrix}a\\ b\\ c\end{bmatrix} = \begin{bmatrix}T & 1_T'X & 1_T'K\\ -Z'1_T & -Z'X & -Z'K\\ K'1_T & K'X & K'K\end{bmatrix}^{-1}\begin{bmatrix}1_T'Y\\ -Z'Y\\ K'Y\end{bmatrix}$$

Equivalence between 3:7 and 3:10 may be demonstrated by considering the normal equations of 3:10:

1/ This equivalence between estimation procedures is discussed by Christ (3; p.402).

3:11

$$\begin{bmatrix} T & 1_T'X & 1_T'K \\ -Z'1_T & -Z'X & -Z'K \\ K'1_T & K'X & K'K \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1'Y \\ -Z'Y \\ K'Y \end{bmatrix}$$

Rewriting 3:11 gives

3:12

$$\begin{bmatrix} T & 1_T'Z & 1_T'K \\ Z'1_T & Z'Z & Z'K \\ K'1_T & K'Z & K'K \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1_T'Y \\ Z'Y \\ K'Y \end{bmatrix} - \begin{bmatrix} 1_T'Y1_N'b \\ Z'Y1_N'b \\ K'Y1_N'b \end{bmatrix}$$

$$= \begin{bmatrix} 1'Y \\ Z'Y \\ K'Y \end{bmatrix} (1-1_N'b)$$

Defining $\hat{a} = a\ (1-1_N'b)^{-1}$, $\hat{b} = b\ (1-1_N'b)^{-1}$, and $\hat{c} = c\ (1-1_N'b)^{-1}$, estimates $\hat{a}$, $\hat{b}$, and $\hat{c}$ derived from 3:12 are identical to estimates $\tilde{a}$, $\tilde{b}$, and $\tilde{c}$ in 3:7. Recall that $\tilde{a}$, $\tilde{b}$, and $\tilde{c}$ are estimates of $\tilde{\alpha} = \alpha\lambda^{-1}$, $\tilde{\beta} = \beta\lambda^{-1}$ and $\tilde{\gamma} = \gamma\lambda^{-1}$, respectively, where $\lambda = (1-1_N'\beta)$. Hence, both procedures give identical estimates of the unknown parameters α, β, and γ and also of the transformed parameters $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$.

The use of $(Y1_N' - X) = (J\delta + P + \epsilon)$ as an instrument for X in 3:10 implicitly assumes that J, P, and ϵ are all linearly independent of the residual ω in 3:9. The covariances between ϵ and ω are specified as zero in 3:1 for this reason. Mundlak (10) proposes an alternative estimation procedure that is similar to 3:7, but uses

$(J\delta + \epsilon) = (Y1_N' - X - P)$ as the instrument for X.[1] Hence, price data are required for the application of Mundlak's procedure.

To complete the estimation of all unknown structural parameters in 3:1, it is necessary to determine values for the constant terms δ_n. Best linear unbiased estimates of these parameters are:

3:13

$$d_n = \bar{y} - \bar{x}_n - \bar{p}_n \ ; \ n = 1,2,\ldots,N,$$

where the bar notation over a variable represents the mean value for all T observations. The estimated variance of 3:13 is

3:14

$$E[(d_n - \delta_n)^2] = s_n^2 \, T^{-1} \quad ; \ n = 1,2,\ldots,N$$

where $s_n^2 = (T-1)^{-1} \, [\, \Sigma_{t=1}^{T} \ (y_t - x_{nt} - p_{nt} - \bar{y} + \bar{x}_n + \bar{p}_n)^2]$ is an unbiased estimate of the residual variance σ_n^2. If current input prices are not recorded for each observation, it is possible to use an approximate value of the mean price $\bar{p}_n$ to obtain an estimate of δ_n, but it is not possible to estimate the corresponding residual variance σ_n^2. This is an important handicap as the expression for predicting net returns to capital R^* given specification 3:1, contains the terms $\exp(-\delta_n)$; $n = 1,2,\ldots,N$.[2] In addition, the expression must include the residual variances or estimates of these variances together with a correction

---

1/ If all current input variables are recorded as expenses rather than in physical units, using $(J\delta + \epsilon)$ as the instrumental variable is justifiable. However, exact equivalence between the reduced form estimates and the instrumental variable estimates no longer exists.

2/ In particular, $\exp(-\delta_n)$ replaces β_n in 2:5. Substituting this into 2:1 gives the corresponding expression for R^*.

factor if prediction unbiasedness is to be maintained (see Bradu and Mundlak (2)).[1] This problem is largely ignored in the empirical section that follows; no simple solution seems possible when available price data are inadequate.

The discussion now shifts to the use of the standard single equation procedure for estimating the transformed coefficients $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$ in 3:4. If maximization of expected profits is chosen as the investment criterion, then ordinary least squares estimates of the untransformed parameters of α, β, and γ are best linear unbiased. With the additional assumption that the residual vector ω is multivariate normal $(0, \sigma_\omega^2 I)$, the distribution of these estimates may be written:

3:14

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} T & 1_T'X & 1_T'K \\ X'1_T & X'X & X'K \\ K'1_T & K'X & K'K \end{bmatrix}^{-1} \begin{bmatrix} 1_T'Y \\ X'Y \\ K'Y \end{bmatrix} \text{ are Normal } \left[\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}, \sigma_\omega^2 \begin{bmatrix} T & 1_T'X & 1_T'K \\ X'1_T & X'X & X'K \\ K'1_T & K'X & K'K \end{bmatrix}^{-1} \right]$$

Since the estimates in 3:14 are also maximum likelihood estimates of α, β, and γ, it is possible to appeal to the invariance property of these estimates and use $a(1-1_N'b)^{-1}$, $b(1-1_N'b)^{-1}$, and $c(1-1_N'b)^{-1}$ to estimate $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$, respectively. However, the optimum properties of these latter estimates only hold asymptotically.

The normality of the estimates in 3:14 implies that the corresponding estimate of any element of $\tilde{\alpha}$, $\tilde{\beta}$, or $\tilde{\gamma}$ is a quotient of two normal variables.[2] If the two variables both have mean zero, the quotient has

---

1/ This statement assumes that the (N+1) residuals in ω and ϵ are multivariate normal.

2/ Any sum of normal variables is also normal, and as a result, the denominator, which in all cases is $(1-1_N'b)$, is normal.

a Cauchy distribution. With non-zero means, the quotient has a considerably more complicated distribution that is asymmetric. Fieller (4) has derived the exact distribution, and in addition, an approximate distribution that is valid only if the denominator is significantly different from zero. If the denominator is not significantly different from zero, confidence intervals for the quotient include infinity and are, consequently, unbounded.[1/] Fieller's approximate distribution for a quotient $Z = UV^{-1}$ implies the following condition:

3:15

$$(\bar{V}Z - \bar{U})[\sqrt{(\sigma_u^2 - 2Z\sigma_{uv} + Z^2\sigma^2)}]^{-1} \quad \text{is Normal } (0,1)$$

where $\begin{bmatrix} U \\ V \end{bmatrix}$ are bivariate Normal $\left[\begin{bmatrix} \bar{U} \\ \bar{V} \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix} \right]$

An equivalent student's t distribution exists if estimates of the variance-covariance components are used in 3:15.[2/] The confidence interval of the quotient Z may be determined by solving the following quadratic equation for Z.

3:16

$$(V^2 - t_o^2\hat{\sigma}_v^2)Z^2 - 2\,(UV - t_o^2\hat{\sigma}_{uv})Z + (U^2 - t_o^2\hat{\sigma}_u^2) = 0$$

where U and V are observed values.

$\hat{\sigma}_u^2$, $\hat{\sigma}_v^2$, and $\hat{\sigma}_{uv}$ are estimates of σ_u^2, σ_v^2, and σ_{uv}, respectively.

t_o is the critical value of t, implying that $P\{t > t_o\} = \alpha/2$

for a $(1-\alpha)$ level of significance.

1/ Fuller (5) gives an empirical application of the following procedure by determining confidence intervals for the isocline of a quadratic production function.

2/ In 3:14, an unbiased estimate of σ_ω^2 is $(T-N-M-1)^{-1}$ [(Y-a-Xb-Kc)' (Y-a-Xb-Kc)], and the resulting t statistic has (T-N-M-1) degrees of freedom.

It is possible to use 3:16 to determine confidence intervals for the single equation estimated $a(1-1_N'b)^{-1}$, $b(1-1_N'b)^{-1}$, and $c(1-1_N'b)^{-1}$. These intervals may be compared with the corresponding intervals for the direct reduced form estimates $\tilde{a}$, $\tilde{b}$, and $\tilde{c}$ in 3:7. As the reduced form estimates are efficient and unbiased, the latter intervals are generally smaller, implying that single equation estimates are relatively inefficient.

4: Empirical Analysis

The following analysis is based on cross-section and time-series data from a survey of 122 dairy farms in California during the years 1960-65. Two methods of milk production, characterized by the type of barn, are identified. Method A uses a stanchion barn, and method B uses a parlor. During the past two decades, a number of milk producers in California have replaced stanchion barns with parlors, but some smaller producers are still using stanchions barns that were built over thirty years ago. In contrast, some large scale firms have built new stanchion barns, and indications are that stanchion barns yield higher returns if the production scale is sufficiently large.

The first objective in this section is to estimate production function parameters for method A (stanchion) and method B (parlor) using both single equation estimation 3:14 (SEE) and reduced form estimation 3:7 (RFE). Estimates of the transformed parameters $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$ are then compared for the two alternative techniques. The quantity of milk produced is the dependent variable in each production function. Three current inputs and five capital inputs are specified. These input variables are:

Current inputs:		Capital Inputs:	
	1. Labor		1. Herd size
	2. Feed		2. Land
	3. Utilities		3. Milk barn
			4. Machinery
			5. Other buildings

Estimated parameters are summarized in Table 1 for both estimation techniques.[1/] All current input and capital input parameters are expected to be positive. Two of the estimated capital coefficients are negative for production method B using SEE, and one is negative using RFE. In contrast, all estimated current input coefficients are negative using RFE, whereas only one coefficient for method B is negative using SEE. However, fewer of the estimated coefficients are significantly different from zero using SEE.[2/]

Predicting net returns to capital requires estimation of the transformed parameters $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$. It is on the basis of these estimates that the two estimation procedures are compared. Both point estimates and interval estimates are summarized in Table 2. Point estimates using SEE are derived from the regression coefficients in Table 1, and the corresponding interval estimates are computed by solving 3:16 for each quotient.[3/] Estimates using RFE are taken directly from Table 1. A comparison of ranges of the interval

---

1/ The estimates are of $\underset{\sim}{\alpha}$, $\underset{\sim}{\beta}$, and $\underset{\sim}{\gamma}$ using single equation estimation, and of $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$ using reduced form estimation.

2/ All statistical tests are based on the assumption that the residual ω is multivariate Normal $(0, \sigma^2_\omega I)$, and that ϵ is given conditionally.

3/ The numerator U is one element of a, b, or c in 3:14, and the denominator V is $(1-1'_N b)$. V is significantly different from zero for both production methods. (The critical t value is 2.576 at the 99% level of significance. The estimated t values are 10.034 and 6.583 for methods A and B, respectively.) The terms σ^2_u, σ^2_v, and σ_{uv} are computed from the variance-covariance matrix of regression coefficients in 3:14.

Table 1

ESTIMATED REGRESSION COEFFICIENTS

Variable and Measurement Units[1]	SINGLE EQUATION ESTIMATION Production Method A	SINGLE EQUATION ESTIMATION Production Method B	REDUCED FORM ESTIMATION Production Method A	REDUCED FORM ESTIMATION Production Method B
Constant term	4.1016	4.6615	2.5512	2.9471
	(31.09)[2]	(24.35)[2]	(25.63)[2]	(22.16)[2]
Labor	0.0710	0.0718*	-0.1964	-0.2259
(Man months x 100)	(3.72)	(2.49)	(15.57)	(12.59)
Feed	0.5285	0.6156	-0.5840	-0.5888
(Pounds of TDN per day)	(17.10)	(13.45)	(31.30)	(21.42)
Utilities	0.0522	-0.0009*	-0.0456	-0.0440
(Expenditure per month)	(4.63)	(0.05)	(5.74)	(3.55)
Herd size	0.3824	.4005	0.9061	0.8987
(Number of cows)	(11.43)	(8.71)	(100.98)	(69.53)
Land	0.0253	-0.0089*	0.0557	-0.0183
(Current value)	(4.94)	(1.01)	(16.59)	(3.11)
Milk barn	0.0319	-0.0382*	0.0270	0.0341
(Current value)	(3.89)	(2.11)	(4.68)	(2.81)
Machinery	0.0297	0.0051*	0.0522	0.0140*
(Current value)	(2.88)	(0.40)	(7.28)	(1.64)
Other buildings	0.0007*	0.0133*	0.0036*	0.0389
(Current value)	(0.21)	(1.65)	(1.58)	(7.39)
Multiple correlation coefficient	.928	.909	.964	.959
Sum of squared residuals	32.8053	29.4131	16.3884	13.2518
Number of observations	1570	944	1570	944

1/ The dependent variable in each regression is the logarithm of the output of milk, standardized to 4% butterfat and measured in pounds per month. The log transformations of all independent variables are used in the regressions.

2/ The numbers in brackets are the absolute values of the 't ratios' for each estimate.

* Implies that the estimated parameter is not significantly different from zero at the 99% level of significance (The critical t value is 2.576).

Table 2

ESTIMATES OF REDUCED FORM COEFFICIENTS

Variable	Single Equation Estimation: Point Estimate 1/	Interval Estimate 3/	Range of Interval	Reduced Form Estimation: Point Estimate 2/	Interval Estimate 3/	Range of Interval	Ratio of Ranges 4/
	PRODUCTION METHOD A			PRODUCTION METHOD A			
Constant term	12.6141	9.6320–15.3098	5.6778	2.5512	2.2949– 2.8075	0.5126	11.08
Labor	0.2038	0.0556– 0.4130	0.3574	-0.1964	-0.1639–-0.2289	0.0650	5.50
Feed	1.5174	1.0445– 2.3067	1.2622	-0.5840	-0.5358–-0.6322	0.0964	13.09
Utilities	0.1499	0.0631– 0.2661	0.2030	-0.0456	-0.0252–-0.0660	0.0408	4.98
Herd size	1.0979	1.0000– 1.2250	0.2250	0.9061	0.8829– 0.9293	0.0464	4.85
Land	0.0726	0.0419– 0.1055	0.0636	0.0557	0.0469– 0.0645	0.0176	3.61
Milk barn	0.0916	0.0349– 0.1523	0.1174	0.0270	0.0120– 0.0420	0.0300	3.91
Machinery	0.0853	0.0106– 0.1650	0.1544	0.0522	0.0336– 0.0708	0.0372	4.15
Other buildings	0.0020	-0.0278– 0.0314	0.0592	0.0036	-0.0023– 0.0095	0.0118	5.02
Scale ($\Sigma\tilde{\gamma}_m$)	1.3494	1.2530– 1.4920	0.2390	1.0446	1.0259– 1.0633	0.0374	6.39
	PRODUCTION METHOD B			PRODUCTION METHOD B			
Constant term	14.8668	11.1982–23.0454	11.8472	2.9471	2.6045– 3.2897	0.6852	17.29
Labor	0.2289	-0.0068– 0.6366	0.6434	-0.2259	-0.1797–-0.2721	0.0924	6.96
Feed	1.9632	1.1816– 3.7106	2.5290	-0.5888	-0.5179–-0.6597	0.1418	17.83
Utilities	-0.0283	-0.1607– 0.1705	0.3312	-0.0440	-0.0121–-0.0759	0.0638	5.19
Herd size	1.2774	1.1005– 1.5977	0.4972	0.8987	0.8654– 0.9320	0.0666	7.47
Land	-0.0284	-0.1088– 0.0480	0.1568	-0.0183	-0.0335–-0.0031	0.0304	5.16
Milk barn	-0.1218	-0.3287– 0.0261	0.3548	0.0341	0.0028– 0.0654	0.0626	5.67
Machinery	0.0164	-0.0969– 0.1305	0.2274	0.0140	-0.0080– 0.0360	0.0440	5.17
Other buildings	0.0424	-0.0271– 0.1131	0.1402	0.0389	0.0253– 0.0525	0.0272	5.15
Scale ($\Sigma\tilde{\gamma}_m$)	1.1860	1.0648– 1.3898	0.3250	0.9674	0.9431– 0.9917	0.0486	6.69

1/ Computed values of a $(1-1_N'b)^{-1}$, b $(1-1_N'b)^{-1}$, and c $(1-1_N'b)^{-1}$ using a, b, and c in 3:14. 2/ Computed values of $\tilde{a}$, $\tilde{b}$, and $\tilde{c}$ in 3:7. 3/ For the 99% level of significance. 4/ Range using single equation estimation/ Range using reduced form estimation.

estimates for SEE and RFE indicates that RFE is clearly superior. The ranges using SEE are generally about five times larger. For production method B, only three of the nine estimates using SEE are significantly different from zero. This contrasts with eight significant estimates using RFE. In addition, point estimates using SEE are only asmptotically unbiased even though the untransformed estimates in Table 1 are unbiased.[1/] Point estimates using RFE are unbiased.

Estimates of returns to scale parameters are also given in Table 2. The estimated values are larger for production method A than for method B using both estimation techniques. This is consistent with the initial observation that many smaller firms have introduced parlors, and that some larger firms have built new stanchion barns. Estimated scale parameters are significantly greater than one using RFE for both production methods. However, using SEE, returns are significantly increasing for method A and significantly decreasing for method B. Since estimated scale parameters are unbiased using RFE and only asymptotically unbiased using SEE, it appears that returns to scale for both production methods are overestimated using SEE.

The negative estimates of $\tilde{\beta}$ using RFE are difficult to interpret in light of economic logic. The implication is that optimum output levels for a given stock of capital inputs increase in response to an increase of current input prices or to a decrease of output price. This is inconsistent with maximizing net returns to capital stock, and suggests that the investment criterion may not be valid. Another criterion such as the maintenance of a specific level of net returns might be considered as a

---

1/ Assuming producers attempt to maximize expected net returns to capital.

more realistic alternative. However, it is also possible that the transformed current input variables $Z = (X - Y1_N')$ in 3:7 are not independent of the residual ω. This dependence implies that the ratios of current inputs to output are related to the unexplained output residual, and consequently, that ordinary least squares estimates of $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$ in 3:7 are biased. One possible procedure for eliminating this bias is to assume that the dependence results from omission of firm and time effects in the original specification 3:1. Respecifying 3:1 to include these effects would be sufficient to insure that estimates corresponding to 3:7 are unbiased.

In conclusion, it seems that although the variances of reduced form coefficients are relatively small using RFE compared with SEE, the RFE estimates for current inputs are not consistent with the economic framework. It is true, however, that the prediction of output for given levels of capital inputs and current prices is still unbiased using RFE even if individual reduced form coefficients are biased. This property does not apply to SEE, as the equivalent expression for predicting output contains quotients with the dependent variable Y in both the numerator and the denominator.

BIBLIOGRAPHY

1. Arrow, K.J, and A.C. Enthoven, "Quasi-concave Programming," Econometrica, 29, 1961, pp. 779-800.

2. Bradu, D., and Y. Mundlak, "Estimation of Lognormal Linear Models," Journal of the American Statistical Association, 65, 1970, pp. 198-211.

3. Christ, C.F., Econometric Models and Methods, Wiley, New York, 1966.

4. Fieller, E.C., "The Distribution of the Index in a Normal Bivariate Population," Biometrika, 24, 1932, pp. 428-440.

5. Fuller, W.A., "Estimating the Reliability of Quantities Derived from Empirical Production Functions," Journal of Farm Economics, 44, 1962, pp. 82-99.

6. Hoch, I., "Simultaneous Equation Bias in the Context of the Cobb-Douglas Production Function," Econometrica, 26, 1958, pp. 566-578.

7. ______, "Estimation of Production Function Parameters Combining Time-Series and Cross-Section Data," Econometrica, 30, 1962, pp. 34-53.

8. Hoch, I., and Y. Mundlak, "Consequences of Alternative Specifications in Estimation of Cobb-Douglas Production Functions," Econometrica, 33, 1965, pp. 814-828.

9. Mundlak, Y., "Empirical Production Functions Free of Management Bias," Journal of Farm Economics, 43, 1961, pp. 44-56.

10. ________, "Estimation of Production and Behavioral Functions from a Combination of Cross-Section and Time-Series Data," in Measurement in Economics, ed. C.F. Christ, Stanford University, Stanford, 1963, pp. 133-166.

11. Salter, W.E.G., Productivity and Technical Change, Cambridge University Press, Cambridge, 2nd Edition, 1966.

12. Smith, V.L., Investment and Production, Harvard University Press, Cambridge, Mass., 1961.

13. Zellner, A., J. Kmenta, and J. Dreze, "Specification of Cobb-Douglas Production Function Models," Econometrica, 34, 1966, pp. 784-795.