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A NOTE ON ELASTICITY ESTIMATION OF CENSORED DEMAND

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Abstract: Estimating censored demand systems using micro-level data has become more pervasive in recent years. However, not enough attention has been paid to the evaluation of the elasticities from the censored systems, and the existent methods used in literatures are usually incorrect. This note proposes a practical procedure on how to obtain the elasticities from a censored AIDS model.

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I. Introduction

In a recent paper by Golan, Perloff, and Shen (2001), the method of maximum entropy was introduced to estimate a *censored* AIDS model. However, in the posterior analysis of the paper, the price and expenditure elasticities were evaluated using the formula for the *uncensored* systems. In this note we show that the way to evaluate elasticity using such a formula is inappropriate for censored model and an appropriate method is developed thereafter.

II. Elasticity of Uncensored AIDS model

We define an uncensored empirical AIDS model as:

$$W^* = \alpha + \gamma \ln P + \beta \ln \frac{Y}{P^*} + \varepsilon = U + \varepsilon\ , \tag{1}$$

where W^* is a (M+1) column vector of expenditure shares, P is a (M + 1) column vector of commodity prices, equation parameters are: α [(M+1) x 1], γ [(M+1) x (M+1)], and β [(M+1) x 1]. ε is a [(M + 1) x 1] vector of equation error terms, Y is total expenditure and P^* is a translog price index defined by:

$$\ln P^* = \alpha_0 + \alpha' \ln P + \frac{1}{2} (\ln P)' \gamma (\ln P)\ , \tag{2}$$

where α_0 is a scalar parameter.

If it is assumed that the error term ε is distributed normal with a mean vector of zeros, the following expected budget share is derived:

$$E(W^*) = \alpha + \gamma \ln P + \beta \ln \frac{Y}{P^*} . \quad (3)$$

Then, the uncompensated (Marshallian) price elasticity is given by:

$$\mathrm{E} = -\Delta + \frac{\gamma - \beta(\alpha + \gamma \ln P)}{E(W^*)} , \quad (4)$$

where E is a [(M+1) x (M+1)] matrix of cross and own price elasticities; Δ is a [(M+1) x (M+1)] diagonal matrix of ones.

The key issue in deriving elasticities is to use expected values of observed shares.

III. Elasticity of Censored AIDS model

Expected values of observed expenditure shares can be obtained from the censored demand system by summing the products of each regimes probability and expected conditional share values over all possible regimes.

Suppose the censored rule for equation (1) is defined as:

$$W_i = \begin{cases} W_i^* / \sum_{j \in S} W_j^* , & \text{if} \quad W_i^* > 0, \\ 0, & \text{if} \quad W_i^* \leq 0, \end{cases} \quad (5)$$

where W^* and W are latent and observed shares respectively, and S is a set of all positive share's subscripts. This mapping makes W: (i) lie between 0 and 1, and (ii) sum to unity (Wales and Woodland).

Let R_k represent the k^{th} demand regime and define it as:

$$R_k = (W_1 = W_2 = \cdots = W_k = 0; W_{k+1} > 0, \cdots, W_{M+1} > 0) . \quad (6)$$

That is, the regime of the first k W's are zeros and the rest are positive. Given k zero W's, other possible regime can be transformed to this pattern by rearranging the ordering of the W's so that the first k are zeros. Then we have the expected share for commodity j as:

$$E(W_j) = \sum_{k=1}^{M+1} \alpha_{R_k} E(W_j \mid R_k), \tag{7}$$

where α_{R_k} is the probability of regime R_k occurring, and

$$\begin{aligned} \alpha_{R_k} &= prob(R_k) = prob(W_1 = W_2 = \cdots = W_k = 0; W_{k+1} > 0, \cdots, W_{M+1} > 0) \\ &= \int_{-\infty}^{-U_1} d\varepsilon_1 \int_{-\infty}^{-U_2} d\varepsilon_2 \cdots \int_{-\infty}^{-U_k} d\varepsilon_k \int_{-U_{k+1}}^{\sum_{i=k+2}^{M+1} U_i - \sum_{i=2}^{k} \varepsilon_i} d\varepsilon_{k+1} \cdots \int_{-U_{M-1}}^{\sum_{i=M}^{M+1} U_i - \sum_{i=2}^{M-2} \varepsilon_i} d\varepsilon_{M-1} \int_{-U_M}^{U_{M+1} - \sum_{i=2}^{M-1} \varepsilon_i} \phi(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_M) d\varepsilon_M \end{aligned} \tag{8}$$

where $\phi(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_M)$ is the multivariate normal pdf with a mean vector of zeros. The expected share value conditional on purchase regime R_k can be represented as:

$$E(W_j \mid R_k) = \begin{cases} \dfrac{E(W_j^* \mid R_k)}{\sum_{i=k+1}^{M+1} E(W_i^* \mid R_k)}, & if \;\; j > k, \\ \\ 0, & if \;\; j \le k; \end{cases} \tag{9}$$

with $E(W_j^* \mid R_k) = U_j + E(\varepsilon_j \mid R_k) = U_j + \dfrac{\alpha_{R_k}^{\varepsilon_j}}{\alpha_{R_k}}$. U_j is the j^{th} row of U given in (1), and

$$\alpha_{R_k}^{\varepsilon_j} = \int_{-\infty}^{-U_1} d\varepsilon_1 \int_{-\infty}^{-U_2} d\varepsilon_2 \cdots \int_{-\infty}^{-U_k} d\varepsilon_k \int_{-U_{k+1}}^{\sum_{i=k+2}^{M+1} U_i - \sum_{i=2}^{k} \varepsilon_i} d\varepsilon_{k+1} \cdots \int_{-U_{M-1}}^{\sum_{i=M}^{M+1} U_i - \sum_{i=2}^{M-2} \varepsilon_i} d\varepsilon_{M-1} \int_{-U_M}^{U_{M+1} - \sum_{i=2}^{M-1} \varepsilon_i} \varepsilon_j \phi(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_M) d\varepsilon_M. \tag{10}$$

The calculation of elasticity is based on equation (7), which involves the evaluation of (8) and (10). Equations (8) and (10) are M-fold integrals and we may approximate them by numerical procedure (Gauss quadrature). Given that there are 2^{M+1}-1 purchase regimes, one needs to evaluate (8) and (10) 2^M times. This would be very time consuming. However, a simulation procedure can be used instead to evaluate elasticities.

Assume we have R replicates of the [M+1] error term vector, e in (1). The r^{th} simulated latent share, $(W^*)_r$, evaluated at sample means of the exogenous variables (indicated by a bar over a variable) is:

$$(W^*)_r = \alpha + \gamma \ln \overline{P} + \beta \ln \frac{\overline{Y}}{\overline{P^*}} + e_r , \tag{11}$$

where e_r is the r^{th} replicate of e. The r^{th} replicate of observed share i given by (5) then is

$$(W_i)_r = \begin{cases} (W_i^*)_r / \sum_{j \in S} (W_j^*)_r , & \text{if} \quad (W_i^*)_r > 0, \\ 0, & \text{if} \quad (W_i^*)_r \leq 0, \end{cases} \tag{12}$$

where the subscript i of W represents the i^{th} element in the vector of W. The expected observed share vector for R replicates is then calculated as simple average of these simulated values:

$$E(W) = \frac{1}{R} \sum_{r=1}^{R} (W)_r \ . \tag{13}$$

Suppose we have a small change in price j, ΔP_j, the elasticity vector with respect to this price change is:

$$\text{(14)} \qquad \eta_j = -\delta_j + \frac{\Delta E(W)}{\Delta P_j} \cdot \frac{P + \Delta P_j / 2}{E(W) + \Delta E(W) / 2} ,$$

where δ_j is a vector of 0's with the j^{th} element 1, and $\Delta E(W)$ is the change of the simulated $E(W)$ given the change of price, ΔP_j.

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