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EXPECTATIONS MODELS OF ELECTRIC UTILITIES' FORECASTS: A CASE STUDY OF ECONOMETRIC ESTIMATION WITH INFLUENTIAL DATA POINTS

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ABSTRACT

This study develops an econometric model for explaining how electric utilities revise their forecasts of future electricity demand each year, and it is based on the senior author's thesis research (Vellutini (1982)).

The model specification is developed from the adaptive expectations hypothesis and it relates forecasted growth rates to actual lagged growth rates of electricity demand. Unlike other studies of the expectations phenomenon, expectations of future demand levels constitute an observable variable and thus can be incorporated explicitly into the model. The data used for the analysis were derived from the published forecasts of the nine National Electric Reliability Councils in the U.S. for the years 1974 to 1980.

Three alternative statistical methods are used for estimation purposes: ordinary least-squares, robust regression and a diagnostic analysis to identify influential observations. The results obtained with the first two methods are very similar, but are both inconsistent with the underlying economic logic of the model. The estimated model obtained from the diagnostics approach after deleting two aberrant observations is consistent with economic logic, and supports the hypothesis that the low growth of demand experienced immediately following the oil embargo in 1973 were disregarded by the industry for forecasting purposes. The model includes transitory effects associated with the oil embargo that gradually disappear over time, the estimated coefficients for the lagged values of actual growth approach a structure with declining positive weights. The general shape of this asymptotic structure is similar to the findings in many economic applications using distributed lag models.

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1. INTRODUCTION

Expectations and Economic Behavior

Expectations arise in many economic situations. Expectations about future outcomes are likely to influence decisions made in the current period; they are also likely to change as new information becomes available. In this sense, expectations are not static, but have embodied a strong dynamic component that is reflected in the updating mechanism. The process by which expectations are updated is an "adaptive" one; as new information becomes available, the "expected" levels of the variable of interest are revised.

The adaptive expectations hypothesis in economics was developed by Nerlove (1958), who examined its implications for the cycles of a cobweb model of a single market. He applied the adaptive expectation model to the problem of instability, concluding that the likelihood of stability is improved when adaptive expectations are assumed. Arrow and Nerlove (1958), in an analysis of multiple markets, showed that under adaptive expectations a dynamic system, stable under static expectations, remains stable no matter what the inertia of the system or the elasticities of expectations are.

Expectations about future economic behavior, although widely used by economic agents, are seldom directly quantifiable. The present study, however, faces an unusual situation where expectations of future outcomes, the forecasted levels of electricity demanded, are published annually. This makes it possible to develop a model that incorporates an explicit adaptive mechanism for the values of the forecasted variable. In other words, the expectations variable can be used as a dependent variable in a regression framework. As such, an objective of this study is to apply the expectations hypothesis to gain insights into how electric utilities forecast electricity demand.

Forecasts made by electric utilities of the future demand for electricity constitute a basis from which decisions to build new generating capacity are made because it takes from five to ten years to build a new plant. The main interest in developing a model of utilities' forecasts is to make it possible to predict utilities construction plans under different economic conditions. It is more relevant to "forecast utilities' forecasts" than to provide an independent forecast of future demand levels, because the utilities' forecasts are the ones that are used to determine the size of capacity additions.

2. ELECTRIC UTILITIES' FORECASTING IN THE LAST DECADE

The electric utility industry was one of the sectors most seriously affected by the oil embargo in 1973. The demand for electricity, which had been showing steady, substantial growth in preceeding years, grew very little and in some states declined in the years immediately following the oil embargo. Demand has grown relatively slowly since.

Most of the disruptions caused by the oil embargo in 1973 and the economic recession which followed were considered by many people to be a temporary phenomenon. It was expected that after a short period of transition in which some adjustments would be made, the economy would be able to return to its normal pace of continuous growth. This optimistic view was apparently shared by electric utility companies, and this meant that their forecasts of future demand implied substantial growth after a certain "delay" caused by an initial disruption of the oil embargo. That is, the growth of electricity was expected to return to rates almost as high as those experienced prior to the oil embargo. This is illustrated in Figure 1 which shows the aggregate forecasts made by the National Electric Reliability Council in the years 1974 to 1982. Thus, the utility companies' revised forecasts were lower than their preceeding ones, but still followed an exponential growth path.

Table 1 presents the forecasts of net energy requirements in the U.S. for 1984 made each year from 1975 to 1981. These forecasts illustrate the implications for capacity planning. In 1975, when actual demand was approximately 1.9 trillion kwh, an increase of 1.65 trillion kwh was anticipated by 1984. In 1981, however, actual demand was still only 2.3 trillion kwh, and the forecast for 1984 had been revised downward from 3.5 to 2.6 trillion kwh. In other words, about a third of the new generating capacity that had, in 1975, appeared necessary to meet increased demand in 1984 was still considered necessary in 1981. Since future levels of generating capacity are based on demand forecasts, the implications of overpredictions on the future amount of excess generating capacity are obvious. Table 2 presents an illustration of this phenomena. When actual generation is compared to the installed generating capacity, the declining average intensity factor shows that generating capacity was used less intensively over time. In other words, the amount of new generating capacity that has been built is greater than the amount required to meet increased demand. This is particularly true in Northern and Eastern states.

One of the immediate consequences of building excess capacity is the financial burden imposed on electric utility companies and their customers. By having to operate plants below efficient levels, average costs are higher than expected. Moreover, if the financial situation of electic utility

Figure 1. Forecasts of Electricity Demand in the U.S., published by the National Electric Reliability Councils.

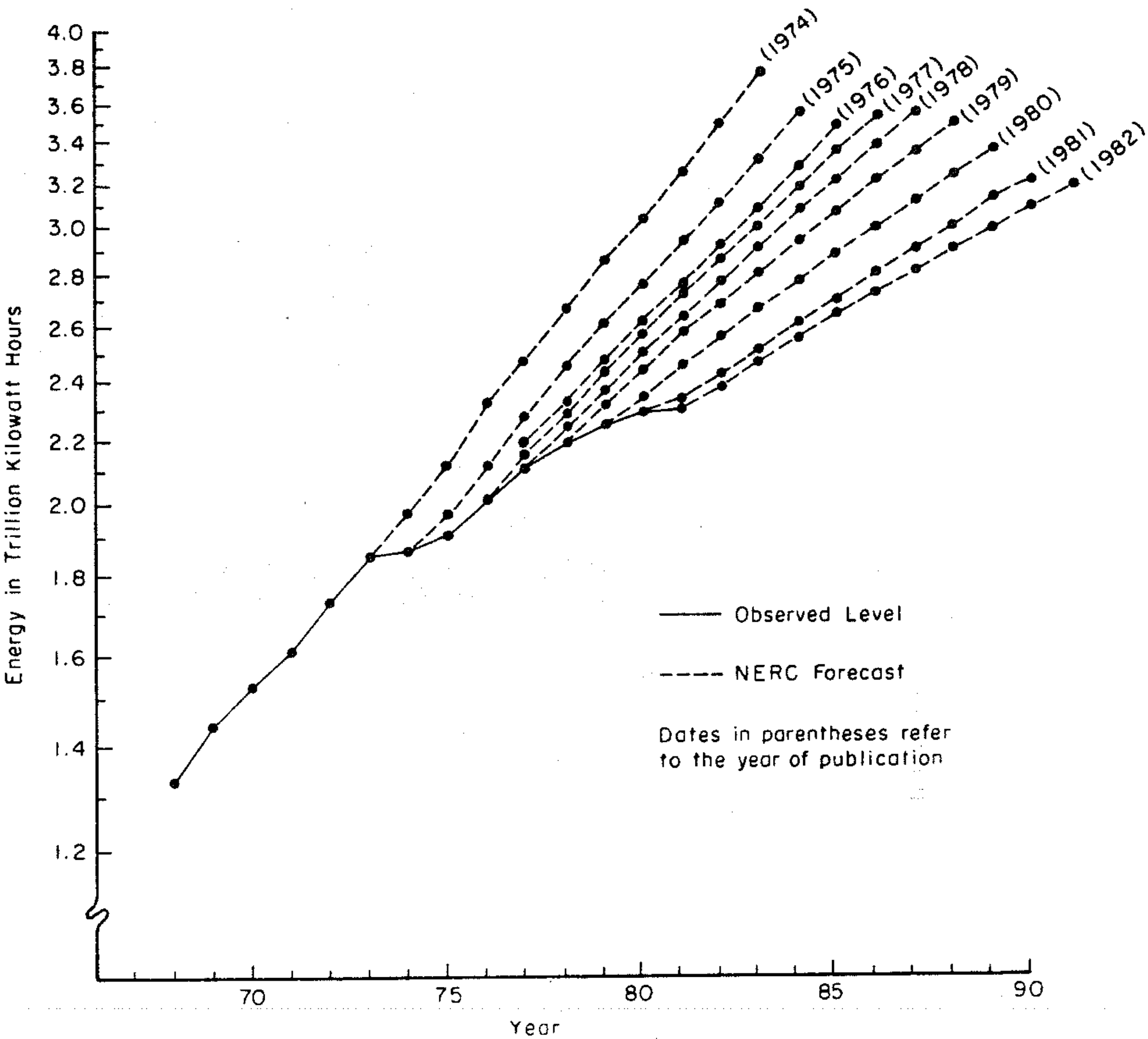

Table 1. Forecasts of Net Annual Energy Requirements for 1984 in the U.S. (48 Contiguous States).

Year When Forecast Was Published	Annual Energy Requirements for 1984
	(billion kwh)
1975	3,555
1976	3,293
1977	3,197
1978	3,080
1979	2,957
1980	2,796
1981	2,637

Source: Department of Energy, Federal Power Commission News Release (1975 to 1977) and Electric Power Supply and Demand for the Contiguous United States (1978 to 1981).

Table 2. Average Intensity Factors for Installed Generating Capacity in the U.S. (48 Contiguous States).

Year	Installed Generating Capacity (1000 MW)	Generation (billions of kwh)	Average Intensity Factor (%)[a/]
1965	236	1,055	51
1968	290	1,327	52
1971	367	1,614	50
1974	476	1,865	45
1977	557	2,124	44
1979	598	2,247	43
1980	614	2,286	43

Source: Edison Electric Institute, Statistical Yearbook of the Electric Utility Industry, several issues.

a/ The intensity factor is the average percentage of time that generating capacity is used during a year; intensity factor = (100 x generation/(capacity x number of hours in a year)).

companies is bad, they may be very reluctant to incur additional capital costs for pollution control devices, for example.

By modeling how demand levels of electricity are forecasted by utility companies, it is possible to predict the amount of planned additions to capacity. Alternative policy situations may be considered, and comparisons made of the amount of capacity that will be installed in the future, and subsequently, of the future financial outlook for electricity utilities and their ability to finance investments to meet air quality standards.

3. DEVELOPMENT OF THE MODEL

A major difficulty with expectations models is that the "expected" variable is generally unobservable. In the case of forecasted electricity demand, however, the relevant variable is observable. Hence, the main characteristic of this study is that the model deals directly with expectations. In more typical expectations models, the expected variable is derived indirectly using, for example, a distributed lag mechanism to relate an unobserved expected price to an observed response in supply. The studies by Schultz and Brownlee (1941-42), Heady and Kaldor (1954), Turnovsky (1970), and Fisher and Tanner (1978) are examples of attempts to quantify expectations variables.

For this study, "new information" used to update expectations refers specifically to actual levels of energy requirements. This is because every year utilities have available the actual levels of energy for the previous year, which can be compared and contrasted with their forecasted (expected) levels for the same period. Since annual growth rates of energy levels are of specific interest in this study, growth rates of actual levels of energy in past years become the relevant information for updating forecasts of expected future growth.

The above discussion implies that utilities' expectations about future outcomes depend on actual experience in previous years. This can be represented mathematically by the following expression:

$$(1) \quad ER_t = f(AR_{t-1}, AR_{t-2}, \ldots, AR_{t-n})$$

where ER_t = expected annual growth rate of future electricity demand made in year t

AR_t = actual annual growth rate of electricity demand from year t-1 to year t.

In this example, the expected growth rate may refer to the average growth rate for a specified number of years into the future (10 years). On the other hand, the actual growth rates are for a single year only. It is assumed that all the relevant information for the purposes of forecasting is contained on the four preceding rates. That is, the length of the lag structure is assumed to be four. There is no _a priori_ reason for selecting four lagged periods. In part, it reflects the limitations of the length of time-series data available for the nine reliability councils, and, from a statistical point of view, avoids losing too many degrees of freedom. Furthermore, there is no indication in the following analysis that this lag length is too short.

With these specifications, the working model may be formally expressed in a linear regression framework as follows:

$$(2) \quad ER_t = \beta_1 AR_{t-1} + \beta_2 AR_{t-2} + \beta_3 AR_{t-3} + \beta_4 AR_{t-4} + e_t$$

where $\beta_1, \ldots, \beta_4$ are unknown parameters and e_t is a stochastic residual.

In this model, the expected growth rate is a weighted average of the actual growth rates in the previous four years. Economic reasoning suggests that the weights should all be positive and sum to one, and that more recent information should receive a relatively big weight. In other words, the weights should decline with the number of periods lagged.

An intercept, together with a set of regional "dummy variables", is included to capture cross-sectional differences that may arise from pooling data from different Electric Reliability Councils. With these additions, the model becomes:

$$(3) \quad ER_{rt} = \beta_0 + \sum_{i=1}^{4} \beta_i AR_{rt-i} + \sum_{i=1}^{8} \alpha_i D_{irt} + e_{rt}$$

where r represents the region and D_{irt} is a zero-one variable that is equal to one if i = r and zero otherwise.[1/]

Since the logic of the model implies that the sum of the coefficients of the lagged growth rates should sum to unity, it is possible to substitute $(1 - (\beta_1 + \beta_2 + \beta_3))$ for β_4. This implies that the model can be simplified to:

$$(4) \quad ER_{rt} - AR_{rt-4} = \beta_0 + \beta_1(AR_{rt-1} - AR_{rt-4}) + \beta_2(AR_{rt-2} - AR_{rt-4}) + \beta_3(AR_{rt-3} - AR_{rt-4}) + \sum_{i=1}^{8} \alpha_i D_{irt} + e_{rt}$$

The sample period used for estimation (1974 to 1980) is a peculiar one because of the oil embargo in 1973, and further developments of the model were made to reflect this. Immediately after the oil embargo, actual growth rates

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1/ Because an overall intercept (β_0) is included in the model, the number of zero-one variables is equal to eight to avoid singularity of the matrix of regressors.

were negative in many regions. Major structural changes occurred in sectors directly or indirectly dependent on oil. Nevertheless, these changes were perceived as temporary phenomena caused by the initial disruptions of the embargo, and it was generally felt that economic recovery would inevitably follow. Consequently, the observed lower negative growth rates were not given much "weight" in deriving long-run forecasts. To incorporate "transitional effects" into the working model, the weights ($\beta_1, \ldots, \beta_4$) are made functions of time. Nevertheless, these time effects are themselves only temporary, and a scheme is needed in which the time effects die away in order to capture the initial disruptions and to allow for a gradual decline of their importance. This is especially important if the model is going to be used for forecasting purposes. A linear shifting scheme, for example, would imply that the changes in the weights over time would be the same in the forecasting period as in the transitional period after the oil embargo.

The implications of the above discussion for the development of the model can be incorporated by making the time effects inverse functions of time. By using an inverse function, the magnitude of these changes depends on the starting point. Since 1974 is treated as the first year, most of the transitional effects occur during the years immediately after 1973. In other words, the weights do not change nearly as much at the end of the sample period as they do at the beginning, as one would expect, and they approach fixed values over time. Under this specification, equations (3) and (4) can be rewritten as

$$(5)\quad ER_{rt} = \beta_0 + \beta_{11}AR_{rt-1} + \beta_{12}\left(\frac{AR_{rt-1}}{t - 1973}\right) + \beta_{21}AR_{t-2} + \beta_{22}\left(\frac{AR_{rt-2}}{t - 1973}\right) + \beta_{31}AR_{t-3} + \beta_{32}\left(\frac{AR_{rt-3}}{t - 1973}\right) + \beta_{41}AR_{t-4} + \beta_{42}\left(\frac{AR_{rt-4}}{t - 1973}\right) + \sum_{i=1}^{8} \alpha_i D_{irt} + e_{rt}.$$

and

$$(6)\quad (ER_{rt} - AR_{rt-4}) = \beta_0 + \beta_{11}(AR_{rt-1} - AR_{rt-4}) + \beta_{12}\left(\frac{AR_{rt-1} - AR_{rt-4}}{t - 1973}\right) + \beta_{21}(AR_{rt-2} - AR_{rt-4}) + \beta_{22}\left(\frac{AR_{rt-2} - AR_{rt-4}}{t - 1973}\right) + \beta_{31}(AR_{rt-3} - AR_{rt-4}) + \beta_{32}\left(\frac{AR_{rt-3} - AR_{rt-4}}{t - 1973}\right) + \sum_{i=1}^{8} \alpha_i D_{irt} + e_{rt}.$$

The implicit constraints in (6), $\beta_{41} = 1 - \sum_{i=1}^{3} \beta_{i1}$ and $\beta_{42} = - \sum_{i=1}^{3} \beta_{i2}$, ensure that the weights always sum to unity for all years after 1973.

Note that the first constraint corresponds to the original specification that the weights sum to unity, and the second constraint implies that the time effects sum to zero. This model is referred to as the restricted model. An unrestricted model is also estimated by dropping the two constraints on the weights.

4. DESCRIPTION OF THE DATA

Both the unrestricted and restricted versions of the model make use of two basic sets of data. The first set contains the expected rates of growth of energy requirements, computed as a compound annual growth rate for a 10 year period. The second set contains the observed rates of growth for preceding years, also computed as a compound annual growth rate. Energy requirements were used rather than peak load because this variable is less sensitive to unusual climatic conditions in the historical period. The forecasted levels of energy requirements were obtained from Federal Power Commission News Release for 1974 to 1977 and Electric Power Supply and Demand for the Contiguous United States, published by the Economic Regulatory Administration of the U.S. Department of Energy for 1978 to 1980. These reports contain the forecasted (expected) levels of energy requirements made by the nine electric reliability councils (members of the NERC) for the ten-year period ahead of the publication date. From these levels, the growth rates were calculated directly. The period covered ranges from 1974 to 1980 for each of the nine councils to provide a total of 63 observations.

Information on the actual levels of energy requirements was not readily available, and some of the data had to be obtained in an indirect way. In the 1980 edition of "Electric Power Supply and Demand for the Contiguous United States," actual levels are reported for 1976 to 1980. Since the estimation period is from 1974 to 1980 and a four-period lagged variable is included in the model, actual levels are needed from 1969 on. For the early years, data on actual energy requirements by state were obtained from the Edison Electric Institute's (EEI) Statistical Yearbook of the Electric Utility Industry. This data set was then aggregated from the state level to the NERC regions using fixed weights for each state on the basis of geographic boundaries and population density. Annual growth rates were then computed directly from these levels.2/

5. METHODS OF ANALYSIS

The use of data for the period 1974-80 from nine different NERC regions can be expected to make the analysis susceptible to influential observations. This is because most of the economic disruptions from the oil embargo in 1973

2/ Another reason for specifying the model in terms of growth rates rather than levels is that the levels of actual energy requirements did not match exactly in the DOE and EEI sources for a common year (1974). There was a much better correspondence using annual growth rates.

were felt in the subsequent years, and hence the sample period includes the transitory period that followed. Since a few bad data points can distort parameter estimates, estimation methods were used that take into account the existence of potential outliers in the data set.

Three alternative statistical methods were used to estimate the model: ordinary least squares (OLS), robust regression using Huber's weighting scheme, and OLS with regression diagnostics to identify influential data points. Among the three estimation procedures outlined above, OLS is the simplest and most commonly used. It is based on the minimization of the sum of squared residuals for all observations, implying that observations with large residuals are important. Robust techniques assign a smaller weight to observations with large residuals than with OLS, using an iterative procedure that may be described as reweighted least-squares. This gives an estimation procedure that is less sensitive to large residuals than OLS. Regression diagnostics is a type of analysis that provides a set of criteria for identifying influential observations and for determining whether they should be deleted from the data set.3/

6. ESTIMATION RESULTS

OLS

Applying ordinary least squares to both equations (3) and (4), the unconstrained and constrained models respectively, yields poor results. Specifically, an oscillating lag structure is found in the former case, which is illogical if we expect a weighted average type of behavior with declining weights. For the restricted model, it yields a relatively flat lag structure with a relatively large coefficient in the fourth lagged period, which is also inconsistent with economic reasoning. Analysis of the computed residuals show behavior which is not randomly distributed around zero. Moreover, there is a consistent tendency to over-predict the endogenous variable in the later periods of the sample.

Inclusion of time effects for the weights, as depicted by equations (5) and (6), produce noticeable changes in the fit and implied lag structure of both models. The unconstrained version, equation (5), had a noticeably better fit than equation (3). Its asymptotic lag structure, however, even though it has a declining weight structure for the first three periods, still includes an upward shift in the last period. Moreover, the asymptotic weights associated with both the third and fourth periods are negative, which is inconsistent with economic logic.4/

3/ All the statistical methods employed in the analysis were performed using the TROLL econometric package (Version 10).

4/ The asymptotic weights or lag structure refer to the values of β_{i1} in (5) and (6), and correspond to the weights when t is large and the time effects are zero.

The constrained version, equation (6), has slightly better fit than equation (4). The most noticeable change, however, is the implied asymtotic lag structure, which shifted from being flat in (4) to one with declining positive weights. Nevertheless, it still has an upward shift in the last period.

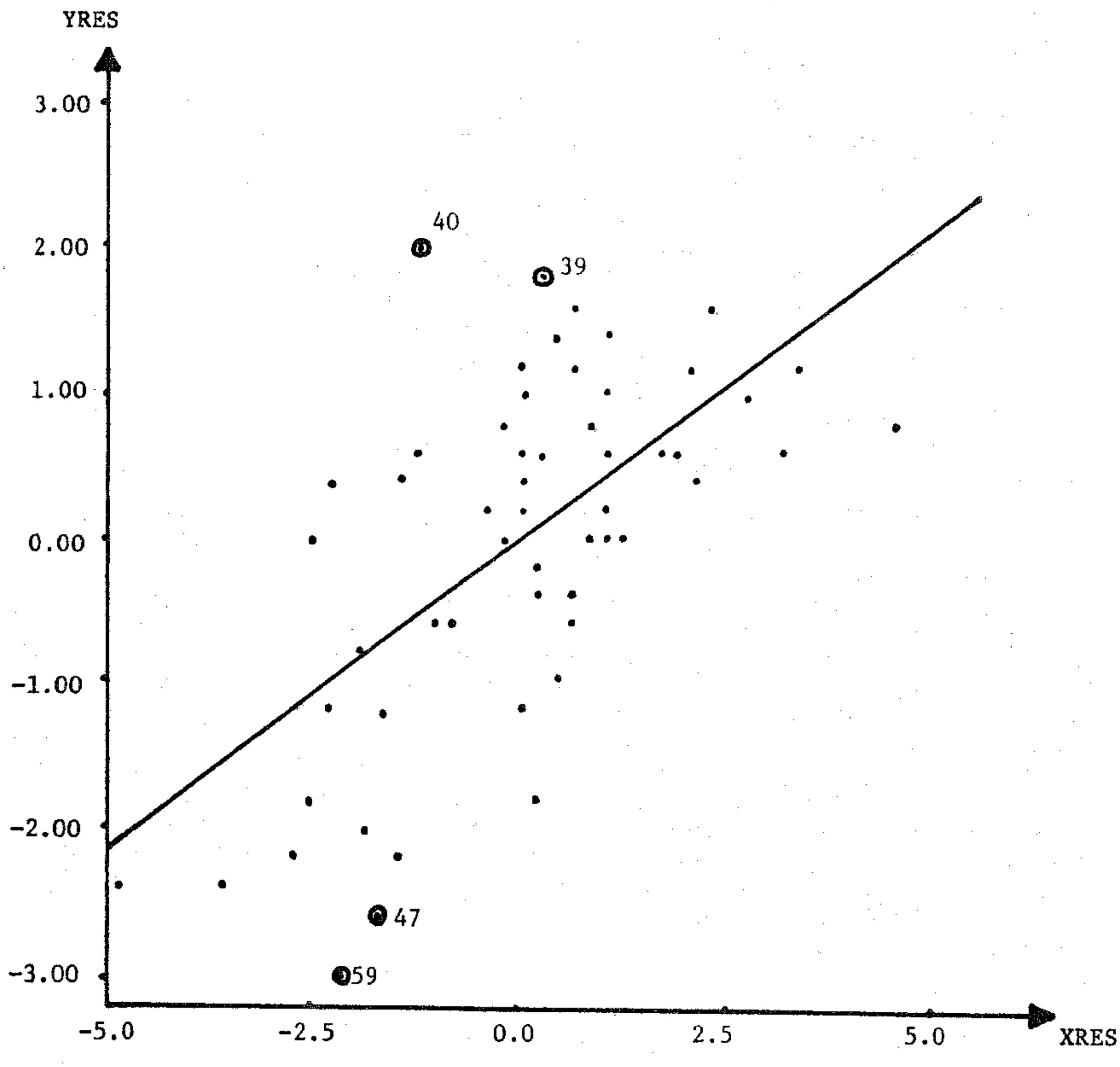
The considerations outlined above suggest that the constrained version of the model with time effects should be chosen for further analysis. In fact, even though the results are not reported, further estimation of the unconstrained version using both robust estimation and diagnostics techniques also give results that are inconsistent with the working hypothesis of declining and positive asymptotic weights for the lag structure.

One of the problems that became apparent with the OLS estimates of equation (6) were that some residuals were relatively large. This led to concern that certain observations may be distorting the estimated coefficients. Consequently, alternative estimation methods were adopted. The results for OLS and for robust estimation are presented in Appendix B. The estimates obtained with robust techniques still exhibit the same problem as the OLS estimates in relation to the asymptotic lag structure. That is, even though it has declining positive weights for three periods, an upward shift in the fourth period is still present. The overall fit, as measured by the coefficient of determination (R^2), is very similar to the value obtained with OLS. The conclusion is that the robust estimates are very similar to the OLS estimates.

A specific set of diagnostic criteria can be computed for the OLS estimates of the model to detect influential data points. These criteria are described in Appendix A. The first criterion is to look for large changes in the coefficient estimates if each observation is deleted. Possible influential points picked up by this criterion are observations 1, 8 and 47. A second criterion searches for large values in the diagonal elements of the "hat-matrix", and the observations with the three largest values are 2, 8 and 38. Analysis of the computed residuals is aimed at identifying observations that are distant from the computed regression line. In this sample, large residuals (positive or negative) are associated with observations 40, 47, 50 and 59. Finally, the partial regression leverage plots5/ for the variables associated with the asympototic lag structure (AR_{t-1}, AR_{t-2} and AR_{t-3}) are presented in Figures 2, 3 and 4.

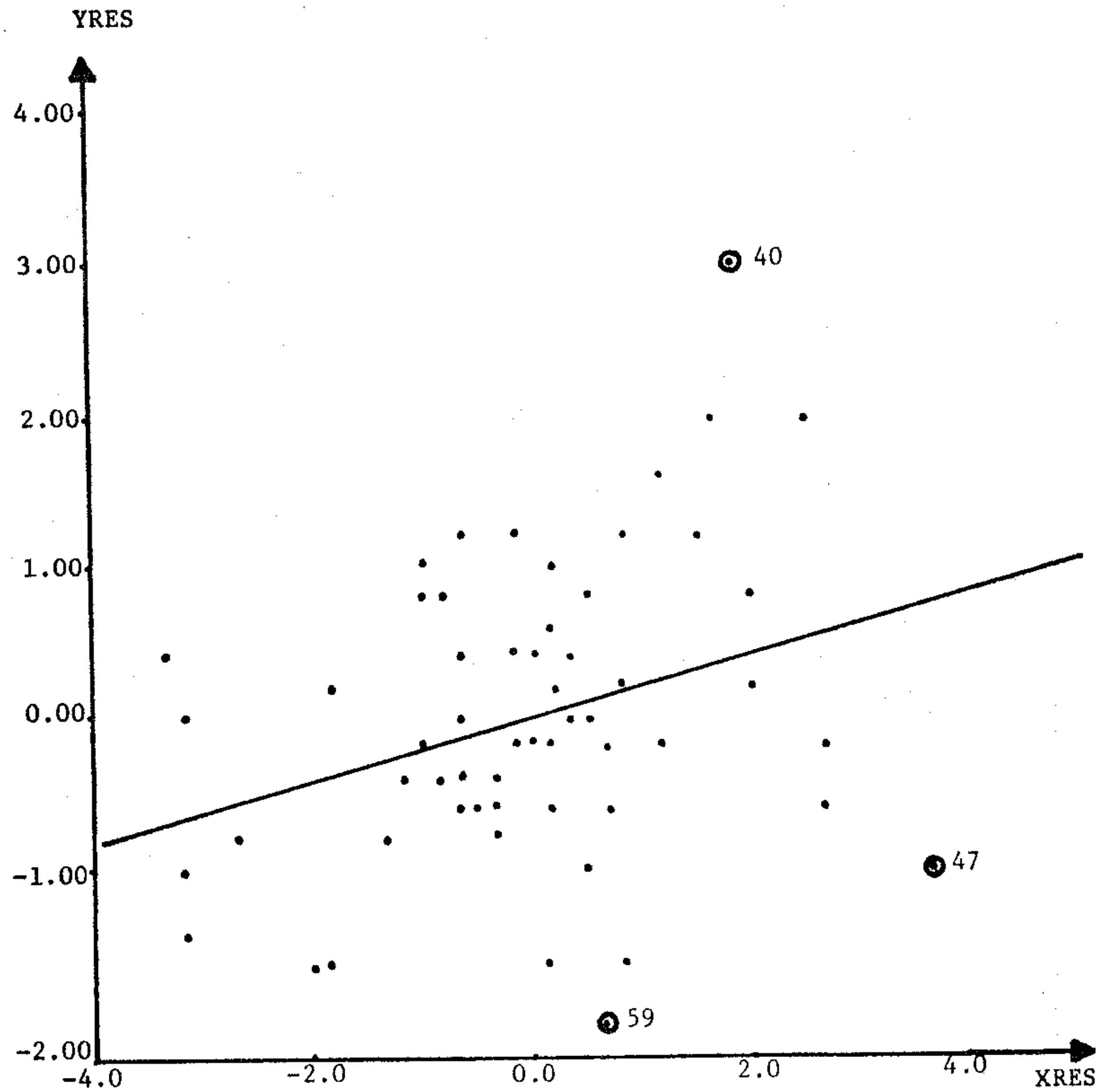
5/ The residuals that result from regressing the exogenous variable under consideration and the endogenous variable on the other exogenous variables are measured on the horizontal (XRES) and vertical (YRES) axes, respectively. The regression line between the two sets of residuals has the same slope as the multiple regression estimate of the coefficient associated with the exogenous variable considered.

Figure 2. Partial Regression Leverage Plot for β_{11} in equation (6).



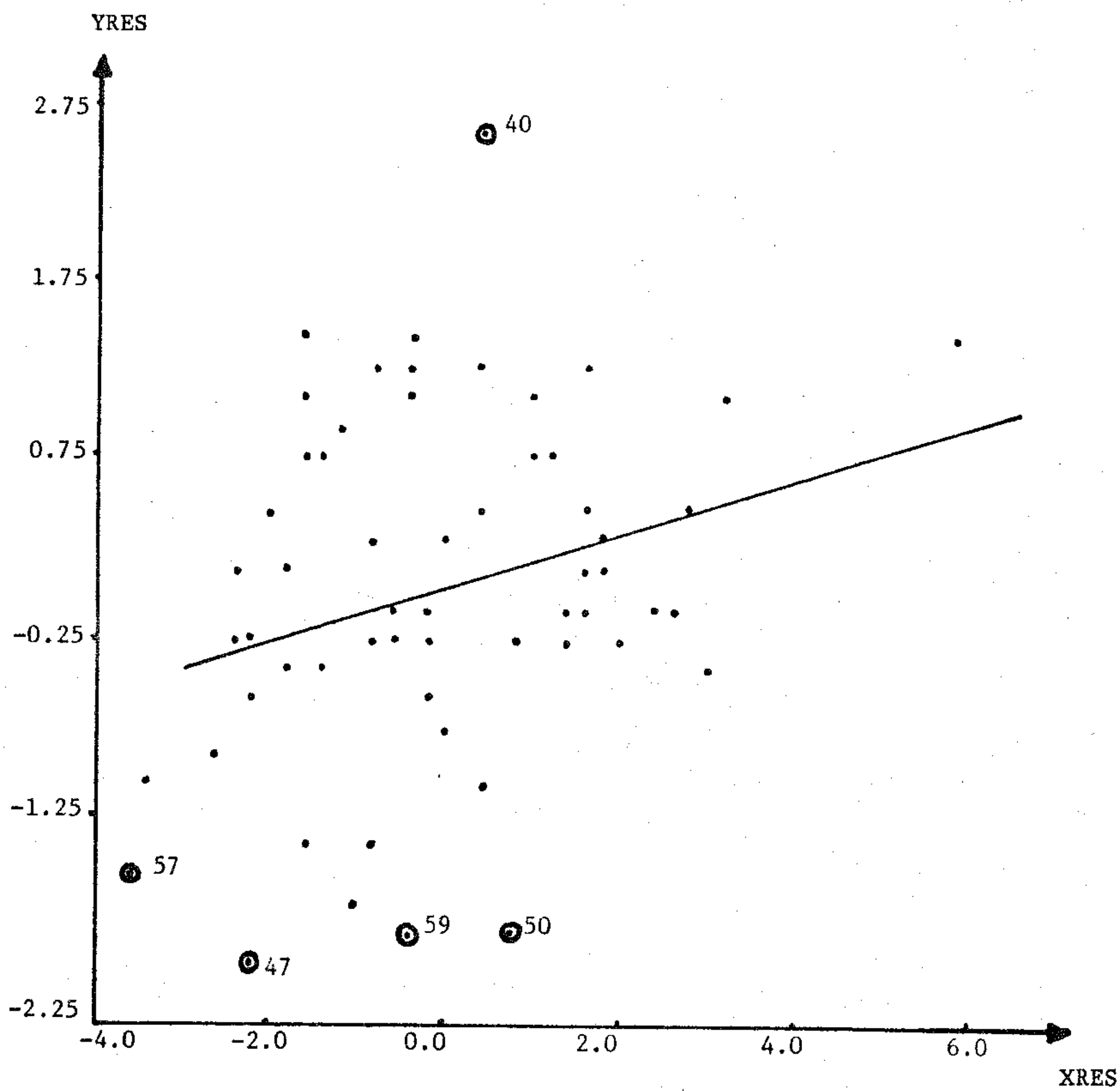
⊙ = Potentially influential points.

Figure 3. Partial Regression Leverage Plot for β_{21} in equation (6).



⊙ = Potentially influential points.

Figure 4. Partial Regression Leverage Plot for β_{31} in equation (6).



⊙ = Potentially influential points.

The observations identified by the different diagnostic criteria are summarized in Table 3. Observations 8, 40, 50 and 59 are the ones that occur more than once, and hence deserve further attention. It is expected that deletion of some of these observations should improve the OLS estimates considerably. After some additional trials, observations 40 and 59 were deleted. It should be noted that deletion of observation 47 made little difference to the lag structure even though this observation appears to be influential in Table 3. The resulting asymptotic lag structure has declining weights, with more recent observations receiving bigger weights, and all weights are positive. Moreover, the fit of the regression is much improved after deletion of these two high leverage data points. The estimated coefficients and summary statistics are presented in full in Appendix B. In addition, Table 4 presents the estimated asymptotic weights and time effects for equation (6) using the three estimation methods employed in this analysis: ordinary least squares, robust regression and a diagnostics analysis. For purposes of illustration, only one robust estimation method (Huber's procedure with $r = 0.5$) is presented. The diagnostics results refer to OLS estimation after deletion of observations 40 and 59, and this is the version used for subsequent analyses.

7. SUMMARY AND CONCLUSIONS

The objective of this study is to develop an appropriate model for explaining how electric utilities revise their forecasts of electricity demand, and the analysis is based on the adaptive expectations hypothesis. Expectations in economic behavior occur in a wide range of economic situations and it is logical to think that expectations of future outcomes will influence the current course of events. It is intuitively appealing that expectations are revised and updated as new information becomes available. Typical studies of the expectations phenomena in its various forms are difficult to undertake because the expectations variables are not observable. In this study, however, utilities' expectations of future demand levels constitute an observable variable and hence can be incorporated explicitly in the model of expectations.

The version of the adaptive expectations model used specifies a mechanism by which electric utilities' forecasts of future demand levels are revised and updated as information on actual levels of demand becomes available. An autoregressive model that relates forecasted to actual lagged growth rates of electricity demand was estimated, using data from the annual reports of the nine National Electric Reliability Council (NERC) from 1974 to 1980. Each of the nine councils prepare a revised forecast of future demand every year.

The forecasted growth rate of demand is explained in the model as a weighted average of past observed growth rates, where more recent observations receive bigger weights. The length of the lag structure is relatively short (four periods). Dummy variables for each region are included in the model specification because they improve the statistical properties of the model, even though it was initially hypothesized that the intercept would be zero. The model also specifies that the weights are inverse functions of the number of years after the oil embargo in 1973. The inclusion of these additional variables captures the effects of a transitional period that followed after

Table 3. Summary of Potentially Influential Data Points

Criteria	Observation Number
Large residuals	40, 47, 50, 59
Large diagonal elements of Hat-matrix	2, 8, 38
Large changes in coefficients	1, 8, 47
Partial Regression Leverage Plots	
Coefficient of $AR_{t-1} - AR_{t-4}$	39, 40, 47, 59
Coefficient of $AR_{t-2} - AR_{t-4}$	40, 47, 59
Coefficient of $AR_{t-3} - AR_{t-4}$	40, 47, 50, 57, 59

Table 4. Estimated Slope Coefficients: Constrained Model.

	Asymptotic Weights				Time Effects			
	AR_{t-1} (β_{11})	AR_{t-2} (β_{21})	AR_{t-3} (β_{31})	AR_{t-4} (β_{41})[a/]	AR_{t-1} (β_{21})	AR_{t-2} (β_{22})	AR_{t-3} (β_{32})	AR_{t-4} (β_{42})[b/]
OLS	0.442	0.213	0.140	0.205	-0.540	-0.068	0.333	0.275
Robust Regresstion[c/]	0.420	0.198	0.143	0.239	-0.474	-0.055	0.335	0.194
Diagnostics Analysis[d/]	0.500	0.250	0.141	0.109	-0.730	-0.242	0.284	0.688

a/ Obtained as $\beta_{41} = 1 - (\beta_{11} + \beta_{21} + \beta_{31})$

b/ Obtained as $\beta_{42} = -(\beta_{12} + \beta_{22} + \beta_{32})$

c/ Huber criterion function with r = 0.5

d/ After deletion of observations 40 and 59.

this economic crisis. During this period, the low growth rates that occurred were initially disregarded by the industry because the economic disruptions were thought to be temporary. This is equivalent to an "inverted" weighting scheme in which more recent observations are given a smaller weight. However, the weighting scheme is an inverse function of the number of years after the oil embargo, and it gradually adjusts to a more logical one with positive declining weights.

Three alternative statistical methods were used for estimation purposes: ordinary least squares, robust regression and a diagnostic analysis to identify influential observations. The latter two are oriented towards estimation in situations in which departures from the classical linear regression model are suspected. OLS is the simplest and most common one, in which all observations are assigned the same weight. Robust regression is an iterative reweighted least-squares in which observations with large residuals are assigned a smaller weight in the next step of the iteration. The diagnostic analysis provides a set of criteria for judging whether certain observations should be deleted from the data set. The results obtained in this study by the procedures described above show a clear superiority of the diagnostic approach. In fact, the results after deleting two observations were far better than the ones obtained by OLS and by different robust regression techniques. The latter methods gave similar results suggesting that robust regression did not lead to any substantial improvement over the OLS in this application.

The lag structure obtained from the diagnostics approach is consistent with economic logic. It exhibits declining weights, and it supports the hypothesis that the low growth rates experienced immediately following the oil embargo were disregarded by the industry. The lag structure associated with the years immediately following the oil embargo (when $t = 1$ and $t = 2$) and the asymptotic lag structure (when $t = \infty$) are summarized in Table 5.

Further improvements in the use of expectations models could be achieved through the use of alternative specifications as well as lag structures. The unavailability of long time-series on "observed expectations" is a serious constraint on the present study. With a longer time-series, it would be possible to broaden the scope on the analysis to include more sophisticated time-series models in which, for example, autocorrelation in the residual terms is considered.

Table 5. Estimated Weights of the Lag Structure.

Variable*	t = 1	t = 2	t = ∞
AR_{t-1}	-0.230	0.135	0.500
AR_{t-2}	0.008	0.129	0.250
AR_{t-3}	0.425	0.283	0.141
AR_{t-4}	0.797	0.453	0.109

* AR_{t-i} refers to actual growth rates lagged i years.

APPENDIX A: METHODS OF ANALYSIS

The issues underlying the different estimation techniques can best be described with the help of the following multiple regression model:

(A.1) $Y = X\beta + U$

where Y is an n×1 vector of the dependent variable, X is a n×p matrix of independent variables, β is a p×1 vector of unknown coefficients and U is a n×1 vector of unobserved random disturbances. The OLS estimate of β is given by

(A.2) $\hat{\beta}_{ols} = (X'X)^{-1}X'Y$

If the assumptions of the classical regression model are met, ($E(Y) = 0$ and $Var(Y) = \sigma^2 I_n$) it can be demonstrated that $\hat{\beta}_{ols}$ is the best linear unbiased estimator (BLUE). The implicit weights assigned to each observation in an OLS regression are all equal. By using the sum of squared residuals as the criterion, the OLS procedure implies that an observation with a large residual has a considerable influence on the computed value of the estimated coefficients. Robust regression and regression diagnostics are designed to deal with problems associated with high leverage points in any data set used for regression analysis.

ROBUST REGRESSION[1/]

Robust regression is basically iteratively reweighted least-squares applied to equation (A.1). It starts with a set of coefficient values $\hat{\beta}^{(0)}$ which are used to compute the corresponding residuals. A set of weights are then computed from the residuals which are then used to re-estimate β to obtain a new vector of coefficients $\hat{\beta}^{(1)}$. This procedure is then repeated (iterated) until convergence is obtained. The second step in this process can be generalized for the mth iteration as follows:

(A.3) $\hat{\beta}^{(m+1)} = (X'W^{(m)}X)^{-1}X'W^{(m)}Y$

where $W^{(m)}$ is an n×n diagonal matrix of weights computed from $\hat{\beta}^{(m)}$.

Note that the residuals used in this procedure are previously scaled so that their magnitude will not depend on the original units. The robust criterion is to determine β to minimize a function of the scaled residuals which can be written:

1/ For a discussion on robust techniques, see Andrews (1973), and Huber (1972).

$$\text{(A.4)} \quad \sum_{i=1}^{n} f\left(\frac{y_i - x_i\beta}{s}\right)$$

where y_i and x_i denotes the ith observation (ith element of Y and the ith row of X), s is a measure of dispersion of the residuals, and f( ) is a criterion function. The residuals for the mth step are given by

$$\text{(A.5)} \quad u_i^{(m)} = y_i - x_i\hat{\beta}^{(m)}$$

The dispersion measure used in the statistical package TROLL is

$$\text{(A.6)} \quad s^{(m)} = \frac{1}{0.6745} \times \text{median of the n-p largest } |u_i^{(m)}|$$ which is a robust estimate of the standard deviation of the residuals.

A necessary condition for the minimum in equation (A.4) is that $\hat{\beta}$ satisfy the normal equations

$$\text{(A.7)} \quad \sum_{i=1}^{n} x_{ij}\, f'\left(\frac{y_i - x_i\hat{\beta}}{s}\right) = 0 \qquad j = 1, \ldots, p$$

where f'( ) is the first derivative of the criterion function, and x_{ij} is the ijth element of X. This can be acheived using iteratively reweighted least squares if the weights are defined as follows:

$$\text{(A.8)} \quad w_i^{(m)} = \frac{f'(u_i^{(m)}/s^{(m)})}{(u_i^{(m)}/s^{(m)})}$$

where $w_i^{(m)}$ are the diagonal elements of $W^{(m)}$.

The criterion function f( ) can be specified three ways in TROLL. In all cases, a user specified parameter, c, is required, and the implications of c are explained below. The two functions used in this analysis are the HUBER function,

$$\text{(A.9)} \quad f'(t) = \max(-c, \min(c,t))$$

and the BISQUARE function

$$\text{(A.10)} \quad f'(t) = t[1 - (t/c)^2]^2 \qquad |t| \leq c$$
$$= 0 \qquad |t| \geq c.$$

The OLS criterion assigns weights to a residual that increase dramatically as the residuals become large. Both Huber and Bisquare functions, on the other hand, assign bigger weights to large residuals up to a certain point, which is determined by the parameter c. Beyond that point, the weights increase linearly (Huber) or do not increase at all (Bisquare). This amounts to smaller weights being assigned relative to OLS observations with large residuals.

Note that if $c = \infty$, the robust criteria correspond to OLS. Hence by choosing a criterion function and varying c, one may determine the sensitivity and stability of the estimated coefficients. In practice, it is convenient to use an index (r) instead of c, given by $r = 1/(1 + c)$, implying that $r = 1$ corresponds to $c = 0$.

Regression Diagnostics[2/]

The main objective of regression diagnostics is to perform different analyses to discover inadequacies of the model formulation, deficiencies in the data and departures from the modeling assumptions. Deficiencies in the data can be assessed by a set of criteria. One criterion is based on the fact that an influential observation is one which, either individually or together with several other observations, has a demonstrably larger impact on the calculated values of the coefficients than is the case for most observations. An obvious way to determine such an impact is to delete each row, one at a time, and determine the resulting changes of the coefficients. Rows whose deletion produces relatively large changes in the calculated values are influential. This change is computed by the formula

$$\text{(A.11)} \quad b - b(i) = \frac{(X'X)^{-1}x_i'u_i}{1 - h_i} \qquad i = 1, 2, \ldots, n$$

where b = estimate of β in A.1, b(i) = estimate of β when ith row of X and Y have been deleted, x_i = ith row of X matrix, u_i = the ith computed residual in the full model, and $h_i = x_i(X'X)^{-1}x_i'$.

From (A.11), it is clear that h_i and u_i are fundamental components of these formulae. The h_i's are the diagonal elements of the least-squares projection matrix, also called the hat matrix, which can be written:

$$\text{(A.12)} \quad H = X(X'X)^{-1}X'$$

The diagonal elements of H (the h_i's) are diagnostic tools themselves as well as being fundamental parts of other criteria. It can be demonstrated that

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2/ The theoretical background of the different criteria presented in this section is based mainly on Belsley, Kuh and Welsch (1980).

$0 < h_i < 1$ and $\sum_{i=1}^{n} h_i = p$. The average size of a diagonal element is then p/n. Since (A.11) implies that values of $(1 - h_i)$ close to zero will be influential, some criterion is needed to decide when a value of h is large enough (away from the average) to deserve some attention, and 2p/n is a rough cutoff value if $p > 10$ and $n - p > 50$. It can also be demonstrated that when $h_i = 1$ the new matrix X_i, formed by deleting the ith row from X, is singular and the least-squares estimates cannot be computed.

The partial regression leverage plot is equivalent to the scatter diagram for a simple regression with a single explanatory variable. It is mostly useful in detecting influential subsets of data that might not be picked up through the use of single-row techniques.

The partial regression leverage plot can be conceptualized as follows. Let X(k) be the nx(p - 1) matrix formed from the data matrix X by removing its kth column, X_k. Let u_k and v_k, respectively, be the residuals that result from regressing Y and X_k on X(k). It is known that the kth regression coefficient of a multiple regression of Y on X can be determined from the simple regression of u_k on v_k. The partial regression leverage plot for b_k is then a scatter plot of u_k against v_k, and a simple linear regression line will have the same slope as the multiple regression estimate of β_k.

APPENDIX B: ESTIMATION RESULTS

Table B.1. OLS Estimation of the Constrained Model (t-ratios in parentheses).

CONST.	$(AR_{t-1} - AR_{t-4})$	$(\frac{AR_{t-1} - AR_{t-4}}{t})$	$(AR_{t-2} - AR_{t-4})$	$(\frac{AR_{t-2} - AR_{t-4}}{t})$	$(AR_{t-3} - AR_{t-4})$	$(\frac{AR_{t-3} - AR_{t-4}}{t})$
0.461 (1.15)	0.442 (5.712)	-0.540 (2.96)	0.213 (2.38)	-0.068 (0.28)	0.140 (1.93)	0.333 (1.52)

D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
-0.193 (0.34)	-1.681 (2.97)	0.494 (0.876)	0.872 (1.55)	-0.784 (1.38)	0.551 (0.98)	0.118 (0.21)	-0.324 (0.57)

R^2	$\bar{R}^2$	SSR	SER
0.896	0.865	52.50	1.046

R^2 Coefficient of multiple determination

$\bar{R}^2$ R^2 corrected for degrees of freedom

SSR Sum of squared residuals

SER Estimated standard error of the regression.

Table B.2. Robust Estimation of the Constrained Model with Huber (H) and Bisquare (B) Weight Functions (t-ratios in parentheses).

		CONST	$(AR_{t-1} - AR_{t-4})$	$(\frac{AR_{t-1} - AR_{t-4}}{t})$	$(AR_{t-2} - AR_{t-4})$	$(\frac{AR_{t-1} - AR_{t-4}}{t})$	$(AR_{t-3} - AR_{t-4})$	$(\frac{AR_{t-3} - AR_{t-4}}{t})$
r = 1.0	(H)	0.504 (53.46)	0.418 (226.28)	-0.418 (83.94)	0.237 (52.12)	-0.175 (15.33)	0.078 (32.62)	0.481 (57.43)
	(B)	0.504 (53.46)	0.418 (226.28)	-0.418 (83.94)	0.237 (52.12)	-0.175 (15.33)	0.078 (32.62)	0.481 (57.43)
r = 0.7	(H)	0.522 (3.67)	0.432 (18.51)	-0.432 (10.25)	0.206 (11.84)	-0.100 (1.99)	0.108 (4.96)	0.428 (5.69)
	(B)	0.641 (24.30)	0.403 (86.30)	-0.374 (28.12)	0.264 (26.60)	-0.232 (9.26)	0.067 (11.11)	0.465 (22.84)
r = 0.5	(H)	0.424 (2.33)	0.420 (14.08)	-0.474 (4.52)	0.198 (8.77)	-0.055 (0.88)	0.143 (4.92)	0.335 (3.35)
	(B)	0.586 (5.76)	0.423 (42.96)	-0.422 (19.82)	0.206 (23.82)	-0.104 (4.57)	0.067 (8.60)	0.510 (23.86)
r = 0.3	(H)	0.458 (1.90)	0.445 (11.34)	-0.546 (6.66)	0.205 (6.89)	-0.051 (0.60)	0.139 (3.35)	0.335 (2.59)
	(B)	0.586 (7.71)	0.406 (31.26)	-0.370 (13.87)	0.204 (21.19)	-0.114 (4.44)	0.105 (8.37)	0.521 (12.13)
r = 0.1	(H)	0.461 (1.88)	0.422 (11.11)	-0.540 (6.49)	0.213 (7.04)	-0.068 (0.79)	0.140 (3.71)	0.333 (2.53)
	(B)	0.453 (1.64)	0.441 (12.89)	-0.538 (7.21)	0.210 (6.71)	-0.063 (0.72)	0.138 (4.53)	0.335 (2.75)

		D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
r = 1.0	(H)	0.017 (0.72)	-0.676 (64.32)	0.273 (26.44)	0.543 (41.64)	-0.351 (23.40)	0.621 (42.87)	0.021 (0.84)	-0.565 (38.20)
	(B)	0.017 (0.72)	-0.676 (64.32)	0.273 (26.44)	0.543 (41.64)	-0.351 (23.40)	0.621 (42.87)	0.021 (0.84)	-0.565 (38.20)
r=0.7	(H)	-0.067 (0.19)	-0.943 (1.28)	0.214 (0.194)	0.431 (0.75)	-0.313 (0.89)	0.542 (1.39)	0.054 (0.15)	-0.453 (0.66)
	(B)	-0.142 (2.49)	-0.746 (23.86)	0.169 (5.50)	0.374 (9.80)	-0.535 (12.40)	0.495 (12.84)	-0.269 (4.24)	-0.740 (18.83)

Table B.2 (continued)

		D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
r=0.5	(H)	-0.081 (0.19)	-1.152 (1.20)	0.360 (0.26)	0.558 (0.75)	-0.252 (0.58)	0.650 (1.35)	0.061 (0.13)	-0.273 (0.31)
	(B)	0.178 (1.78)	-0.757 (6.87)	0.119 (1.04)	0.359 (2.88)	-0.498 (3.11)	0.432 (3.84)	-0.183 (1.37)	-0.651 (6.18)
r=0.3	(H)	-0.194 (0.34)	-1.675 (1.34)	0.499 (0.28)	0.785 (0.39)	-0.767 (1.31)	0.555 (0.86)	0.121 (0.194)	-0.319 (0.28)
	(B)	-0.147 (0.86)	-0.672 (1.59)	0.199 (0.31)	0.306 (0.95)	0.206 (1.11)	0.591 (3.00)	-0.108 (0.57)	-0.623 (1.62)
r=0.1	(H)	-0.193 (0.33)	-1.681 (1.32)	0.494 (0.28)	0.872 (0.87)	-0.784 (1.32)	0.551 (0.84)	0.118 (0.18)	-0.324 (0.28)
	(B)	-0.183 (0.22)	-1.645 (1.25)	0.486 (0.42)	0.795 (0.78)	-0.715 (1.10)	0.570 (0.63)	0.121 (0.15)	-0.312 (0.25)

		R^2	WR^2	SSR	WSSR	SER	WSER
r=1.0	(H)	0.873	0.999	63.88	0.0042	1.154	0.009
	(B)	0.873	0.999	63.88	0.0042	1.154	0.009
r=0.7	(H)	0.879	0.961	60.65	9.835	1.124	0.453
	(B)	0.869	0.999	65.95	0.073	1.172	0.039
r=0.5	(H)	0.886	0.935	57.145	23.46	1.091	0.699
	(B)	0.867	0.998	66.86	0.341	1.180	0.084
r=0.3	(H)	0.896	0.897	52.57	50.80	1.046	1.029
	(B)	0.856	0.987	72.51	2.87	1.229	0.245
r=0.1	(H)	0.896	0.896	52.50	52.50	1.046	1.046
	(B)	0.895	0.900	52.58	47.78	1.047	0.998

R^2, SSR and SER are defined in Table B.1.

WR^2, WSSR and WSER are the same measures in terms of the weighted residuals.

Table B.3. OLS Estimation of the Constrained Model After Deleting Two Observations (t-ratios in parentheses).

CONST	$(AR_{t-1}-AR_{t-4})$	$(\frac{AR_{t-1}-AR_{t-4}}{t})$	$(AR_{t-2}-AR_{t-4})$	$(\frac{AR_{t-2}-AR_{t-4}}{t})$	$(AR_{t-3}-AR_{t-4})$	$(\frac{AR_{t-3}-AR_{t-4}}{t})$
0.415 (1.34)	0.500 (7.14)	-0.730 (3.92)	0.250 (3.02)	-0.242 (1.02)	0.141 (2.47)	0.284 (1.53)

D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
-0.384 (0.85)	-1.54 (3.3)	0.447 (1.02)	0.515 (1.14)	-0.026 (0.05)	0.753 (1.65)	0.181 (0.411)	-0.319 (0.73)

R^2	SER	SSR
0.94	0.80	27.25

R^2, SER and SSR are defined in Table B.1.

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